STATUS REPORT

Time Frequency representation deformation

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Contents

Abstract

This report has been made for the project TFR Deform which its purpose was to create a processing chain in order to analyse the electromagnetic field of object which its form change trough time. According to the recommendation of the supervisors, this project has considered only cylinder which its radius change through time. The deliverable wanted was:

- Create a script which can provide the field scatter by the cylinder, the incident wave and all the geometric parameters of the simulation.
- Implement or use different time frequency representation in order to see the modification of the electromagnetic signature of the object through time.
- Implement methods in order to interpolate the law frequency of the electromagnetic signature and, if there was enough time, find correlation between those modifications and the deformation of those objects.

In the end, the code for the generation of signal is working very well and, even if we implemented our own functions for the time frequency representation at the beginning of the project, we found a library which provide the same functions with a lot of other tools which provide a lot of help during the project. The modelization of the movement is also working and provide excellent result on the time frequency representation. Nevertheless, there are still some work to do in order to see the impact of the variation of the form of the object. The processing chain is working but it remains a little work in order to finalize the study of the signal.

Introduction

The purpose of this project is to analyse the electromagnetic signature of an object when its form changes during time. The electromagnetic signature analysis is a common subject for people who work in the field of radar detection. However, the study of object signature which form changes through time is still full of many questions remain in. The study of these objects imply a very strict analyse of the electronic signature thanks to the time frequency analysis tools.

During this project, we have to look at the different tools which can help us to analyse the electronic signature of an object and to help us to understand the effects of change of form of an object. The final deliverable expected is a set of programs which allow us to implement an entire loop of treatment. This loop has to help us to understand how an object deformed according to the modification of its electronic signature. **Coding had to be done in python**.

The whole project code and result can be found here :
https://github.com/chuzelph-ENSTA-Bretagne/TFR _ Deform.git

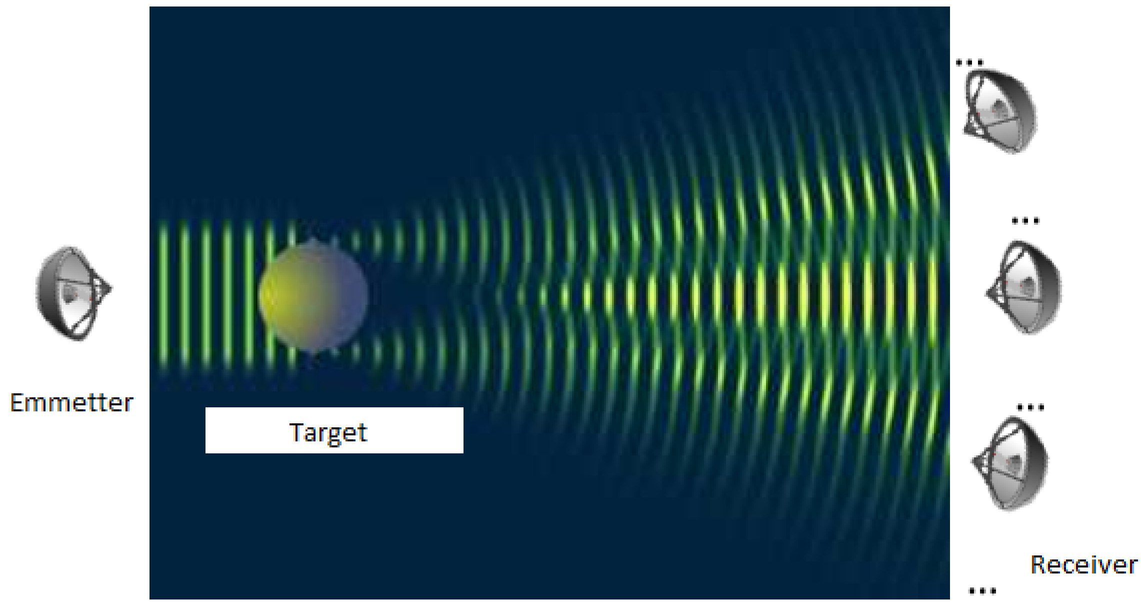

Figure 1: General situation of our subject.

The first part will explain which equations have to be understood in order to get the analytic expression of the electromagnetic field coming from the object. The second part will expose the different tools which have been created for the time frequency analysis that can be used for this project. In this part, some simple signals will be used in order to represent the results that can be obtained with these tools. The last part will present the tools which have been developed in order to extract information from the time/frequency representation in order to find a correlation between the deformation of the object and the variation of the electromagnetic field.

Part I

Generation of the signal

Chapter 1

Explication of the problem

During this project, we will only study cylinder objects. A lot of information given here come from this document [4].

1.1 Explication of the context

Lets consider here a cylinder object which is excited by a plane wave. This problem is independent of the z-dimension so we can consider a two-dimension problem. We will need to switch between a Cartesian landmark and a polar landmark.

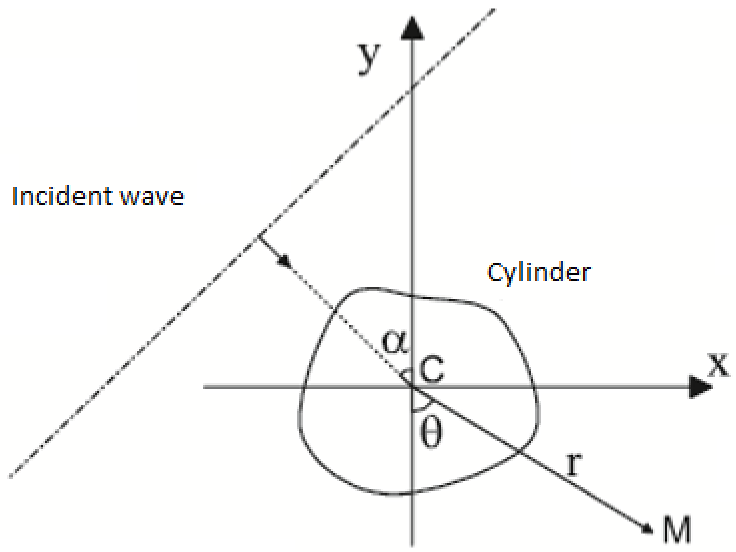

Figure 1.1: General situation of the problem.

Here, α is the incident angle of the wave and r and θ are the polar position of the point M. Furthermore, the center of the cylinder is at the same place as the origin.

