A robust medical image watermarking framework based on SVD and DE in Integer DCT domain (Workshop Paper)

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Abstract—A secure medical image watermarking framework using SVD, DE, step space filling curve and IntDCT is proposed. The proposed algorithm starts with partition of the host medical image into 8×8 blocks and then IntDCT is applied in each block. By collecting integer DC coefficients, a feeble resolution medical image is obtained. SVD is applied on this feeble resolution image. For maintaining a strong relationship between imperceptibility and robustness, the popular differential evolution (DE) technique is chosen to examine the most suitable scaling factors. The singular values of the host image are modified with the singular values of the watermark to insert the watermark in the host image. Here, DE is playing a very important role to determine the most suitable scaling factors to the embedded process for maintaining the dilemma between robustness and imperceptibility without degrading the nobility of medical images. To increase the security of the watermark image, first it is scuffled with the help of step space filling curve before the embedding process. The experimental observations and results proved the efficiency of the proposed framework. It maintains the quality of the watermarked images and the watermark can be extracted from the seriously malformed images.

Keywords-Differential evolution; Integer discrete cosine transform; step space filling curve; singular value decomposition; watermarking

I. Introduction

The advanced clinical research has moved into the digital age because of significant manifestation in medical research and technology. Almost all the medical imaging detectors gives the information in the shape of digital images such as ultrasound, CT, MRI, X-ray, and PET [1]. This data in the form of medical images furnished the valuable information about the shape, position and visual representation of the internal human body organs. These information is very useful in medical treatment and clinical analysis. It always helps to detect the disease in the body by X-ray image, measuring discrepancies in the MRI images and segmenting organs in CT images. Similarly, former quantitative information (like shape and scale) are beneficial in the detection of tumor and brain cancer. Therefore, medical images essentially carry valuable, confidential and sensible information associated to a sick person's health condition. The storage, security and communication of these images are still a big problem due to the importance for various purpose. Generally, these symptomatic images are picked, storages and transmitted over the several insecure public channels through internet technology for various applications such as compression, denoising, segmentation and data hiding [2, 3, 4]. However, due to the deficiency of the security measures, this medical data may be destroyed or disclosed against various threats like privacy leakage, data integrity and illegal manipulation. Therefore, the security of the medical data is very crucial and need to be addressed.

Generally, the image watermarking techniques are categorized in two main category based on the selection of the domain in which the watermark is embedded, namely (1) spatial domain watermarking algorithms and (2) frequency domain watermarking algorithms [5]. The watermark is directly inserted by altering the pixel values of the cover image in spatial domain. The spatial domain algorithms have the reward of easy implementation with minimum cost of process but are very sensitive against geometric distortions and different kind of attacks. While the frequency domain techniques first transform the data into frequency domain from the spatial domina. The watermark is embedded by modifying the frequency coefficients. There are so many methods which are adopted to transform the information from the spatial domain into the frequency domain e.g. FT and QFT [6], DWT[7, 8, 9], QWT [10, 11], DCT [5] and integer discrete cosine transform (IntDCT) [12]. SVD is utilized to pull out the geometric symptoms with combining one or more other transformation [13, 14, 15]. Due to less computation cost, high robustness and high compression, DCT is frequently chosen as a foundation of watermarking algorithms [16]. But, for quantizing the transforming coefficients, it is quite difficult to utilize for lossless compression [17]. So, lossless coding algorithms are hardly available on the basis of the DCTs [1]. Therefore, integer transforms and transforms without float-point multiplications have attracted to the researchers towards them. Integer transforms become more popular rather than float-point transforms due to the better features. These features include the decorrelation attributes of low computational cost.

In medical image encryption algorithms, first the host image is encrypted by altering the pixel values in such a way that the encrypted image should not loss any relevant information. However, an authorized person can get back the original medical image from the encrypted image. Various encryption algorithms are available in the literature [4, 18, 19]. Some of them are falling short to supply the essential requirements with the exposure against various malicious attacks. To heighten the security of medical image encryption algorithms chaos theory is introduced [20]. But due to the week key management, these algorithms are not completely robust. In general, the security of the encryption techniques purely banks on the associated keys. If these keys are copied or stolen, then an unauthorized person can reveal all the information from the medical images. While, on the other hand, if the keys are corrupted or destroyed then the original data can not access back. Hence, the key management must be robust, secure and untraceable.

