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| **Course Code** | **:** | **MCSE-004** |
| **Course Title** | **:** | **Numerical and Statistical Computing** |
| **Assignment Number** | **:** | **MCA(V)-E004/Assignment/2018-19** |
| **Maximum Marks** | **:** | **100** |
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**Question 1: Obtain the positive root of the equation $x^2 - 1 = 0$ by Regular Falsi method.**

Ans.

Let $f(x) = x^2 - 1$ .

Since $f(0) = 0^2 - 1 = -1, f(2) = 2^2 - 1 = 4 - 1 = 3$

Let us take that that root lies in (0,2).We have $x_0 = 0$ ,$x_1 = 2$.

Then ,using (2),we get

$$X_2 = \frac{x_0 f(2) - x_1 f(0)}{f(2) - f(0)}$$

$$= \frac{0-2(-1)}{3+1} = 0.5$$

$f(0.5) = -0.75$

The root lies in (0.5,2.0) we get

$$X_3 = \frac{0.5f(2) - 2.0f(0.5)}{f(2) - f(0.5)}$$

$$= \frac{0.5(3)-2.0(-0.75)}{3 + 0.75} = 0.8$$

$f(0.8) = -0.36$

The root lies in (0.8,2) The next approximation

$$X_4 = \frac{0.8f(3) - 2.0f(-0.36)}{3 + 0.36}$$

$= 0.9286$

$f(0.9286) = -0.1377$

We obtaion the next approximation as $x_5 = 0.9757$, $x_6 = 0.9973$, $x_7 = 0.9973$, $x_8 = 0.9990$.Since $|x_8 - x_7| = 0.0017 < 0.005$,the approximation **$x_8$ =0.9990 is correct to decimal places.**

**Question 2: Apply Gauss – Eliminaiton method to solve the following sets of equation**
**$x + 4y - z = -5$ ; $x + y - 6z = -12$ ; $3x - y - z = -4$**

Ans.

Ans 2 Gauss Elimination method

$$x + 4y - z = -5$$
$$x + y - 6z = -12$$
$$3x - y - z = -4$$

In augmented form we write this system as

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -5 \\ 1 & 1 & -6 & -12 \\ 3 & -1 & -1 & -4 \end{array}\right]$$ subtracting third row from row second

$$\left[\begin{array}{ccc|c} 1 & 4 & -1 & -5 \\ 1 & 1 & -6 & -12 \\ -2 & 2 & -5 & -8 \end{array}\right]$$ 2 times in first row and subtract from second row

$$\left[\begin{array}{ccc|c} 1 & 4 & -1 & -5 \\ 0 & 3 & 5 & -12 \\ -2 & 2 & -5 & -4 \end{array}\right]$$ add 3rd row from 2nd row



$$\left[\begin{array}{ccc|c} 1 & 4 & -1 & -5 \\ 0 & 3 & 5 & -12 \\ -2 & 2 & 5 & 4 \end{array}\right]$$ subtracting 3rd row form second row

$$\left[\begin{array}{ccc|c} 1 & 4 & 1 & -5 \\ 0 & 3 & 5 & -12 \\ -1 & -1 & -1 & -9 \end{array}\right]$$ add third ~~of~~ & second row

$$\left[\begin{array}{ccc|c} 0 & 4 & -1 & -5 \\ 0 & 3 & 5 & -12 \\ 0 & 3 & -2 & 3 \end{array}\right]$$

solving equation

$$x_3 = +\frac{2}{3}$$
$$x_2 = +\frac{12}{5}$$
$$x_1 = -5 + \frac{2}{3} + \frac{12}{5}$$
$$x_1 = \frac{-75 + 10 + 36}{15}$$
$$x_1 = \frac{-29}{15}$$

**Question 3: Use method of Lagrange's interpolation to find f(0.16), for Given function f( x) = sin(x) where f (0.1) = 0.09983,**
**f (0.2) =0.19867. Also, Find error in f (0.16).**

Ans.

Using the Lagrange's interpolation formula,we obtain

$$F(0.16) = \frac{0.16 - 0.2}{0.1 - 0.2}(0.09983) + \frac{0.16 - 0.1}{0.2 - 0.1}(0.19867)$$

= 0.039932 + 0.119202

=0.159134

The Error in the Lagrange linear interpolation formula is given by

= (0.00125)(0.19867)

=0.00025

**Question 4: Evaluate $\int_0^1 \frac{dx}{1+x}$ Use Gauss-Legendre three point formula.**

Ans.

**Solution:** First we transform the interval [0, 1] to the interval [−1, 1].

Let $t = a x + b$. We have,

For $x = 0$ : $\quad -1 = b$

For $x = 1$ : $\quad 1 = a + b = a - 1$, or $a = 2$

Therefore, $t = 2x - 1$, or $x = (1 + t)/2$.



Hence, $I = \int_0^1 \frac{dx}{1+x} = \int_{-1}^1 \frac{dt}{t+3} = \int_{-1}^1 F(t)\,dt$

Guass – Lengendre three point formula gives

$$I = \frac{1}{9}\left[5F(-\sqrt{0.6}) + 8F(0) + 5F(\sqrt{0.6})\right]$$

$$\frac{1}{9}\left[5\left\{\frac{1}{3-\sqrt{0.6}} + \frac{.1}{3+0.6}\right\} + \frac{8}{3}\right] = 0.693122$$

The exact solution is $I = In2 = 0.693147$

**Question 5: Find Newtons Forward difference interpolating polynomial for the following data :**

| X | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
|---|---|---|---|---|---|
| f (x) | 1.40 | 1.56 | 1.76 | 2.00 | 2.28 |

Ans.



