Simulation of the Fraunhofer Diffraction in the Far Field

Physics 77 Final Capstone Project

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1 Abstract

Young's famous double slit experiment unveils the wave nature of light. The objective of this project is to create an interactive simulation of Young's experiment in the regime of Fraunhofer, also known as Far-Field, Diffraction. The simulation also extends beyond the double-slit case by introducing single-slit diffractions and multiple-slit diffractions. The user can choose which form of diffraction they want and can alter different parameters such as the wavenumber, the distance between the slits, etc.; the simulation would then return the resultant image of how the detector screen would look.

2 Introduction

The motivation for this project comes from the final unit of the course Physics 5B: Introductory Electromagnetism, Waves, and Optics. In the last section of the class, Maxwell's equations were solved and the concept of light as an electromagnetic wave was introduced. This knowledge was then applied to solve the double-slit experiment. However, most of the understanding comes from mathematical equations and there are not many visual demonstrations included. Therefore, a simulation could help to build the physical intuition regarding the parameters' impact on the diffraction pattern

3 Theory

3.1 Fraunhofer Diffraction

The diffraction patterns and the interference effects of Young's Double Slit experiment (YDS) unveil light's wave nature. Fraunhofer Diffraction is implementing YDS under two conditions. First of which is the requirement that the wavefronts arriving at the double slit are plane waves and the second requires the distance between the double slit and the detector to exceed a minimal length.

3.1.1 Plane Wave and Intensity

The concept of plane waves comes from solving the decoupled Maxwell's Equations. Mathematically, it is expressed in the form of:

$$\tilde{E}(x,t) = \tilde{E}_0 e^{i(kx-\omega t)} \tag{1}$$

Where $\tilde{E}(x,t)$ is the Electric-Field at a given point, at a given time. The term $\tilde{E}_0$ is the initial amplitude of the electric field and the $(kx - \omega t)$ term represents its spacial and temporal phase. The solution for the magnetic field is not shown because it is irrelevant to the scope of this project. The objective is to find the diffraction pattern, which is essentially the distribution of intensity on the detector. The intensity (irradiance) of an EM wave is calculated by taking the time average of the Poynting vector, which could be expressed as:

$$I = \langle \vec{S}(x,t) \rangle_t = \frac{1}{2}\epsilon_0 c E^2 \tag{2}$$

$$E = \langle Re[\tilde{E}(x,t)] \rangle_t \tag{3}$$

where ϵ_0 is the permittivity of free space, c is the speed of light, and E is the time average of the real part of the function in equation 1.

3.1.2 Distance Requirement

The second condition for Fraunhofer Diffraction places a minimal distance between the detector and the slits. The requirement implies that the distance between the slits and the detector, L, would be much greater than the ratio between the size of the slit b squared divided by the wavelength, λ. Hence, the minimal distance would be:

$$L_{min} = \frac{b^2}{2\lambda} \tag{4}$$

Generalizing the above requirement, as long as:

$$L >> \frac{Area\, of\; Aperture}{\lambda} \tag{5}$$

Fraunhofer, or Far-Field, Diffraction is applicable.

The final job is to find the diffraction pattern (intensity). According to Equation 2, it amounts to finding the time average of the electric field at a point with coordinate (x, y) on the detector and generalizing it to the entire screen. Invoking Huygens's principle, the E-field could be found by superposing the E-fields emanating from each slit (Ling, 2018).

3.2 Phase Difference

Although the process sounds simple, one has to consider the phase difference that would arise when the plane wave arrives at the double slit. The most simple case would be the ordinary double-slit diffraction where the phase difference comes from the wave number multiplied by an additional distance required for one of the sources to travel.

According to Figure 3, the phase difference would be:

$$\delta = k\Delta = ka\sin\theta \tag{6}$$

There are many other cases that may cause a shift in the phase. Another factor that may cause a phase shift would be the change in optical path length (OPL). This occurs when the wave travels through a medium consisting of a