In this work, an efficient and robust medical image watermarking framework is proposed based on integer discrete transform (IntDCT), differential evolution (DE) algorithm and SVD. The proposed watermarking framework begins with the partition of the host medical image into 8×8 blocks. IntDCT is applied on each blocks to obtain the DC coefficients and these coefficients are altered in the SVD domain. DE is helped to determine the most suitable multiple scaling factors in the shape of diagonal matrix and their vales lie in the interval [0, 1]. In SVD domain, already DE is used for determining an optimized solution for image watermarking [21]. In our work, an another scope is provided by utilizing DE to pull out the effectiveness of watermark in the unite domain of IntDCT and SVD. The robustness and security of the watermark is also enhanced by adding one more protection level i.e., scrambling the watermark image. Watermark is scuffled using the step space filling curve before the embedding process. It gives an extra security and protection to the watermark image.

II. PRELIMINARY

A. Step space filling curve

The space-filling curve (SFC) [22, 23] is a technique for uninterrupted scanning in which every picture element of the image traversed just once. The consequential sequence of picture elements is further refined. After processing, the obtained image with modified sequence of picture elements pull back along with the same step space filling curve in a picture. Many image space techniques bank on the spatial cohesion of neighboring pixels. The main idea of step space filling curve is to break a matrix into two sub-matrices according to its main diagonal. First matrix is an upper triangular matrix including the main diagonal elements of the original matrix and second matrix is a lower triangular matrix including all zero elements at its main diagonal

elements. The step space filling curve strictly follow the property of matrices that the elements in the super-diagonal as well as sub-diagonal are perpendicular to the nearby elements of the diagonal elements. Hence, the elements of the super-diagonal and sub-diagonal can be inserted between the nearby elements of the diagonal elements. The detailed arrangement of step space filling curve can be seen in [23].

B. Singular value decomposition (SVD)

The singular value decomposition (SVD) is an algorithm for factorization of a rectangular matrix into three matrices U, S and V^T such that the matrices U and V are orthogonal square matrices and whose columns are known as left and right singular vectors, respectively. S is a diagonal rectangular matrix whose enterers in the decreasing order are called the singular values. The SVD of a rectangular matrix B of size $m \times n$ is represented mathematically as

$$B = U * S * V^T \tag{1}$$

The columns of U are orthonormal eigenvectors of $B*B^T$, the columns of V are orthonormal eigenvectors of B^T*B and $S=diag(\sigma_1,\sigma_2,\sigma_3,...,\sigma_k)$, where σ_k are the singular values of the matrix B with k=min(m,n) and satisfying the relation $\sigma_1 \geq \sigma_2 \geq \sigma_3 \geq ... \geq \sigma_k$.

C. Integer discrete cosine transform (IntDCT)

IntDCT is a linear mapping which transforms integer input to integer output in the way that this inversion is simple as well as fast [1, 12]. So far, various integer linear transformations are available for video as well as image coding applications. These transformation are purely bank on the lifting factorization of the standard Fourier and Cosine transform. IntDCT finds numerous applications where lossless coding is an integral component. It is obtained based on different DCT kernels and among them type II DCT kernels is of utmost importance. The details of the IntDCT algorithm used in this work can be seen in [12].

D. Differential evolution (DE)

DE is a fast yet simple and substantial evolutionary algorithm was presented first time by Storn and Price [24]. It begins with a preparatory population of NP mortals $Y_{i,G}, i=1,2,...,NP$, where i represents the i^{th} results of the population at generation G. Three important components of DE are explained as below:

Mutation: To develop the rattled mortals $R_{i,G}$ corresponding to each destination mortals $Y_{i,G}$, the mutation process is started by randomly picking up three different mortals $\{Y_{r1}, Y_{r2}, Y_{r3}\}$ form the present population that must be different from the destination mortals $Y_{i,G}(i.e., r1 \neq r2 \neq r3 \neq i)$. It indicates the vector conflict between the existing population mortals in finding both direction as well as degree of the disruption. The conflict between two mortals with the

scaling factor (α) lies in the interval [,1], is summed to third mortal. It is evaluated as:

$$R_{i,G} = Y_{r1,G} + \alpha \times (Y_{r2,G} - Y_{r3,G}) \tag{2}$$

where i has the ranges through the number of mortals.