1.2 Equations

The wave equation can be written as follows:

$$(\nabla + k^2).p = 0 \tag{1.1}$$

In cylinder landmark, this result becomes :

$$\frac{1}{r}\frac{\partial}{\partial r}(\frac{r\partial p}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 p}{\partial \theta^2} + \frac{\partial^2 p}{\partial z^2} + k^2 p = 0 \tag{1.2}$$

The problem is independent of the z-dimension so we obtain :

$$\frac{1}{r}\frac{\partial}{\partial r}(\frac{r\partial p}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 p}{\partial \theta^2} + k^2 p = 0 \tag{1.3}$$

This problem allows us to use the separation of variables (also known as the Fourier method) and allows to rewrite an equation so that each of two variables occurs on a different side of the equation. The set of solution p can be written as:

$$p(r,\theta) = R(r).\Theta(\theta) \tag{1.4}$$

Thanks to the separation of variable, we know that we have a set of solutions which is countable so we can write:

$$p_n(r,\theta) = R_n(r).\Theta_n(\theta) \tag{1.5}$$

The function Θ is 2π periodic so it can be written as:

$$\Theta_n(\theta) = a_n.e^{in\theta} + b_n.e^{-in\theta} \tag{1.6}$$

Thanks to the Bessel and Hankel functions, we can express the solution:

$$R_n = c_n.J_n(kr) + d_n.Y_n(kr) \tag{1.7}$$

Or:

$$R_n = c_n.H_n^{(1)}(kr) + d_n.H_n^{(2)}(kr) \tag{1.8}$$

The set of solutions can be expressed by the functions:

$$J_n(kr)e^{in\theta} \qquad Y_n(kr)e^{-in\theta} \qquad n \in \mathbb{Z} \tag{1.9}$$

$$H_n^{(1)}(kr)e^{in\theta} \qquad H_n^{(2)}(kr)e^{-in\theta} \qquad n \in \mathbb{Z} \tag{1.10}$$

We will have two waves:

- An incident wave p_{inc}.
- A diffused wave p_{dif} resulted from the reaction of the incident wave and the object.

Chapter 2

Expression of the wave resulting from this situation.

Both the incident wave and the diffused wave have to be defined at (0,0) and mustn't diverge when $r \to +\infty$.

The consequences are that the incident wave can be written as :

$$p_{inc} = \sum_{n=-\infty}^{\infty} a_n J_n(kr) e^{in\theta} \tag{2.1}$$

If we choose to use the Hankel functions in order to express the diffused wave, we get the result:

$$p_{dif} = \sum_{n=-\infty}^{\infty} b_n H_n^{(1)}(kr) e^{in\theta} \tag{2.2}$$

The limit condition which applies here is the SOMMERFELD condition. That means that:

$$p_{inc}(r,\theta) = p_{dif}(r,\theta) \qquad \forall (r,\theta) \in \{Border\, of\, the\, object\} \tag{2.3}$$

Thanks to the fact every term are independent, we have:

$$b_n.H_n^{(1)}(kr) = a_n J_n(kr) \qquad \forall n \in \mathbb{N} \qquad \forall (r,\theta) \in \{Border\, of\, the\, object\} \tag{2.4}$$

So we have a relation between the coefficient a_n and b_n. We still have to find an expression for a_n. The wave p_{inc} is a plane wave which means that it can be write as:

$$p_{inc} = e^{ik_{inc}x} \qquad or \qquad p_{inc} = e^{ik_{inc}r.\cos(\theta-\alpha)} \tag{2.5}$$

If we develop the term $\cos(\theta - \alpha)$ as a Fourier series, we get the equation:

$$p_{inc} = \sum_{n=-\infty}^{\infty} i^n e^{-in\alpha} J_n(kr) e^{in\theta} \tag{2.6}$$

Which means $a_n = i^n e^{-in\alpha}$. With this, we can express the wave result from the diffraction of the incident wave on the object.

Chapter 3

Examples of situations encountered.

During this project, we will have only two situations.

We will in a first step consider a unique cylinder which radius changes through time and we will study the magnetic field at the point M:

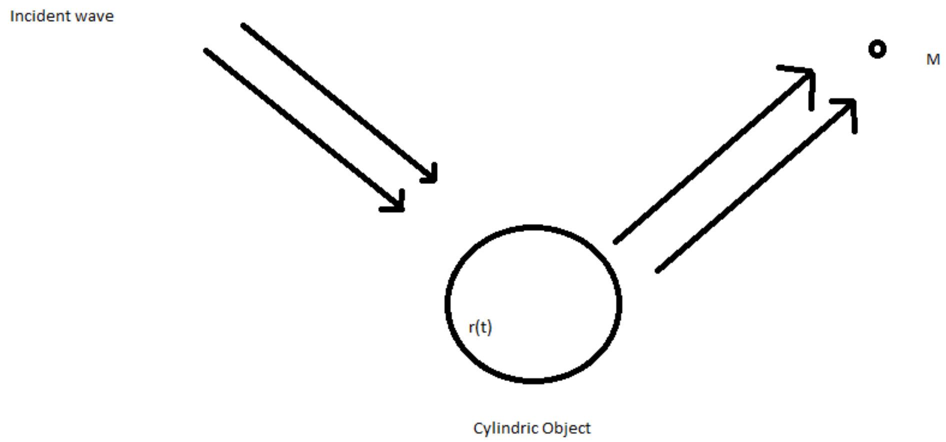

Figure 3.1: Study with a unique object.

In a second step, we will consider a lot of cylinders which can move through time and study the magnetic field at the point M:

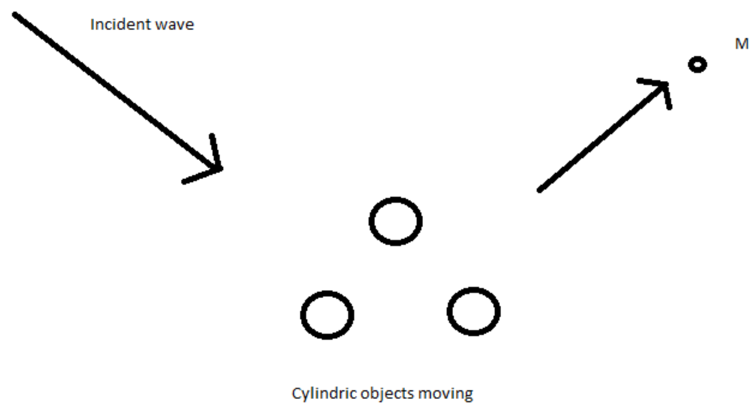

Figure 3.2: Study with a set of objects.

For this situation, we have to estimate the phase shift between those objects and an origin. Thanks to some geometric theorems, this can be easily obtained.

Conclusion

The code needed in order to generate these signals is already working and all we need is to simulate the movement and the deformation of one or two cylinders and see the effect of the movement and the form deformation on the electromagnetic signature.

You will have to use the script Objectdiff.py in order to generate a signal.

Part II

Time/frequency analysis

Chapter 4

Spectrogram

The spectrogram is the most common tools used in order to do a time-frequency representation. This representation uses the short-time-Fourier transformation in order to apply a fast-Fourier transformation on a slippery window of the signal. In the end, we get an image which represent the frequency which composes the signal at a given moment.

Lets consider the signal chirp below which corresponds to a signal which frequency increases from 0 to 300Hz.