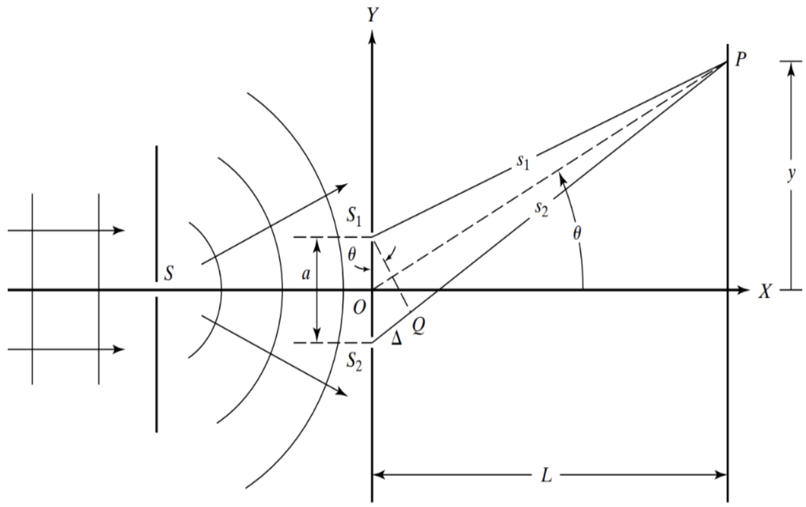

Figure 1: As shown by the image, there is an additional distance of Δ necessary for the source at the bottom of this picture to travel. Image adapted from (Pedrotti, 2019a)

different material. These materials can be characterized by the index of refraction n, where $n \geq 1$. This number dictates how fast would light travel through such material given by the equation (Pedrotti, 2019b):

$$v_{inside\ material} = c/n \tag{7}$$

where c is the speed of light. Since the velocity is different, there would naturally be a difference in the length traveled within a fixed amount of time. For instance, if a thin film of thickness p and index of refraction n was placed at the bottom source in Figure 3, the phase difference would be (Pedrotti, 2019b):

$$\delta = ka \sin\theta + (knp - kp) \tag{8}$$

As shown by the equation, the thin film augments the difference in the spatial phase shift. On the other hand, if the thin film is placed at the top source in Figure 3, the phase difference would then be (Pedrotti, 2019b):

$$\delta = ka \sin\theta - (knp - kp) \tag{9}$$

In this case, the innate phase difference gets diminished as the top source now traverses a longer OPL.

3.3 Double Slit Diffraction

Putting everything together, the equation that models the double slit diffraction would be (Pedrotti, 2019a):

$$\frac{I(\theta,\phi)}{I_0} = sinc^2(\beta)\cos^2(\alpha)sinc^2(\gamma)\cos^2(\zeta) \tag{10}$$

$$\beta \equiv \frac{1}{2}kb\sin\theta$$

$$\alpha \equiv \frac{1}{2}kd\sin\theta$$

$$\gamma \equiv \frac{1}{2}ka\sin\phi$$

$$\zeta \equiv \frac{1}{2}kd\sin\phi$$

In the equations, θ is the azimuthal angle relative to the double slits, and ϕ is the angle relative to a horizontal plane connecting the double slit and the detector. d is the distance between the two slits, a and b are respectively the width and the length of the slits, and k is the wave number.

3.4 Single Slit and Multiple Slit Diffraction

Single-Slit and Multiple-Slit Diffraction can then be derived from the equations of double-slit diffraction. For a single-slit with width a and length b, the equation would be (Pedrotti, 2019a):

$$\frac{I(\theta,\phi)}{I_0} = sinc^2(\kappa)sinc^2(\mu) \tag{11}$$

$$\kappa \equiv \frac{1}{2}kb\sin\theta$$

$$\mu \equiv \frac{1}{2}ka\sin\phi$$

all variables are the same as that of equation 10. A variant of this would be a single-slit with a circular aperture of diameter D. The equation becomes (Pedrotti, 2019a):

$$\frac{I(\theta,\phi)}{I_0} = (\frac{2J_1(\eta)}{\eta})^2(\frac{2J_1(\iota)}{\iota})^2 \tag{12}$$

$$\eta \equiv \frac{1}{2}kD\sin\theta$$

$$\iota \equiv \frac{1}{2}kD\sin\phi$$

The function J_1 is the Bessel function of the first kind of the first order. All other variables are the same as indicated in Equation 10. For the case of multiple-slit diffraction, the modeling equation would become (Pedrotti, 2019a):

$$\frac{I(\theta,\phi)}{I_0} = sinc^2(\nu)(\frac{\sin N\xi}{\xi})^2 sinc^2(\upsilon)(\frac{\sin N\chi}{\chi})^2 \tag{13}$$

$$\nu = \frac{1}{2}kb\sin\theta$$

$$\xi = \frac{1}{2}kd\sin\theta$$

$$\upsilon = \frac{1}{2}ka\sin\phi$$

$$\chi = \frac{1}{2}kd\sin\phi$$

all variables are the same with equation 10.