Crossover: Crossover procedure is executed between rattled mortal $R_{i,G}$ and destination mortal $Y_{i,G}$ to produce the trial mortal $T_{i,G}$. This procedure relies on a crossover chance (Cr) lies in the interval [0,1] that finalize the factors of trial mortal. It also assures that trial mortal will be distinct in at least one factor. Mathematically, the equation of trial mortal generation is given as:

$$t_{l,i,G} = \begin{cases} r_{l,i,G}, & \text{if } rand_l \leq C_r \text{ for all } l = k; \\ y_{l,i,G}, & \text{otherwise.} \end{cases}$$
 (3)

where $k \in \{1,2,...,D\}$ and l has the range through the dimension of the problem.

Selection: The fitness is calculated after replication of the trial mortal and equate it to its corresponding destination mortal. It is achieved by the selection process of the best mortal from the destination and trial mortals. It is shown mathematically by the equation

$$Y_{i,G+1} = \begin{cases} T_{i,G}, & \text{if } f(T_{i,G}) \le f(Y_{i,G}); \\ Y_{l,i,G}, & \text{otherwise.} \end{cases}$$
 (4)

If the trial mortal is better than the destination mortal then it exchanges the destination mortal in the next generation otherwise it will continue with the current destination mortal. Thus, each mortal of the trial population is equated with its similitude in the present population.

III. PROPOSED FRAMEWORK

A. Embedding Process

- 1) The host medical image I is partitioned into 8×8 blocks. IntDCT is applied on each block and DC values are collected from each IntDCT coefficients matrix C(r,s) to build the matrix B, called the feeble resolution approximation image.
- 2) Apply SVD on the matrix B and obtain three matrices $U,\,S$ and V such that

$$B = U * S * V^T \tag{5}$$

3) The watermark image W is scrambled by step space filling curve and embedded into the matrix S by hiring the scaling factor α with the help of DE algorithm as

$$S_1 = S + \alpha * W \tag{6}$$

 α is also in the shape of a diagonal matrix.

4) Again SVD is applied to the matrix S_1 and receive three new matrices namely U_w , S_w and V_w such that

$$S_1 = U_w * S_w * V_w^T \tag{7}$$

5) The matrices U, S_w and V^T are multiplied together to determine a matrix B_w with the modified DC values as

$$B_w = U * S_w * V^T \tag{8}$$

6) Perform the inverse IntDCT after substituting the modified DC values back to the relevant blocks and finally merge all the blocks to achieve the watermarked image I_W .

B. Extraction Process

If the owner provides the values of U_w , S, V_w and α while I_w^* is the distorted watermarked image, then by doing the reverse process, a distorted watermark W^* can be extracted. The following steps are involved in the extraction process:

1) Perform the step 1 of embedding process on the distorted watermarked image I_w^* to obtain B_w^* and following the step 2 on B_w^* to partitioned it into three matrices such as

$$B_w^* = U^* * S_w^* * V^{*T}$$
 (9)

2) Evaluate the likely distorted diagonal matrix S_1^* as

$$S_1^* = U_w * S_w^* * V_w^T \tag{10}$$

3) Extract the likely distorted scuffled watermark W^* as

$$W^* = \frac{S_1^* - S}{\alpha} \tag{11}$$

4) Perform the inverse step space filling curve on W^* to obtain the finally extracted watermark image.

IV. EXPERIMENTAL RESULTS AND DISCUSSIONS

To analyze the execution and robustness of the proposed algorithm, various experiments are executed on the different medical images. The experimental medical images along with watermark images are shown in the figure 1. Three gray-scale medical images with size 256×256 namely, MRI of brain, X-ray image of chest and CT scan of chest are taken as the host images. Two gray-scale images (logo and text) of size 32×32 are taken as watermark images. The watermarked images are presented in the figure 2. The parameters which are used in DE are mentioned in the Table I.

Table I: Parameters of DE.

| Scaling factor F | 0.5 |
|------------------------|-----|
| Crossover rate Cr | 0.5 |
| Population size NP | 10 |
| Maximum generation G | 100 |

The peak signal noise ratio (PSNR) is adopted to measure the optic caliber and normalized cross-correlation coefficients (NCC) is taken as similarity measure between the

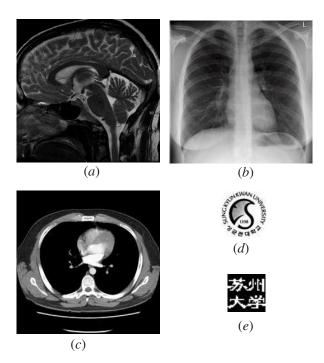


Figure 1: (a) - (c) Host images: MRI, X-ray, and CT Scan and (d)-(e) watermark images.