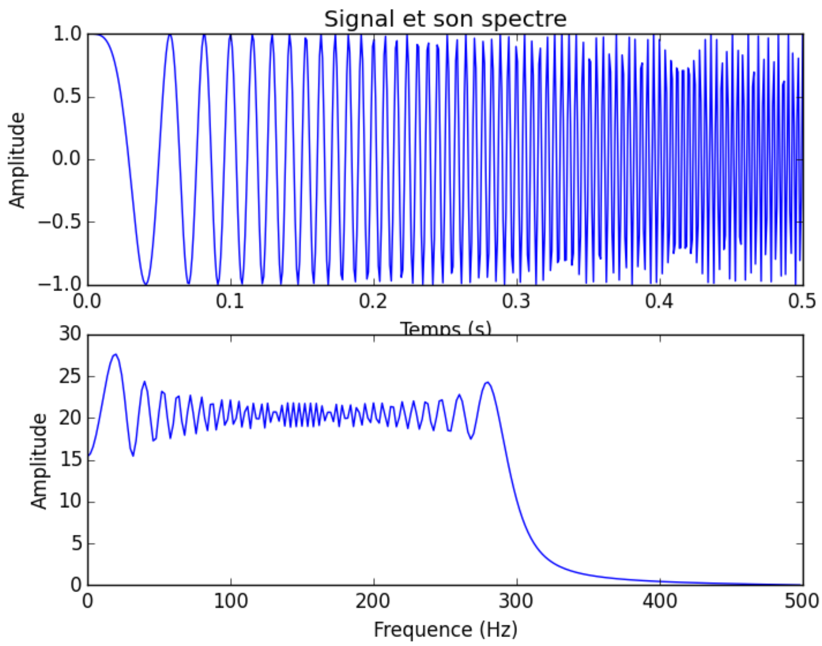

Figure 4.1: Time and frequency representation of a chirp signal.

The figure below shows the results we can get:

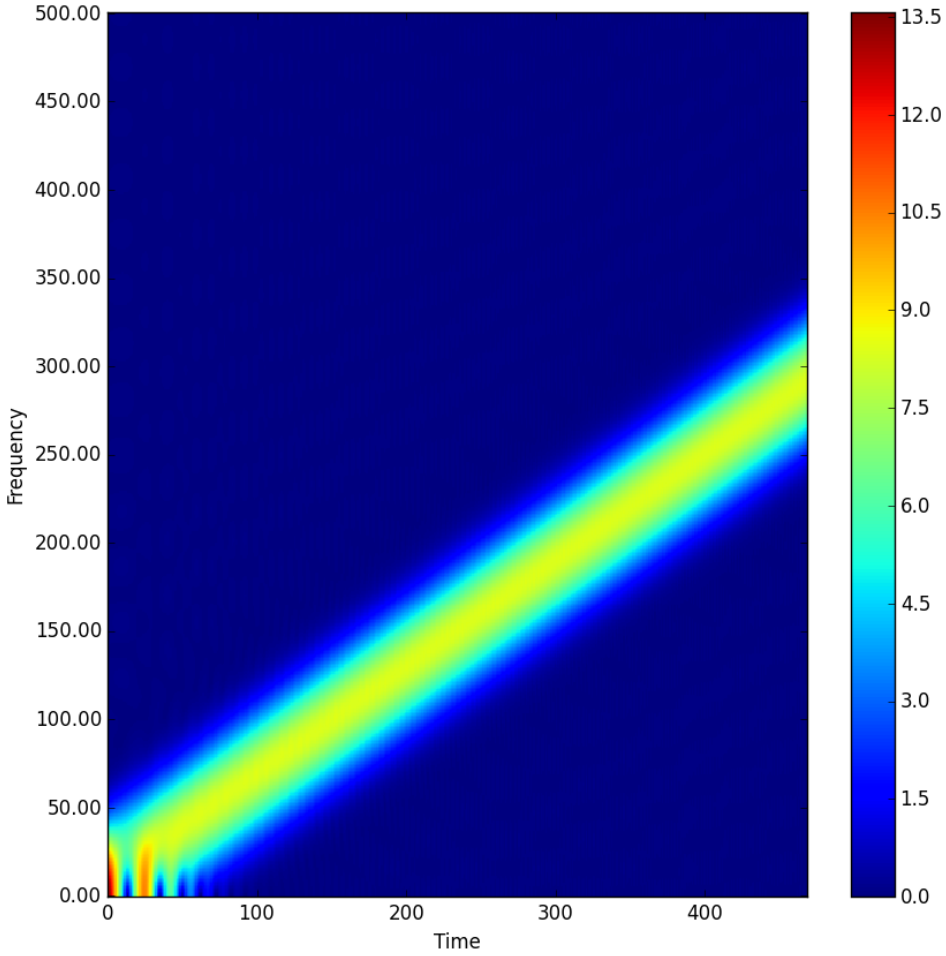

Figure 4.2: STFT/spectrogram of a chirp signal.

This figure clearly shows that the frequency of the signal increases through time but the result is a little blured and isn't very accurate.

Chapter 5

Wigner-Ville

The Wigner-Ville distribution allows us to get a far more accurate time frequency representation. Nevertheless, this distribution creates some interferences if the signal is a combination of different frequency laws.

The Wigner-Ville Distribution (WVD) of a signal y(t), denoted by $W_z(t, f)$, is defined as :

$$W_z(t, f) = \int_{n=-\infty}^{\infty} z(t + \tau/2)z^*(t - \tau/2)e^{-j2\pi f\tau} d\tau \tag{5.1}$$

The following figure shows the result of the Wigner-Ville representation on the previous signal:

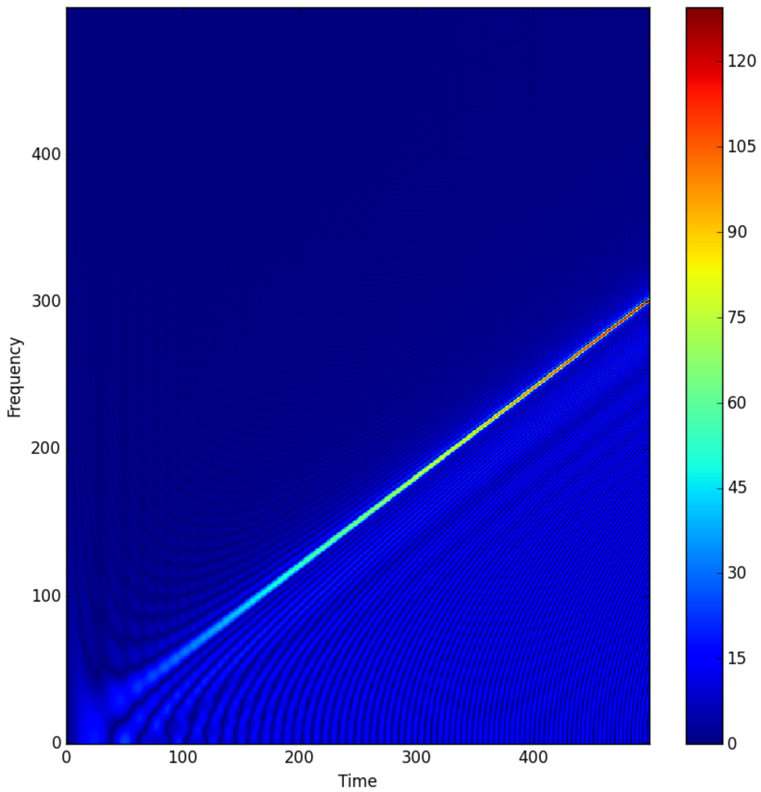

Figure 5.1: Wigner-Ville of a chirp signal.