4 Method

4.1 Libraries

The libraries used for this project include:

1. NumPy
2. SciPy.special
3. Matplotlib.pyplot
4. Matplotlib.gridspec
5. IPython.display
6. ipywidgets

The first four libraries are used to turn the above equations into codes and plot and format the outputted image. The last two libraries are used for interface construction where users can interact with the simulation. The usage of Matplotlib.gridspec for formatting is taken from (Hunter, 2017).

4.2 Flowchart

The logic of the program is shown in Figure 2. When the program starts running, the user first decides whether they want the case of Single-Slit or Double-Slit Diffraction. If Single-Slit diffraction is chosen (proceed to the right of the decision), the program would then ask the user to choose one of the three cases for single-slit diffraction. The three cases correspond to 3 types of apertures: circular, rectangular (width and length have a similar order of

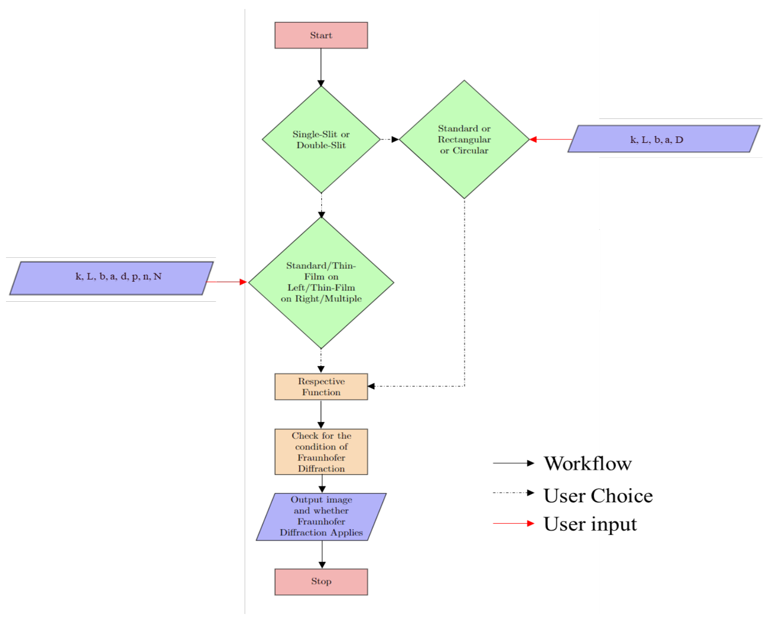

Figure 2: This is the Workflow Chart for the simulation. Pink rectangles represent the start and stop, green diamonds represent a decision, purple parallelograms represent input, and the orange rectangles represent a certain process.

magnitude), and standard (where the length is much larger than the width). Based on the type of aperture chosen, the user would be asked to input different kinds of parameters. The possible parameters include wave number (k), the distance between slits and detector (L), the width of slit (a), the length of slit (d), and the diameter of the slit (D).

On the other hand, if the choice of 'double slit' was chosen, the user would then need to choose one of the four cases contained in 'double-slit'. The cases are 'standard,' 'left,' 'right,' and 'multiple slit.' The first corresponds to the ordinary scenario of a double-slit experiment, 'left' and 'right' places a thin film respectively on the left or the right slit, and the last choice creates a multiple-slit diffraction pattern. In addition to the mentioned parameters above, the user would also be asked to enter the value for the thickness of the thin film (p), the index of refraction of the material (n), and the number of slits (N). Therefore, there will be 7 outcomes in total. The outcome and its asked inputs are listed in the chart below.

Outcome	Asked Input
Single-Slit, Standard	k, L, b, a
Single-Slit, Rectangular	k, L, b, a
Single-Slit, Circular	k, L, D
Double-Slit, Standard	k, L, b, a, d
Double-Slit, Right	k, L, b, a, d, p, n
Double-Slit, Left	k, L, b, a, d, p, n
Multiple Slit	k, L, b, a, d, N

Values entered by the user would be inputted into the respective function designated for each outcome. The parameters k, L, and b (or D for the case of circular aperture), would also be used to run a check function that uses equation 5 to examine whether Fraunhofer diffraction is applicable. Finally, the program would return the diffraction pattern along with horizontal and vertical intensity distribution.