original and the distorted image. Mathematically, PSNR is defined as

$$PSNR = 10 \log_{10} \left(\frac{\max(I_{rs})^2}{\frac{1}{mn} \sum_{r=1}^m \sum_{s=1}^n ||I_{rs} - I_{rs}^*||^2} \right)$$
 (12)

where I_{rs} is the original host image and I_{rs}^* is the water-marked iamge. Similarly, NCC is defined as

$$NCC = \frac{\sum_{r=0}^{M-1} \sum_{s=0}^{N-1} W_{rs} W_{rs}^*}{MN\sqrt{\sum_{r=0}^{M-1} \sum_{s=0}^{N-1} W_{rs}^2} \sqrt{\sum_{r=0}^{M-1} \sum_{s=0}^{N-1} W_{rs}^{*2}}}$$
(13)

where W_{rs} is the original watermark image and W_{rs}^* is the extracted watermark image. The PSNR and NCC values between the host image and watermarked image (logo and text) are mentioned in the Table II. The NCC values are near about 1 and PSNR values are approximately equal to 56. The visual quality of the watermarked images are shwon in the figure 2. From the figure, it can be observe that there is no degradion in the optic caliber of an image.

Table II: NCC and PSNR between original image and watermarked images.

| Host Images | Watermarked Image (logo) | | Watermarked Image (text) | | |
|----------------|--------------------------|-------|--------------------------|-------|--|
| 110st Illiages | NCC | PSNR | NCC | PSNR | |
| MRI | 0.9999 | 55.72 | 0.9999 | 58.84 | |
| X-ray | 0.9999 | 55.17 | 0.9999 | 56.56 | |
| CT Scan | 0.9999 | 55.94 | 0.9999 | 55.80 | |

To check the performance and robustness of the proposed

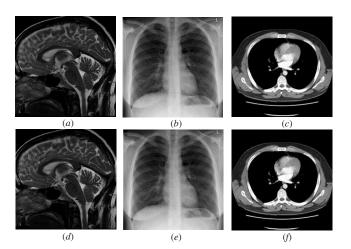


Figure 2: Watermarked images: (a) MRI, (b) X-ray and (c) CT Scan with watermark (logo) and (d) MRI, (e) X-ray and (f) CT Scan with watermark (text).

algorithm against the attacks, watermarked image is distorted using several attacks such as (1) median filtering (MF) with 3×3 neighboring pixels, (2) average filtering (AF) with 3×3 neighboring pixels, (3) translation (TR) with 20×20 pixels, (4) one fourth cropping (CR) from left side, (5) JPEG compression with quality factor 50, (6) histogram equalization (HE), (7) rescaling (RS) $256 \rightarrow 128 \rightarrow 256$, (8) gamma correction (GC) with 0.6, (9) Gaussian noise (GN) with 0 mean and 0.01 variance and (10) rotation (RO) with 30° anticlockwise.

Usually, to fit an image in some specific application, it can be cropped or rescaled. One fourth of the image is cropped and the results are shown in figure 4(a) and 3(a). The extracted watermark is still robust with good NCC values. Sometimes, the size of the image may be enlarged or reduced. In that case, the information of the embedded watermark can be lost. First the size of the image is reduced from 256×256 to 128×128 and then enlarged back to 256×256 . The results are shown in the figures 4(e) and 3(e). Gamma correction transformed the image using gamma correction factor 0.6. During the image capturing, displaying and printing, the encountered nonlinearity can be corrected using gamma correction. The results are mentioned in the figures 4(b) and 3(b). Histogram equalization technique is adopted for contrast adaptation using the histogram of the image. The impression of histogram equalization and extracted watermark are presented in figures 4(d) and 3(d).

