Lecture 6 Worksheet

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Every worksheet will work as follows.

1. You will be entered into a Zoom breakout session with other students in the class.
2. Read through the worksheet, discussing any questions with the other participants in your breakout session.
   - You can call me using the "Ask for help" button.
   - Keep in mind that I will be going through all rooms during the session so it might take me a while to get to you.
3. Answer each question (preferably in the order provided) to the best of your knowledge.
4. While collaboration between students in a breakout session is highly encouraged and expected, each student has to submit their own version.
5. You will have 24 hours (see Compass) to submit your work.

Worksheet 1: Selling on craigslist

You have decided to sell an item on craigslist. You have given no price in your ad: instead, you ask the interested people reading the ad to submit an offer. Their offers will be uniformly distributed between 0 and 99 dollars. [1] You would like to make at least \$75 from the item, so any offer that is greater than or equal to \$75 dollars would be acceptable. In addition, you receive *exactly* one offer every day since posting the ad.

[1] You may assume that only whole dollar amount offers can be given, such as \$37 or \$73

Problem 1

What is the probability a random offer you receive is higher than or equal to \$75?

Answer to Problem 1.

$P(\text{offer} \geq \$75) =$

Problem 2

What is the probability you sell the item on the 3rd day? What distribution does this follow? [2]

[2] Hint: consider each offer/day a success or a failure...

Answer to Problem 2.

Problem 3

What is the probability you sell the item in one of the first 3 days?

Answer to Problem 3.

Problem 4

You decide not to look at your email for 10 days. When opening your email again to check on the (10) offers you have received, what is the probability that at least one of them offers you $75 or more? What distribution does the number of "successful" offers follow? [3]

[3] Hint: consider as if you get 10 "tries" and need at least one of them be a "success"...

Answer to Problem 4.

Problem 5

You decide not to look at your email for 3 days. What is the probability that you sell the item on the 6th day, given that in the first 3 days you receive no satisfying offers? Contrast this answer to your answer in Problem 2. [4]

[4] Hint: use conditional probabilities and the geometric distribution probability mass function.

Answer to Problem 5.

Will you look at this. *Knowing that the first three days are failed* does not change (either increase or decrease) the probability of getting a success in the next three days! This is observed because our answers in Problems 2 and 5 are the same. We say that the geometric distribution is **memoryless**.

Worksheet 2: Practice with the Poisson distribution

Problem 6

A transportation engineer is collecting data during rush hour on the intersection of University and Neal in Champaign. They have noticed that the number of vehicles during rush hour follows a Poisson distribution with a rate of 7.5 per minute. What is the probability that exactly 10 vehicles pass through the intersection in the next minute?

Answer to Problem 6.

Problem 7

What is the probability that exactly 10 vehicles pass through the intersection in the **next 2 minutes**?

Answer to Problem 7.

Problem 8

During a pandemic, patients are in need of a special treatment with rate of 1 patient every 4 hours. Assuming a hospital can offer 5 of those treatments per day (24 hours), what is the probability the hospital runs out of available treatments during a day (24 hours) of operations?

Answer to Problem 8.

Problem 9

Scientists in the midwestern states have observed an increase in the frequency of floods and river overflows. The most recent estimate is that a devastating flood in some location in the midwest may happen with a rate of 1 every 50 years, whereas earlier estimates had placed that number in a rate of 1 every 200 years. Quantify the change in probability of a devastating flood in the midwest between the two estimates. What is the probability of seeing at least one devastating flood in the next year with the earlier and the more recent estimates?

Answer to Problem 9.

Worksheet 3: Interesting Poisson distribution properties

Problem 10

We saw in class the probability mass function for a Poisson distributed random variable with rate λ.[5] Assume that $\lambda = 3$ per year. What is the probability that there will be no events in the next year? Can you say that this means that the next event will happen more than a year from now? Let T be the time of the next event: what is $P(T > 1 \text{ year})$?

[5] It is $P(X = x) = e^{-\lambda} \cdot \frac{\lambda^x}{x!}$.

Answer to Problem 10.

$P(T > 1 \text{ year}) =$

So, the **time to the next event** is related to the number of events.. But, time is of a continuous nature whereas the number of events is discrete (has to be integer). *Interesting*: let's keep that in mind for our next lecture on continuous random variables.

Worksheet 4: "Truncated" Poisson distributed random variables

Problem 11

Assume that X is a Poisson-distributed discrete random variable with rate $\lambda = 2$ per 30 minutes. However, if X becomes bigger than or equal to 3, it stays equal to 3. Practically, X follows a Poisson distribution for $x \leq 3$, but is equal to 3 whenever $x > 3$. In essence, you are asked to compute the pmf and the cdf of $Y = \min\{X, 3\}$. Note that the values that Y is allowed to take are 0, 1, 2, and 3. [6]

[6] As a hint, we are providing a way to start the problem in the answer box.

Answer to Problem 11.

$$P(Y = y) = \begin{cases} P(Y = 0) = P(X = 0) \\ P(Y = 1) = P(X = 1) \\ P(Y = 2) = P(X = 2) \\ P(Y = 3) = P(X \geq 3) \end{cases}$$

Problem 12

Similarly, consider the opposite problem in which discrete random variable X is Poisson distributed with rate $\lambda = 2$, but it now is equal to 3 whenever $x \leq 3$. In essence, you are asked to compute the pmf and the cdf of $Y = \max\{X, 3\}$.

Answer to Problem 12.