Nevertheless, if we consider the following signal which contains two sinusoids, with different frequency and with one which lasts for only a portion of the signal, we get the result below:

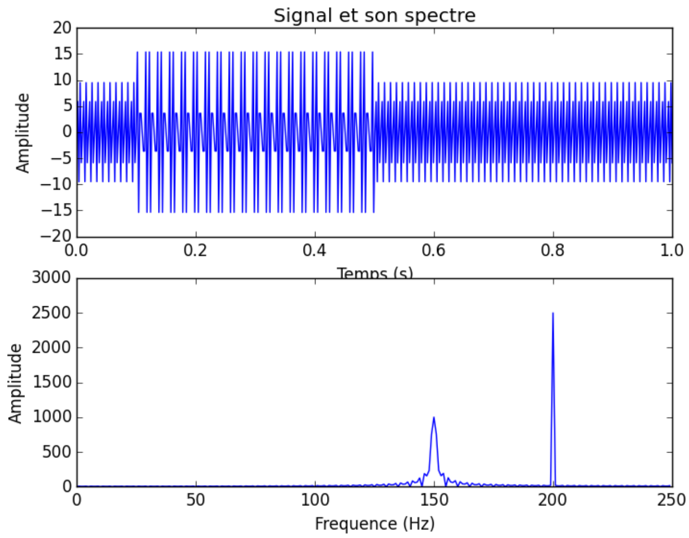

Figure 5.2: Time and frequency representation of the signal with two frequency laws.

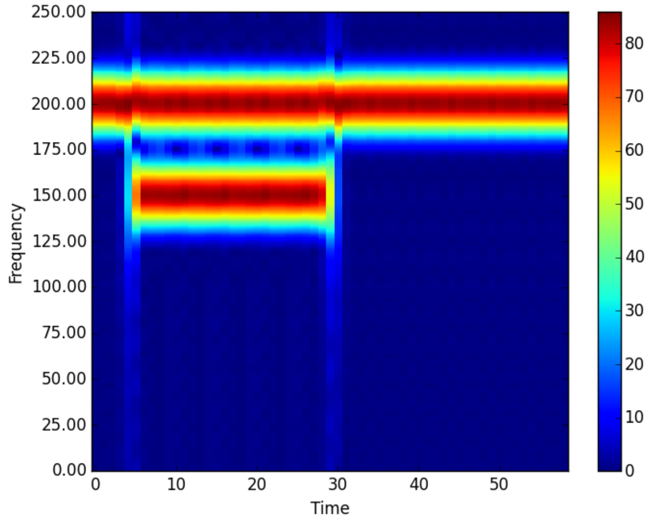

Figure 5.3: Spectrogram of the signal.

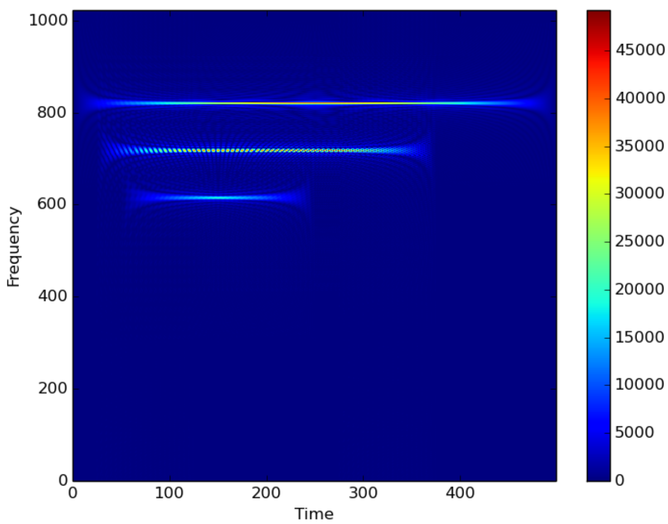

Figure 5.4: Wigner-Ville of the signal.

As you can see, there is an interference which implies to analyse this figure if we want to extract relevant information.

Chapter 6

Pseudo Smooth Wigner-Ville

The Pseudo-Wigner-Ville Distribution is defined as:

$$W_z(t,f) = \int_{n=-\infty}^{\infty} h(\tau) z(t+\tau/2) z^*(t-\tau/2) e^{-j2\pi f \tau} d\tau \tag{6.1}$$

where h is a regular window. This windowing is equivalent to a frequency smoothing of the WVD so It leads to the attenuation of the interference terms but it will damage the signal representation.

The result for the previous signal is below:

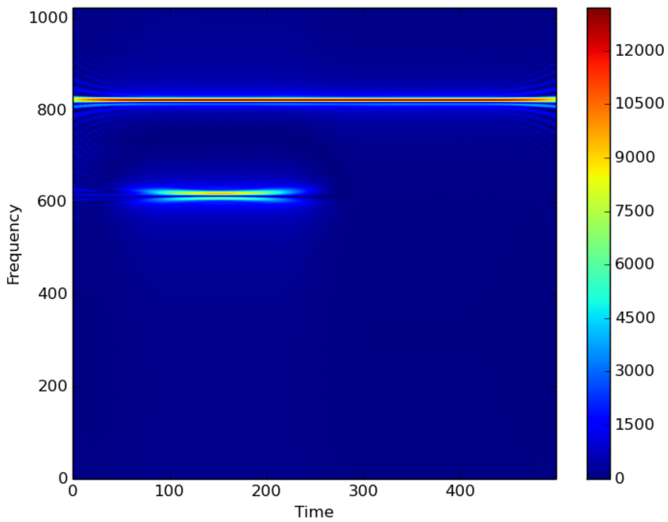

Figure 6.1: Pseudo Smooth Wigner-Ville of the signal.

Conclusion

All these representations are of interest but have disadvantages. Nevertheless, if we want to get as much information as we can, we will need to switch from a representation to another according to the kind of signal we have. At the beginning of the project, an implementation of the Flandrin library coming from Matlab was implemented but we found afterwards a python library which was already doing this part. The following link shows the function of this library and the second one explain how it can be install on your computer. The previous pictures came from our own functions but the next pictures will use the function of this library.

https://github.com/scikit-signal/pytftb

http://pytftb.readthedocs.org/en/master/

All the time frequency representation of this project now use those function which provide a much more reliable result than the function we tried to implemented.

Part III

Time frequency analysis

Chapter 7

Seam carving algorithm.

7.1 Presentation of the algorithm.

At the beginning, the seam carving algorithm is used in order to reduce the size of an image and keep some proportion of the image. The main idea of this algorithm is to find the path in an image which have the least importance and automatically remove them in order to resize the image [1].

The following figure show what kind of result can be expected and the different step of the algorithm:

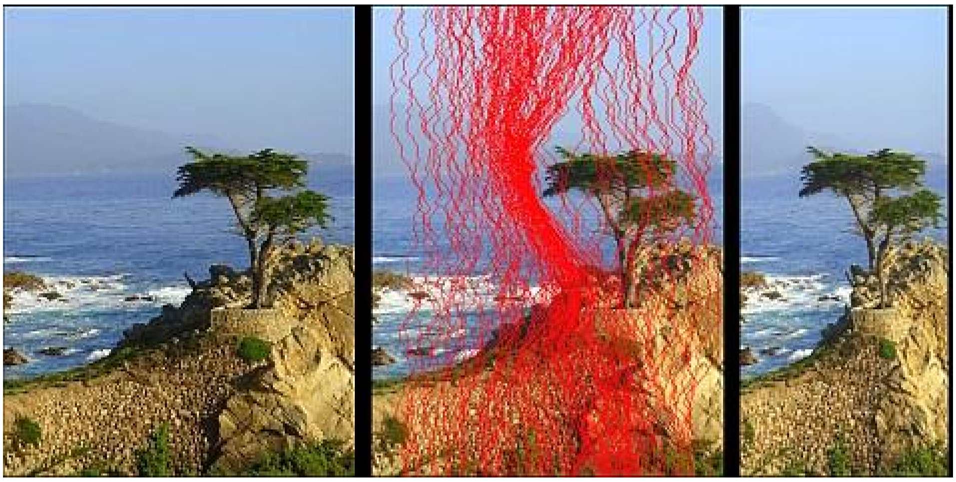

Figure 7.1: Example of the seam carving algorithm, coming from the website *http://courses.cs.washington.edu/courses/cse557/08wi/projects/final/artifacts.html*.