The average and median filtering are used to remove the noise and smoothen the images. Average filters takes the average of the values from its neighbors pixels to replace the current value of the corresponding pixel. The experimental results shows that the extracted watermark obtained by the proposed method has the robustness against median filtering (figure 4(g)-(h) and 3(g)-(h)). Another effective attacks in

| Attack | Watermarked Images | Extracted Watermark | Attack | Watermarked Images | Extracted Watermark |
|-----------|-----------------------|------------------------|-------------|-----------------------|------------------------|
| (a) CR | | 赤州 ナ学 | (f) JPEG | | 芬州 大学 |
| (b) GC | | 苏州 大学 | (g) AF | | 苏州 大学 |
| (c) GN | | 苏州 大学 | (h) MF | | 芬州 大学 |
| (d) HE | | 苏州 大学 | (i) RO | | 苏州 大学 |
| (e) RE | | 苏州 大学 | (j) TR | | 芬州 大学 |

Figure 3: Distorted watermarked images MRI with corresponding extracted text watermarks after attacks (a) CR, (b) GC, (c) GN, (d) HE, (e) Re, (f) JPEG, (g) MF, (h) AF, (i) RO and (j) TR.

watermarking is JPEG compression. The robustness is examined against JPEG compression with quality index 50 and the results are shown in figures 4(f) and 3(f). Rotation is another very popular geometrical attacks. This attacks is applied to destroy the watermark by breaking the bonding of the spatial relationship between the host image and watermarked image. Here, a rotation of 30° anticlockwise is applied to the watermarked image. The extracted watermarks with watermarked images are shown in the figures 4(i) and 3(i). At the last, translated attack is applied on the watermarked image by translating it to 20×20 pixels. The results for this are shown in the figures 4(j) and 3(j). NCC values in each case also estimated for both the watermark images. It is mentioned in the Table III for logo watermark. In case of cropping attack, the results are not so good compare to other attacks. In Table IV, NCC values are mentioned for text watermark. From the table, it can be observed that the results in cropping, histogram equalization and rotation are degraded compare to the other attacks.

V. Conclusion

In the proposed framework, the singular values of a feeble resolution medical image obtained by DC values of blocks of the host medical image are replaced with the singular values of watermark image. The main contribution of the proposed work is the application of IntDCT along with SVD,

Table III: NCC between original watermark (logo) and corresponding extracted watermark images.

| Attacks | MRI | X-ray | CT Scan |
|---------|--------|--------|---------|
| MF | 0.9245 | 0.9280 | 0.9579 |
| AF | 0.9196 | 0.9256 | 0.9570 |
| TR | 0.9025 | 0.8935 | 0.9475 |
| CR | 0.7684 | 0.5974 | 0.9313 |
| JPEG | 0.9236 | 0.9292 | 0.9579 |
| HE | 0.9287 | 0.9529 | 0.9324 |
| RS | 0.9252 | 0.9274 | 0.9580 |
| GC | 0.9440 | 0.9098 | 0.9559 |
| GN | 0.9014 | 0.9034 | 0.9448 |
| RO | 0.9060 | 0.9117 | 0.9474 |

| Attack | Watermarked Images | Extracted Watermark | Attack | Watermarked Images | Extracted Watermark |
|-----------|-----------------------|------------------------|-------------|-----------------------|------------------------|
| (a) CR | | | (f) JPEG | | © |
| (b) GC | | Ø | (g) AF | | (() |
| (c) GN | | (S) | (h) MF | | (5) |
| (d) HE | | Ĭ. | (i) RO | | (B) |
| (e) RE | | | (j) TR | | (B) |

Figure 4: Distorted watermarked images MRI with corresponding extracted logo watermarks after attacks (a) CR, (b) GC, (c) GN, (d) HE, (e) Re, (f) JPEG, (g) MF, (h) AF, (i) RO and (j) TR.

DE technique for determining the suited multiple scaling factors with scrambling the watermark by step space filling curve. The experimental results shows that the proposed work generates an effective robustness to the geometrical and other image processing attacks. The watermark can be extracted from the corrupted images after the most common attacks are used with high NCC values.

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Table IV: NCC between original watermark (text) and corresponding extracted watermark images.

| Attacks | MRI | X-ray | CT Scan |
|---------|--------|--------|---------|
| MF | 0.9702 | 0.9796 | 0.9664 |
| AF | 0.9739 | 0.9796 | 0.9719 |
| TR | 0.9498 | 0.9647 | 0.9700 |
| CR | 0.8273 | 0.6200 | 0.6966 |
| JPEG | 0.9739 | 0.9759 | 0.9626 |
| HE | 0.9275 | 0.9778 | 0.7169 |
| RS | 0.9740 | 0.9796 | 0.9700 |
| GC | 0.9650 | 0.9727 | 0.9796 |
| GN | 0.9681 | 0.9564 | 0.9555 |
| RO | 0.8273 | 0.9483 | 0.9160 |

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