5 Results

The result for the interface that users can interact with and the 7 outcomes are shown in figures 3 to 10. Apart from the Standard Single-Slit Diffraction Case, all images come along with a horizontal and vertical density distribution.

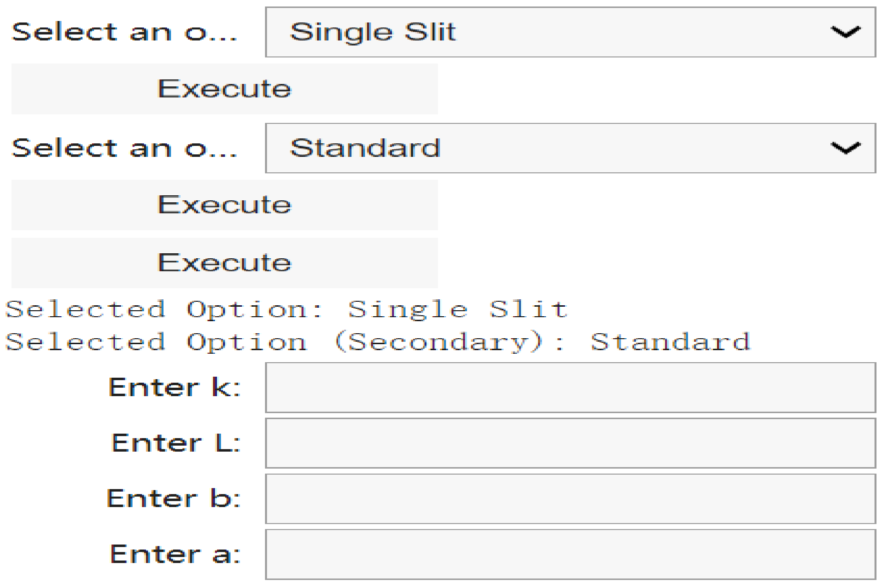

Figure 3: Interface for the Standard Single-Slit Diffraction Case

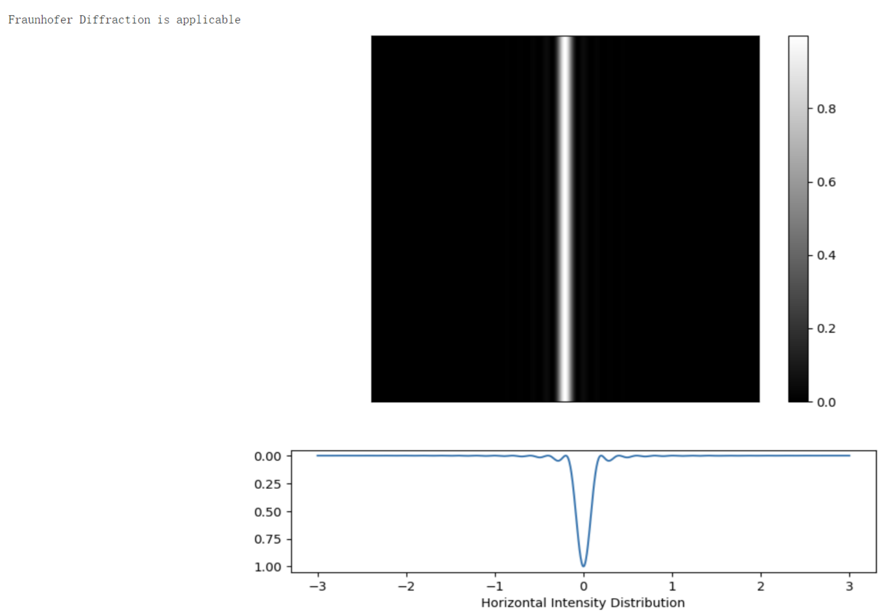

Figure 4: Single-Slit Diffraction, Standard case with input parameters: k = 100000, L = 10, b = 0.001, a = 0.00000001

