Lecture 12 Worksheet

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Every worksheet will work as follows.

- 1. You will be entered into a Zoom breakout session with other students in the class.
- 2. Read through the worksheet, discussing any questions with the other participants in your breakout session.
 - You can call me using the "Ask for help" button.
 - Keep in mind that I will be going through all rooms during the session so it might take me a while to get to you.
- 3. Answer each question (preferably in the order provided) to the best of your knowledge.
- 4. While collaboration between students in a breakout session is highly encouraged and expected, each student has to submit their own version.
- 5. You will have 24 hours (see Compass) to submit your work.

Worksheet 1: Jointly distributed discrete random variables

During our last worksheet, we saw two discrete random variables $X = \{1,2,3\}$ and $Y = \{10,20\}$ that were jointly distributed with probability mass function:

$$f_{XY}(x,y) = \frac{20}{27} \cdot \frac{x+1}{y}.$$

Problem 1: Expectations and variances

What is the expectation of X and what is the variance of X? ¹

Answer to Problem 1.

¹ Recall that the expectation of one of two jointly distributed random variables can be found by properly summing (if discrete, as is the case here) or integrating (when continuous) its marginal distribution.

Problem 2: Conditional expectations and variances

What is the expectation of *X* and what is the variance of *X given that* $Y = 10?^{2}$

Answer to Problem 2.	

² The previous hint still applies! However, we now replace the marginal with the conditional distribution.

Problem 3: Independent?

Can we make the claim that *X* and *Y* are two independent random variables? Why/Why not?

Answer to Problem 3.	

Problem 4: Covariance

Based on your answer in Problem 3, what is the covariance? What is the correlation? 3

Answer to Problem 4.

$$\sigma_{XY} = Cov[X, Y] =$$

$$\rho_{XY} = Corr[X, Y] =$$

³ Recall that two independent random variables have zero covariance and, consequently, no correlation.

Worksheet 2: Jointly distributed continuous random variables

Consider two jointly distributed *continuous* random variables $0 \le$ $X \le 2$ and $0 \le Y \le 1$ with joint probability density function equal to:

$$f_{XY}(x,y) = \frac{3}{4}x^3y^2.$$

Answer the following questions.

Problem 5: Marginal distributions

Let's repeat what we had done during our previous lecture. What are the marginal distributions of X and Y? ⁴

Answer	to	Pro	h	lem.	5

⁴ As a reminder, the marginal distribution of X will be a function of x and the marginal distribution of Y will be a function of *y*.

Problem 6: Getting the expectatio	าทร
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١	What are the expectations of X and Y ?	Don't forget that they are
(defined over different domains! 5	

Answer to Problem 6.

 5 X is defined over [0,2], whereas Y is defined over the range [0,1].

Problem 7: Independent?

Are *X* and *Y* independent? Why/Why not?

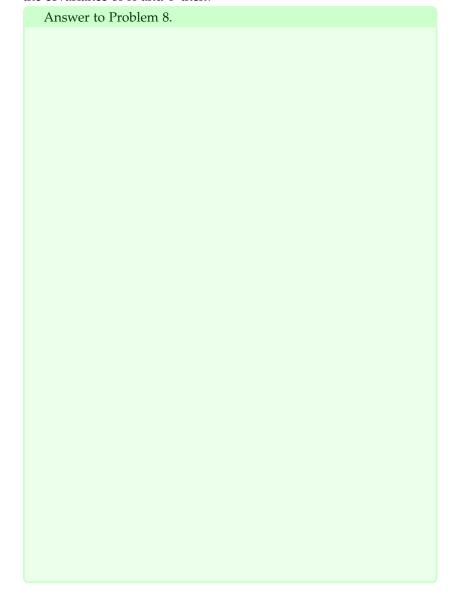
Answer to Problem 7.

Worksheet 3: When X and Y restrict each other

Assume that random variables *X* and *Y* are jointly distributed with probability density function $f_{XY}(x,y) = \frac{1}{4}(x+y)$ defined over $0 \le X \le Y \le 2$. Note how random variable X always takes values that are at most as big as the value of random variable Y. ⁶ Answer the following questions.

Problem 8: Independent?

X and *Y* are not independent; this is clear from their definition, as knowing the one restricts the values the other one may take. What is the covariance of X and Y then? ⁷



⁶ If you are wondering how this is a valid pdf, we may show that the double integration is equal to 1. Be very careful with how you are integrating this. Following are the two correct ways (for an incorrect way, look at the notes!):

$$\int_{0}^{2} \int_{0}^{y} \frac{1}{4} (x+y) dx dy = 1$$
$$\int_{0}^{2} \int_{0}^{2} \frac{1}{4} (x+y) dy dx = 1$$

- ⁷ You will need to calculate a lot of things. Namely you will need:
- 1. the marginal distributions $f_X(x), f_Y(y);$
- 2. the expectation E[X], E[Y];
- 3. the expectation of function $X \cdot Y$: $E[X \cdot Y];$
- 4. finally, you'll get $Cov[X,Y] = E[X \cdot Y] - E[X] \cdot E[Y].$

Problem 9: A small conditional pd

Assume that we are given that y = 1. What is the probability that Xis smaller than or equal to $\frac{1}{2}$?

Answer to Problem 9.	

Problem 10: A tougher one

What is the probability that $X \leq \frac{Y}{2}$?

Answer to Problem 10.	