Experiments, sample spaces, and events

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Lecture 1

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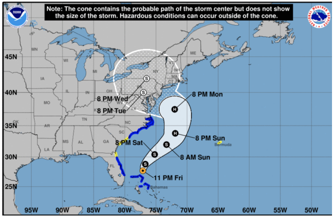
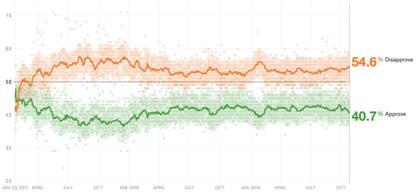
ISE | Industrial & Enterprise Systems Engineering
GRAINGER COLLEGE OF ENGINEERING

Motivation


MONOPOLY
Property Trading Game from Parker Brothers
TENNESSEE AVENUE
COMMUNITY CHEST
FOLLOW INSTRUCTIONS ON TOP CARD
ST. JAMES PLACE
PRICE $180
PENNSYLVANIA RAILROAD
PRICE $200
VIRGINIA AVENUE
PRICE $160
STATES AVENUE
ELECTRIC COMPANY
CHANCE
VERMONT AVENUE
CONNECTICUT AVENUE
ORIENTAL AVENUE
INCOME

Motivation





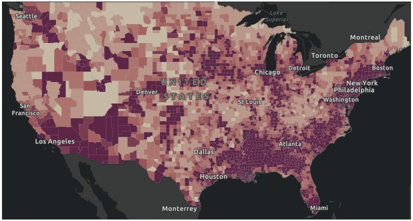


Image taken by https://coronavirus.jhu.edu/us-map.

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Experiments and sample spaces

- Experiment: a situation where the outcome is uncertain.
  - A lottery; a coin toss; the weather.
  - A football game; course performance; etc.
- Sample space (S): the set of all possible outcomes of an experiment.
  - The sample space can have a finite number or countably infinite number of outcomes..

    Discrete random experiments.
    - Coin toss: $S = \{Heads, Tails\}$;
    - Grade in class: $S = \{A, A-, B+, B, B-, \dots\}$;
    - Think of a positive integer number: $\{1, 2, 3, \dots, \}$.
  - The sample space can be an interval between two real numbers..

    Continuous random experiments.
    - The pressure of a gas;
    - The time the next bus arrives;
    - Think of a real number between 0 and 4: $[0, 4]$.

We will focus on

$\boxed{\textbf{discrete random experiments}}$

for the coming lectures.

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Events

- Event: a set of one or more outcomes in the sample space.
  - Examples: get *Heads* in a coin toss; no snow tomorrow; $grade \geq B$ in a class.
  - **simple** events.
  - **compound** events.

Examples:

- Two coins are tossed. What is the sample space? What is an event?
  - $S = \{HH, TT, HT, TH\}$.
  - An event could be to get at least one H.
- A dice is rolled. What is the sample space? What is an event?
  - $S = \{1, 2, 3, 4, 5, 6\}$.
  - An event is to get a 5.
- Two dice are rolled. What is the sample space? What is an event?
  - $S = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), \dots, (6, 6)\}$.
  - An event is to have both dies show the same number.

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Set operations

We define some operations for sets and, by extension, events:

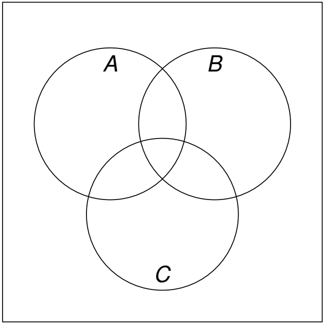
- Union of two events E_1, E_2 as $\boxed{E_1 \cup E_2}$:
  either event E_1 **or** E_2 (or both!) should happen;
- Intersection of two events E_1, E_2 as $\boxed{E_1 \cap E_2}$:
  both events E_1 **and** E_2 should happen;
- Complement of an event E as $\boxed{\overline{E}}$ (sometimes can also be written as E^c or E'): **any** event **but** E.
- Relative complement of an event E_2 from event E_1 as $\boxed{E_1 \setminus E_2}$:
  all outcomes in E_1 that are not also in E_2.

  sometimes termed as a *difference*.

Operations

Operations are very nicely explained with the use of Venn diagrams.

In the following examples, assume that S is described by events A, B, and C (that is at least one of these three events will happen):



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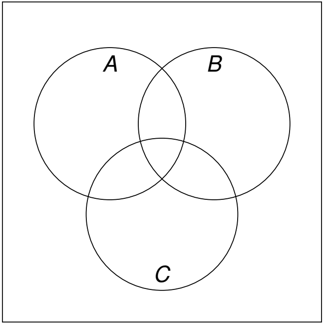
A and B should happen



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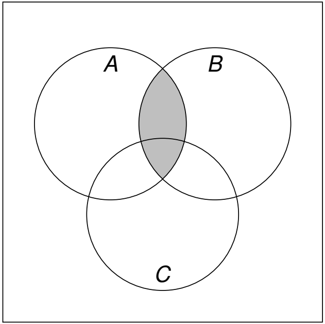
In the following examples, assume that S is described by events A, B, and C (that is at least one of these three events will happen):