Nevertheless, it can be used in this project in order to find a approximation of the frequency law which compose a signal. In the time frequency representation, the different laws which compose the signal represent a path in the image and we can use this algorithm in order to find an empiric expression $f(t)$. The main purpose of those articles [2] [5] [3] is to present how this algorithm can be implement in order to find the speed of an object thanks to the Doppler effect but it can be use here in order to find the frequency laws which compose the signal.

The seam carving algorithm can be express by the following line:

Algorithm 1 SEAM CARVING algorithm

Inputs

Matrix TFR
Vector T time
Vector F frequency

Output

Vector Value $f(t)$ of the first frequency law found

Initialization

$init \leftarrow 0$
$i \leftarrow 0$
$k \leftarrow zeros(len(t), 1)$

Algorithm

while i<len of T **do**
    if i==0 or init == 0 **then**
        $peakind \leftarrow$ peak of the vector TFR$[:, i]$
        if $len(peakind)! = 0$ **then**
            $k[i] \leftarrow peakind[0]$
            $init \leftarrow 1$
        else
            if max(TFR[:,i])!=0) **then**
                $k[i] \leftarrow$ indice of the maximum of the vector TFR[:,i]
                $init \leftarrow 1$
            end if
        end if
    else
        $Indice \leftarrow$ the value of j, when the values TFR[k[i-1]+j,i] is maximize, for j in $[-1, 0, 1]$
        $k[i] = k[i-1] + Indice$
    end if
    $i++$
end while
return k

7.2 Application on a Doppler signal.

We can apply easily this algorithm on a Doppler signal. Let suppose that an object is moving toward an observer and it is sending a wave (sound for example). The frequency of the signal is changed because of the movement of this object. The following figure shows how the signal is affected by the movement:

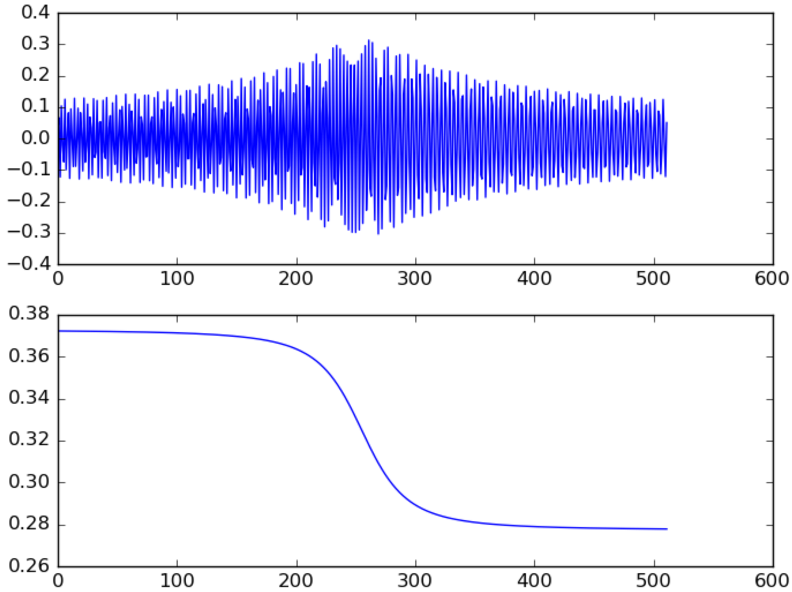

Figure 7.2: Time representation and instantaneous frequency of the signal send.

The next figure show the time-frequency representation of the signal:

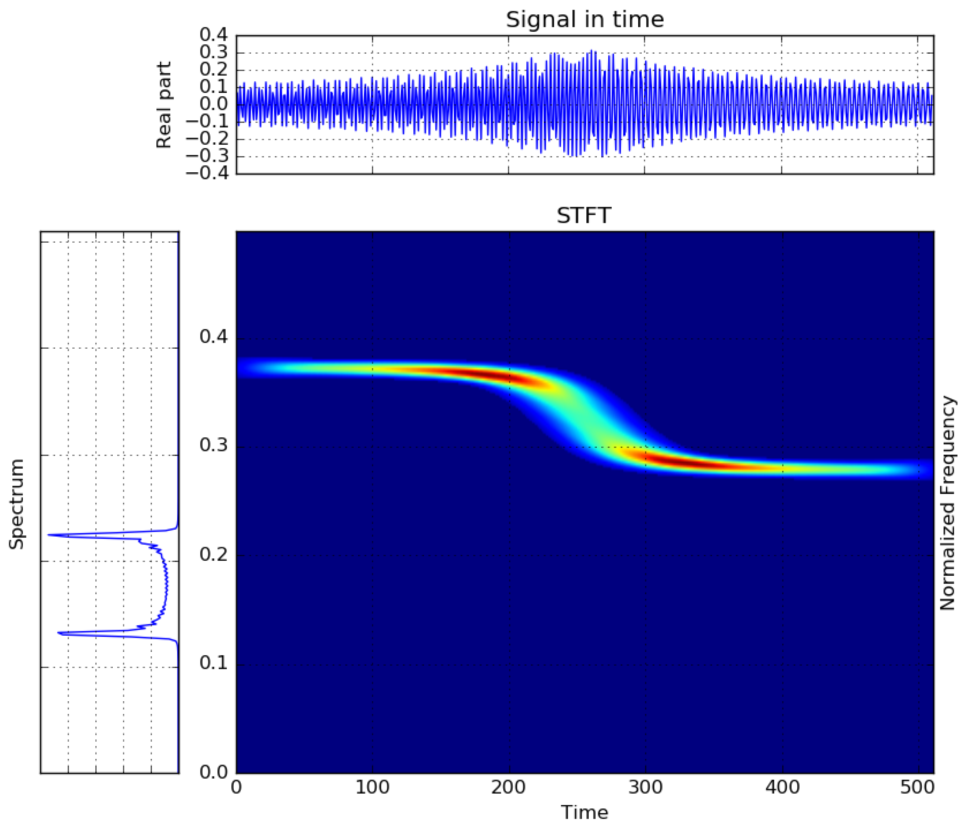

Figure 7.3: Time/Frequency representation of the signal send.