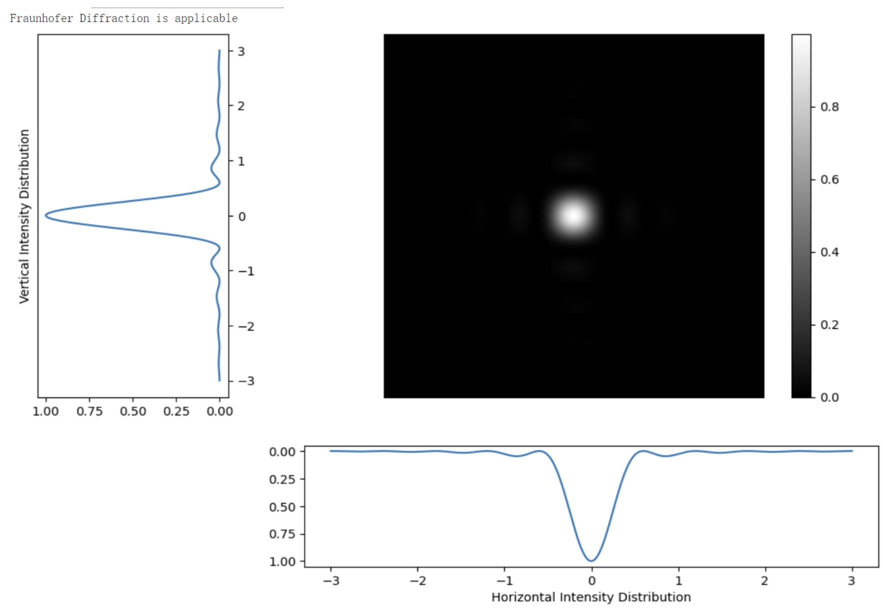

Figure 5: Single-Slit Diffraction, Rectangular case with input parameters: k = 100000, L = 300, b = 0.01, a = 0.01

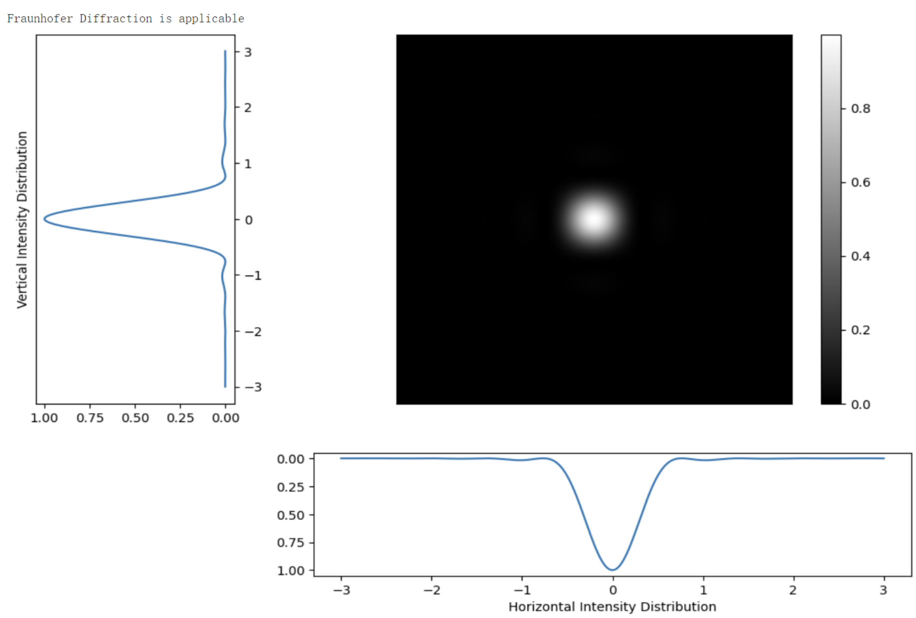

Figure 6: Single-Slit Diffraction, Circular case with input parameters: k = 500000, L = 50, D = 0.001