$A \cap B$



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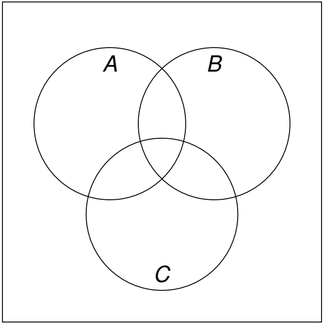
In the following examples, assume that S is described by events A, B, and C (that is at least one of these three events will happen):

any of B or C should happen



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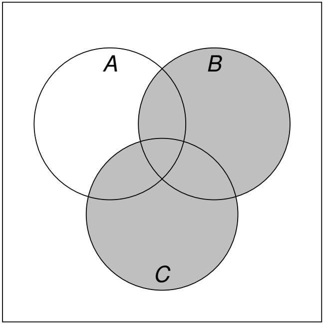
$B \cup C$



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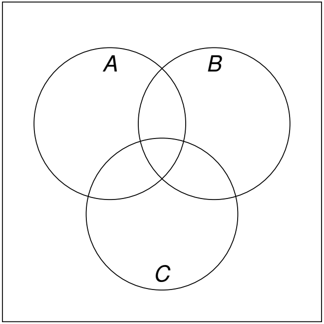
In the following examples, assume that S is described by events A, B, and C (that is at least one of these three events will happen):

anything but A or B should happen



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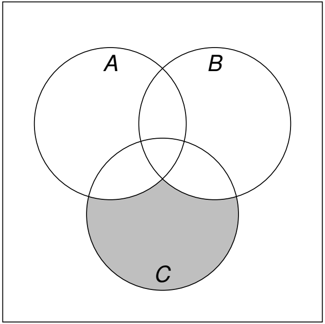
$\overline{(A \cup B)}$



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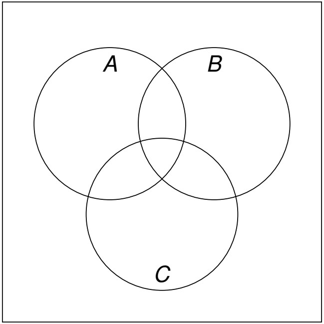
A, B, and C should happen



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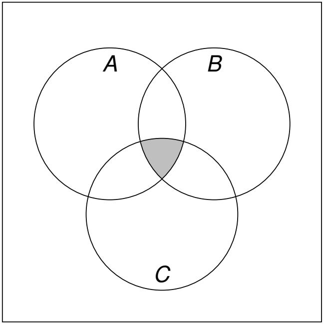
$A \cap B \cap C$



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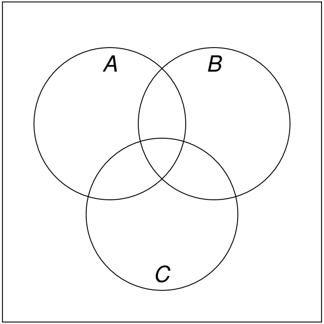
A but not B should happen



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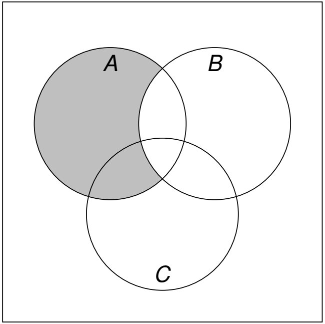
In the following examples, assume that S is described by events A, B, and C (that is at least one of these three events will happen):

$A \setminus B$



Mutually exclusive events

- A set that contains no elements is called an *empty* or *null* set.
- A null set is represented by $\emptyset$.

Mutually exclusive events:

- Two events are called **mutually exclusive** if they contain no common outcomes.
- For example, you cannot get *grade* $\geq$ *B* and *fail*.
- In other words, two events E_1, E_2 are mutually exclusive if $E_1 \cap E_2 = \emptyset$.

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Laws for set operations

Assume S is the sample space, and A, B, C some events.

1. $A \cup \overline{A} = S, \;\; A \cap \overline{A} = \emptyset, \;\; \overline{\overline{A}} = A.$
2. $A \cup B = B \cup A$ and $A \cap B = B \cap A.$
3. De Morgan's laws:
   - $\overline{(A \cup B)} = \overline{A} \cap \overline{B}.$
   - $\overline{(A \cap B)} = \overline{A} \cup \overline{B}.$
4. $A \cup (B \cup C) = (A \cup B) \cup C, \;\; A \cap (B \cap C) = (A \cap B) \cap C.$
5. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C), \;\; A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$

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Cardinality

The cardinality of an event E is the number of outcomes that it contains and it is denoted by $|E|$.

In the letter grade example (with $S = \{A, A-, B+, B, B-, \ldots\}$, what is the cardinality of $E = grade \geq B$?

$$|E| = 4.$$

- Consider $E = \emptyset$. Then, we have that $|E| = 0$.
- Consider an event E_1 that is contained in event E_2. We write that $E_1 \subseteq E_2$. For the cardinalities of E_1 and E_2, we certainly have that $|E_1| \leq |E_2|$.

During class, we will see how we can derive many more useful properties to help us count...

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