Thanks to some tools coming from the image processing, we can improve this result in order to keep only the relevant points which contain information:

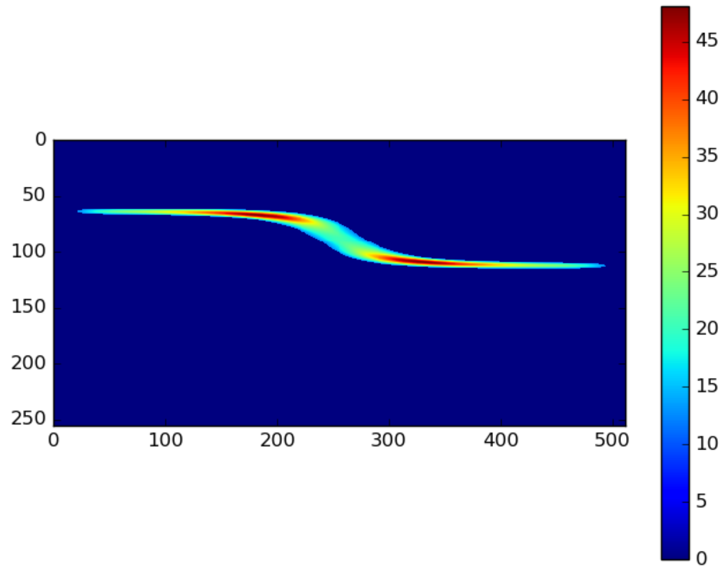

Figure 7.4: Time/Frequency representation of the signal send after treatment.

Finally, when we use the seam carving algorithm, we achieve to get this result:

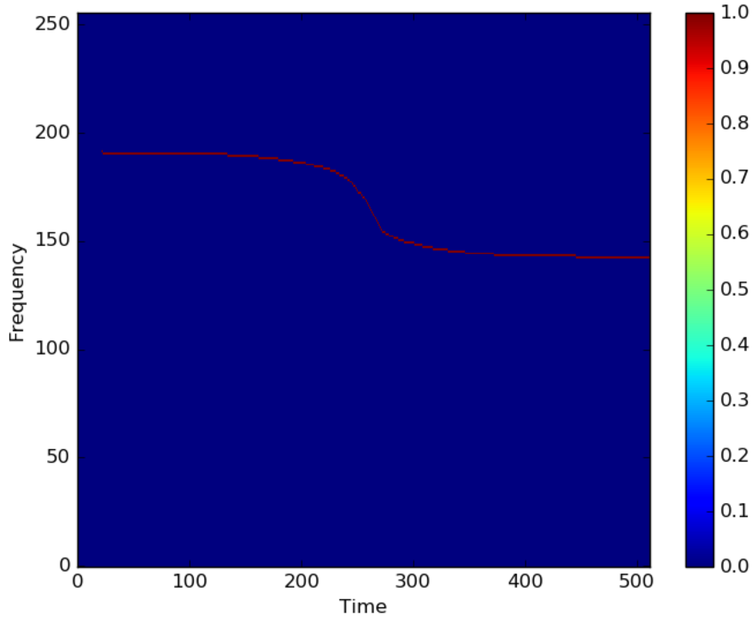

Figure 7.5: Time/Frequency representation of the signal send after treatment.

The last part is to use interpolation algorithm such as fit spline interpolation in order to find an approximation of the law found. The following figure shows how the fit spline interpolation can provide some results:

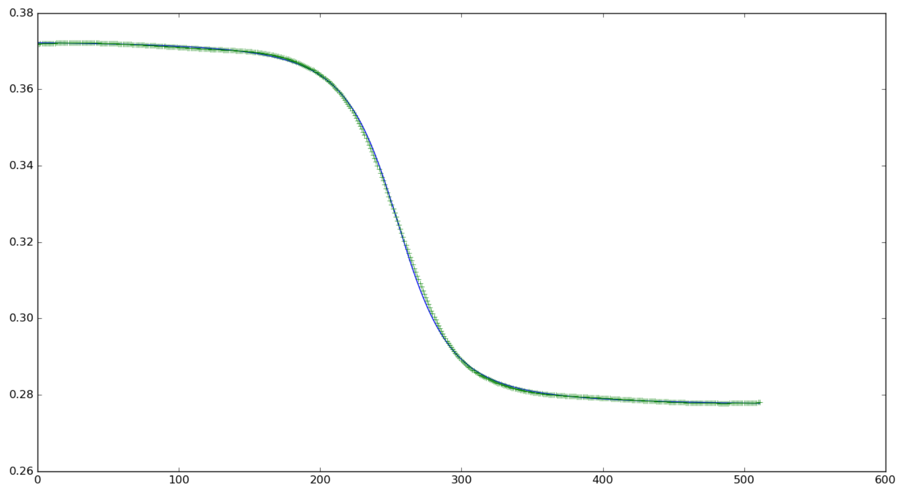

Figure 7.6: Spline interpolation result. In green the value of the interpolation and in blue the value of the law found

Nevertheless, the result given by this method can be disturbed by the beginning of the signal. For example, the following figure shows how the first values of the law found can impact the interpolation.

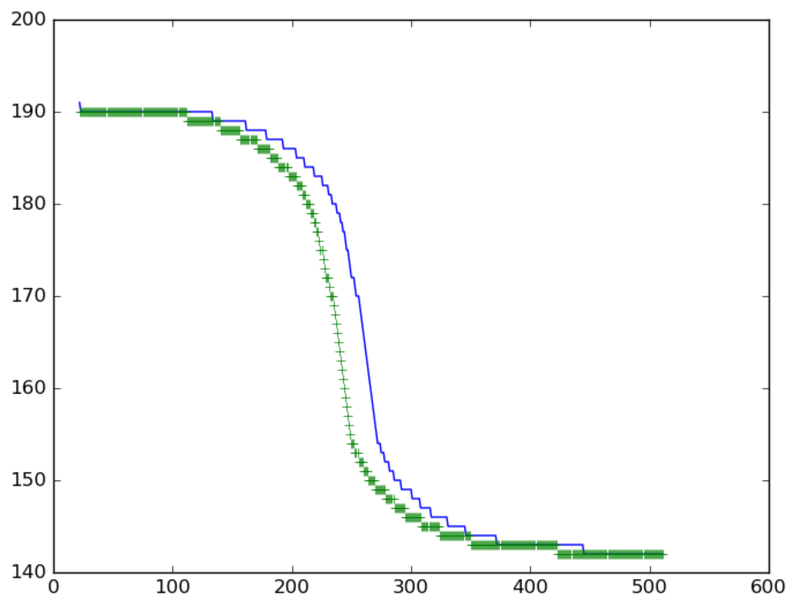

Figure 7.7: Spline interpolation with bad result. In green the value of the interpolation and in blue the value of the law found

As the different part have shown, we have now all the tools needed in order to create and then analyse any kind of signal, including the signal details in the introduction.

Chapter 8

Application to our project.

In this part, we will apply all the result of the previous part in order to analyse understand the signal generate by the cylinder as it was explain in the introduction.

8.1 A first application.

Let consider a cylinder which **radius don't change through time** but it's moving according to the following figure.