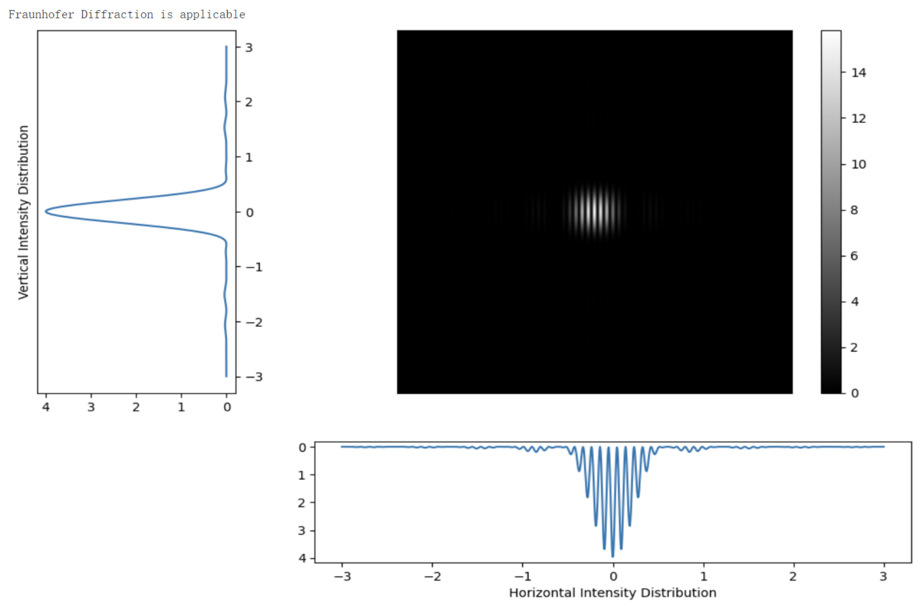

Figure 7: Double-Slit Diffraction, Standard case with input parameters: k = 100000, L = 30, b = 0.02, a = 0.001, d = 0.001

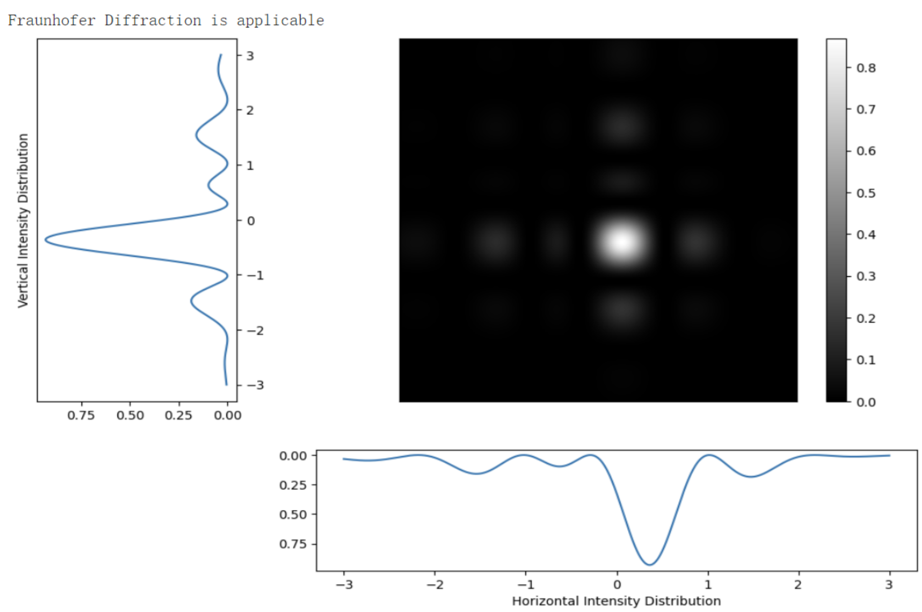

Figure 8: Double-Slit Diffraction, thin film on the right slit with input parameters: k = 10000, L = 5, b = 0.002, a = 0.01, d = 0.001, p = 0.02, n = 2

Fraunhofer Diffraction is applicable

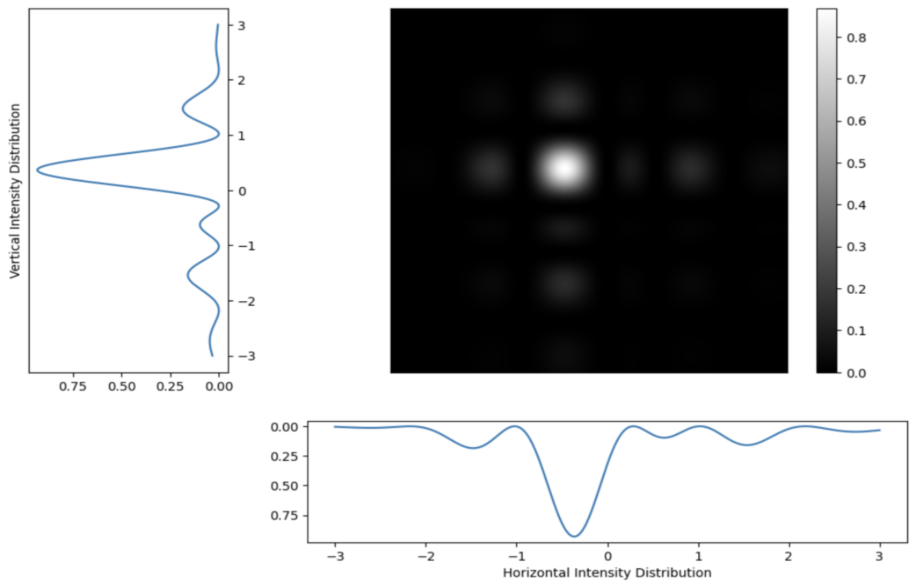

Figure 9: Double-Slit Diffraction, thin film on the left slit with input parameters: k = 10000, L = 5, b = 0.002, a = 0.01, d = 0.001, p = 0.02, n = 2

Fraunhofer Diffraction is applicable

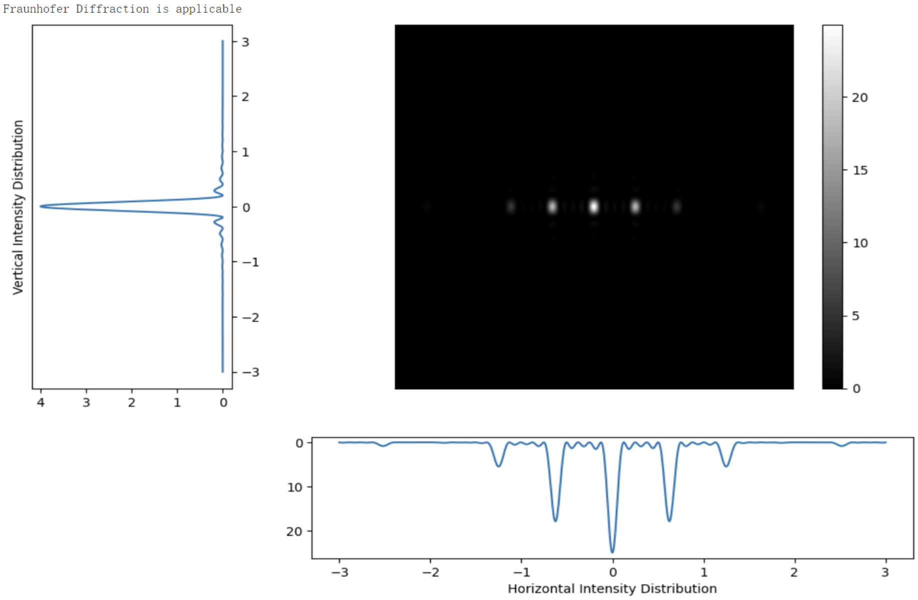

Figure 10: Multiple Slit Case with parameters: k = 100000, L = 100, b = 0.001, a = 0.01, d = 0.01, N = 5

6 Conclusion

The result of the simulation consists of successes and failures. It successfully takes into consideration of 7 cases that produce a spatial phase difference between the sources; it also generates the correct intensity distribution along two different axes and a 2-dimensional diffraction pattern. However, the simulation could become more exhaustive by considering factors that would introduce temporal phase differences such as relativistic effects from a moving source. Furthermore, the returned image can also have a higher resolution or contrast. The reason is that, for cases of Single-Slit diffraction, the color map makes it hard to notice the secondary interference patterns; resulting in difficulties in distinguishing between rectangular (Figure 5) and circular apertures (Figure 6). Additionally, another aspect that could be considered in the future would be making the simulation more animate in the sense that the image automatically updates when a parameter is altered.

7 Acknowledgment

I would like to thank my project partner Madhav Agrawal for diligently working to create the interface and turning this simulation into reality. I would also like to thank Professor Jazaeri for helping us to find the source for the physics used in this simulation.

8 References

1. Hunter, J. (2017). Customizing location of subplot using GridSpec¶. Customizing Location of Subplot Using GridSpec - Matplotlib 2.1.1 documentation. https://matplotlib.org/2.1.1/tutorials/intermediate/gridspec.html

2. Ling, S. J., Sanny, J., & Moebs, W. (2018). Huygens's Principle. In University physics (Vol. 3). essay, OpenStax, Rice University.

3. Pedrotti, F. L., Pedrotti, L. M., & Pedrotti, L. S. (2019a). Fraunhofer Diffraction. In Introduction to optics. essay, Cambridge University Press.

4. Pedrotti, F. L., Pedrotti, L. M., & Pedrotti, L. S. (2019b). Interference in Dielectric Films. In Introduction to optics (pp. 175–180). essay, Cambridge University Press.