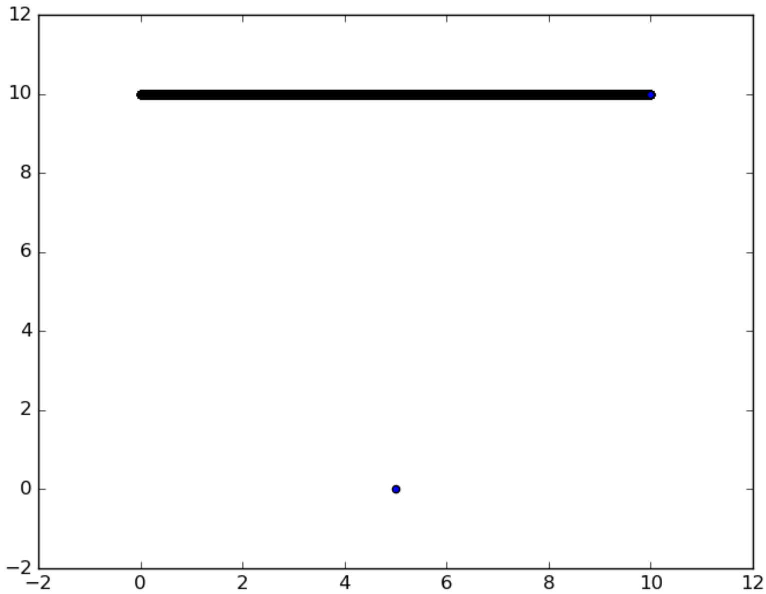

Figure 8.1: First situation studied. The object is moving linearly.

The following figure shows the spectrogram obtain after we analyse the signal without its means.

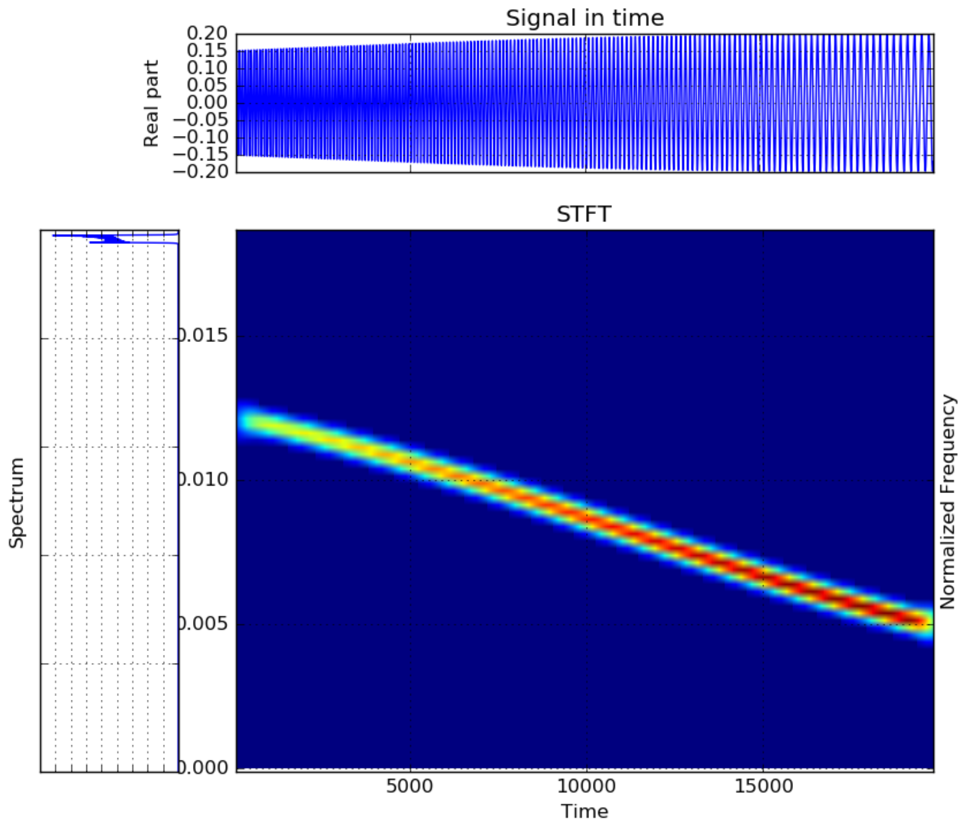

Figure 8.2: Spectrogram of an object moving linearly.

And this figure show the result of the seam carving algorithm:

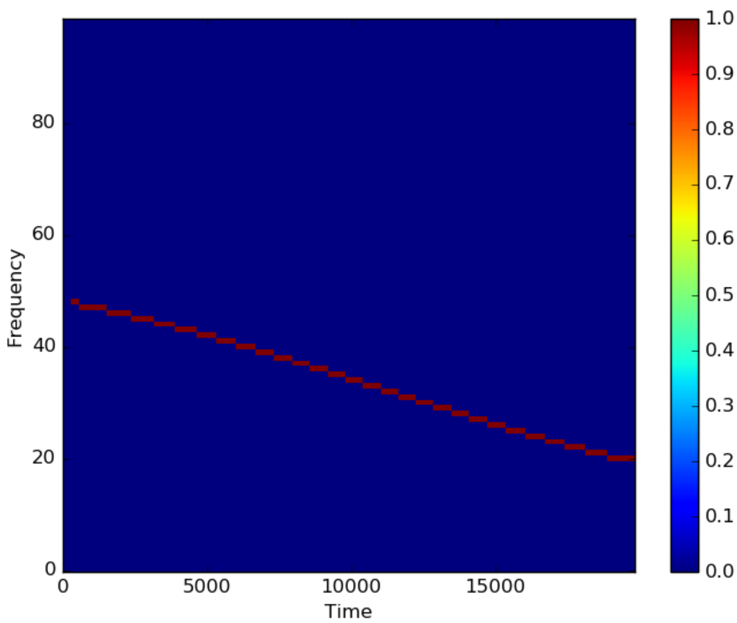

Figure 8.3: Seam carving application.

As you can see, we are in the same configuration as the Doppler signal we have studied in the previous part. There is a frequency which value received by the observer is disturbed by the movement. Futhermore, the seam carving apply perfectly to this kind of figure as the figure shows it.

8.2 Application in view to a verification in the anechoic chamber.

The school ENSTA Bretagne has a anechoic chamber in which we can simulate the field generate and scatter by object but we can only give a circular movement to those object. Thanks to the code we have created, we know that the wave scatter and it's time frequency representation will be:

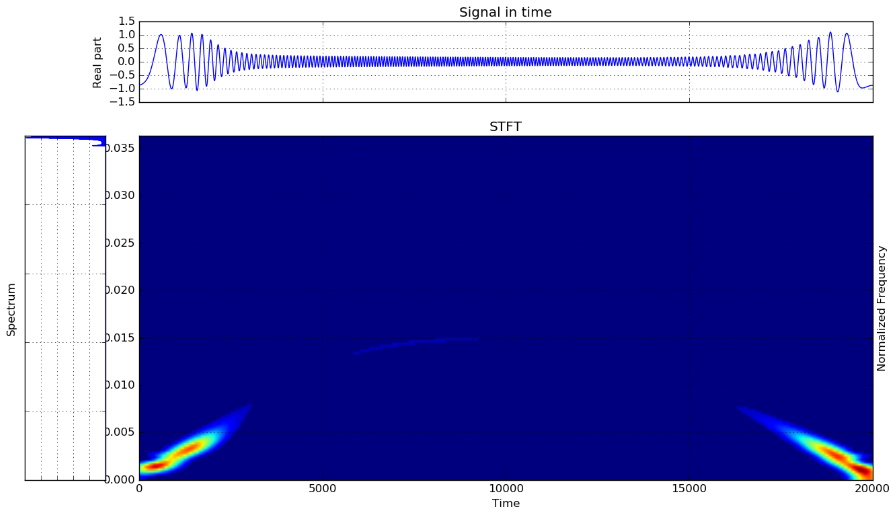

Figure 8.4: Time frequency representation of a cylinder with a circular movement.

The partial sinusoid we can see in this time frequency representation is conformed to the theory but the signal contains too much energy when it's close to the observer which explains why we don't see all the curve.

Thanks to this code, we can generate easily any signal for any object and we can check if the theoretical results are corroborated by the experimental results.

8.3 Application to object which form change through time.

According to the document coming from the supervisors, if we consider a single cylinder which form change through time, we should get time-frequency representation like this one:

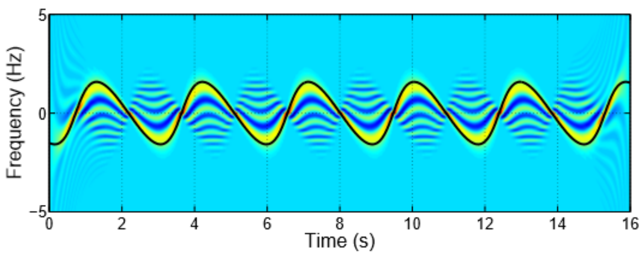

Figure 8.5: Time frequency representation expect. The frequency received have a periodic behaviour.

Nevertheless, even with a lot of test, we only get result like this one:

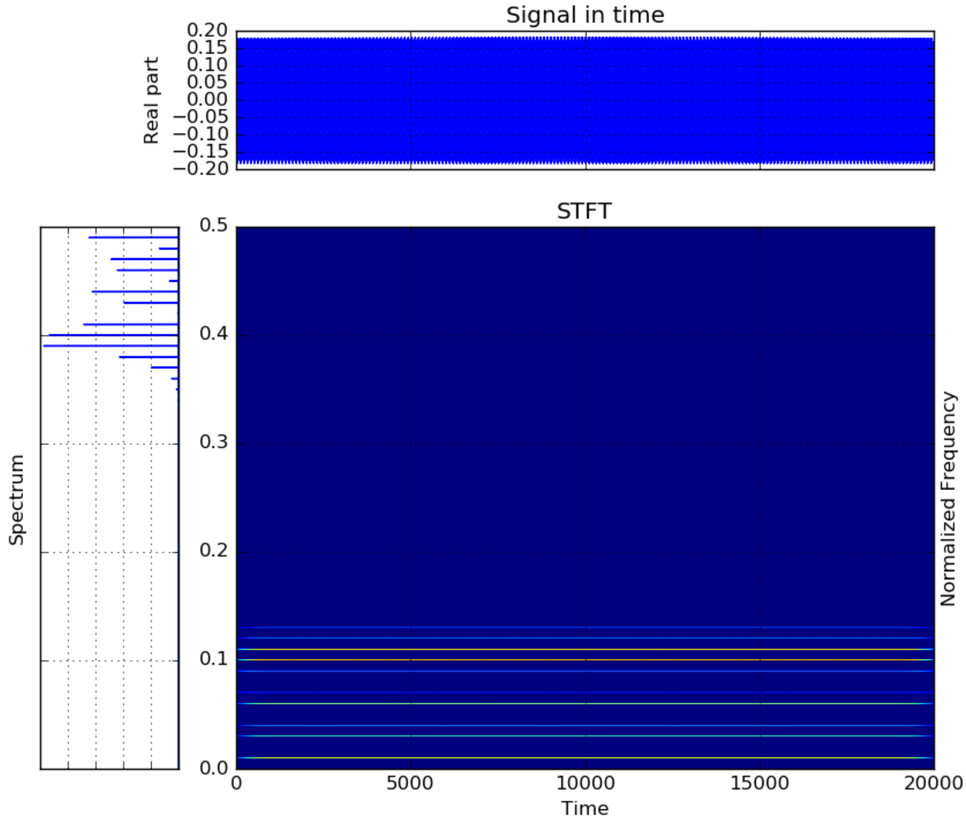

Figure 8.6: Time frequency representation of a cylinder with a sinusoidal variation of its radius.

As you can see, we have the fundamental and the harmonic of the original signal which are present in this time frequency representation. But the form changing seems to don't have impact on those law frequency during time.

Conclusion

All the different part which have been realized during this project very good result and can be easily reused by anybody. Most of them can be easily use for the signal we wanted to analyse. Nevertheless, the fact that the impact of the variation of the radius of the cylinder have not been seen is regretful.

Part IV

Appendix

Gaant diagram

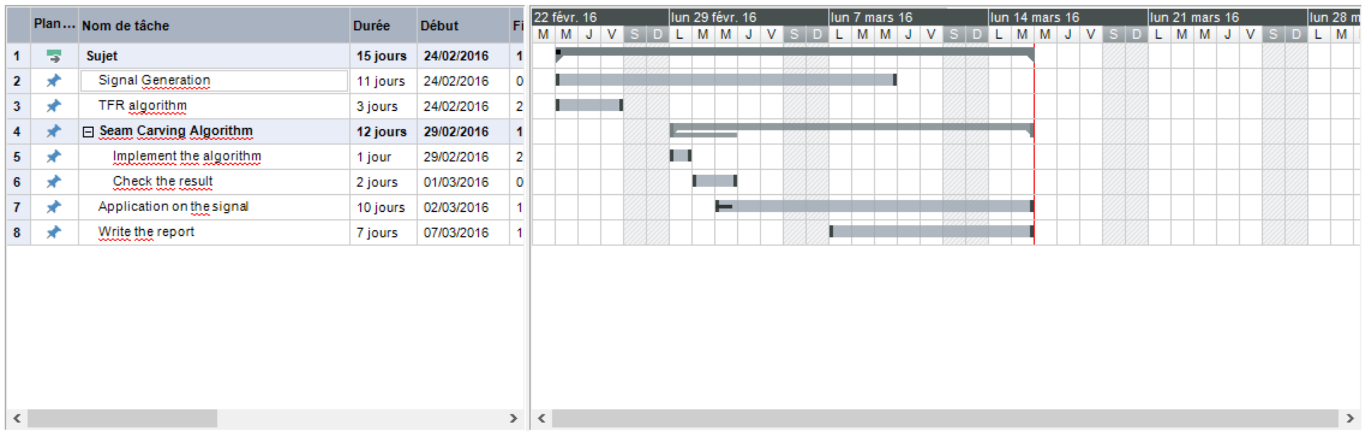

Figure 7: Gaant diagram.

There are a lot of differences between what I expected and what I have done. I had to do a lot of verification in order to check if the code which generate the signal was correct. The seam carving was implemented pretty quickly but the time-frequency representation didn't give the result expected and it took me a lot of time.

The program which has been used in order to generate this diagram is **MindView 6.0**

Bibliography

[1] Shai Avidan and Ariel Shamir. Seam carving for content-aware image resizing. In *ACM Transactions on graphics (TOG)*, volume 26, page 10. ACM, 2007.

[2] Shubhranshu Barnwal, Rajesh Barnwal, Rajeshwari Hegde, Rajdeep Singh, and Bhiksha Raj. Doppler based speed estimation of vehicles using passive sensor. In *Multimedia and Expo Workshops (ICMEW), 2013 IEEE International Conference on*, pages 1–4. IEEE, 2013.

[3] Volkan Cevher, Rama Chellappa, and James H McClellan. Vehicle speed estimation using acoustic wave patterns. *Signal Processing, IEEE Transactions on*, 57(1):30–47, 2009.

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