Ans 5

Newton forword difference

| $x$ | $f(x)$ | $\Delta f$ | $\Delta^2 f$ | $\Delta^3 f$ | $\Delta^4 f$ |
|---|---|---|---|---|---|
| 0.1 | 1.40 | | | | |
| 0.2 | 1.56 | 0.16 | 0.4 | | |
| 0.3 | 1.76 | 0.20 | | 0 | |
| 0.4 | 2.00 | 0.24 | 0.4 | | 0 |
| 0.5 | 2.28 | 0.28 | 0.4 | 0 | |

Last three value of f(x) for $x = 0.3 + 0.4 + 0.5$ are taken to consideration so that $= \frac{1.2}{2} = 0.6$

Hence, $a = 0.3,\ h = 1,\ x = 0.6$

$\therefore\ a + hu = 0.6$

$0.3 + 1 \times u = 0.6$ or $u = 0.3$



$$f(0.6) = f(0.3) + u\,\Delta f(0.3) + \frac{u(u-1)}{2!}\Delta^2 f(0.3) + \frac{u(u-1)(u-2)}{3!}\Delta^3 f + (0.3)$$

$$\Rightarrow 1.76 + 0.3 \times 1.76 + \frac{0.3(0.3-1)}{2!} + 0.4 + \frac{0.3(0.3-1)(0.3-2)}{3!} + 0.4$$

$$\Rightarrow 1.76 + 2.06 + (-0.0573) + 0.4 + (-0.1152) + 0.4$$

$$\Rightarrow 1.76 + 2.06 + (-0.0573) + 0.4 + (-0.1152) + 0.4$$

$\Rightarrow 2.6875$ solved

**Question 6: Calculate the value of integral $\int_0^6 \frac{dx}{1+x^2}$ by**
**(i) Simpson's 1/3 rule.**

**(ii) Simpson's 3/8 rule.**
Ans.

Divide the interval (0, 6) into six parts each of width $h = 1$.
The values of $f(x) = \frac{1}{1+x^2}$
are given below:
**Solution:-**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| X: | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| f(x) | 1 | 0.5 | 0.2 | 0.1 | 1/17 | 1/26 | 1/37 |
| | y0 | y1 | y2 | y3 | y4 | y5 | y6 |

(*i*) **By Simpson's one-third rule,**

$$\int_0^6 \frac{dx}{1+x^2} = \frac{h}{3}\ [(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)]$$

$$= \frac{1}{3}\left[\left(1+\frac{1}{37}\right)+4\left(0.5+0.1+\frac{1}{26}\right)+2\left(0.2+\frac{1}{17}\right)\right]$$

$$= 1.366173413.$$

(*ii*) **By Simpson's three-eighth rule,**

$$\int_0^6 \frac{dx}{1+x^2} = \frac{3h}{8}\ [(y_0 + y_6) + 3(y_1 + y_2 + y_4 + y_5) + 2y_3]$$

$$= \frac{3}{8}\left[\left(1+\frac{1}{37}\right)+3\left(.5+.2+\frac{1}{17}+\frac{1}{26}\right)+2(.1)\right]$$

$$= 1.357080836.$$

**Question 7: Given $dy/dx$ = y - x, where y(0) = 2. Find y(0.1) and y(0.2), correct to four decimal places, using Runge-Kutta Second Order method.**

Ans.

*To find y(0.1)*

Here $\quad y' = f(x, y) = y - x, x_0 = 0, y_0 = 2$ and $h = 0.1$

Now, $\quad k_1 = hf(x_0, y_0) = 0.1(2 - 0) = 0.2$

$\quad k_2 = hf(x_0 + h, y_0 + k_1) = 0.21$

$\therefore \quad \Delta y = \frac{1}{2}(k_1 + k_2) = 0.205$

Thus, $\quad x_1 = x_0 + h = 0.1$ and $y_1 = y_0 + \Delta y = 2.205$

*To find y(0.2)* we note that,

$$x_1 = 0.1, y_1 = 2.205, h = 0.1$$

For interval II, we have

$$k_1 = hf(x_1, y_1) = 0.2105$$

$$k_2 = hf(x_1 + h, y_1 + k_1) = 0.22155$$

$\therefore \quad \Delta y = \frac{1}{2}(k_1 + k_2) = 0.216025$

Thus, $\quad x_2 = x_1 + h = 0.2$ and $y_2 = y_1 + \Delta y = 2.4210$

Hence $\quad y(0.1) = 2.205, \quad y(0.2) = 2.421.$

**Question 8: A farmer buys a quantity of cabbage seeds from a company that claims that approximately 90% of the seeds will germinate if planted properly. If four seeds are planted, what is the probability that exactly two will germinate?**

Ans.

This situation follows the binomial distribution with n = 4 and p = 90/100 = 9/10.The random variable X is the number of seeds that generate. We have to calculate the probability that exactly two of the four seeds will germinate. That is P[X = 2].

By applying Binomial Formula ,we get

$P[X = 2] = {}^4C_2 * (9/10)^2 * (1/10)^2$

= 6 * (81/100) * (1/100)

=486/10000

= 0.0486

**So ,the required probability is 0.0486**

**Question 9: Suppose that the amount of time one spends in a bank to withdraw cash from an evening counter is exponentially distributed with mean ten minutes, that is λ = 1/10. What is the probability that the customer will spend more than 15 minutes in the counter?**

Ans.

If X represents the amount of time that the customer spends in the counter than we need to find P(X>15).Therfore,

$P(X>15) = \int_{15}^{\infty} e^{-15} \lambda$

$= \mathbf{e^{3/2}}$

$= 0.223$

P(X>15) = 0.223 represents that there is a 22.3% chance that the customer has th wait more than 15 minutes.

**Question 10: Fit a straight line to the following data by the method of least square.**

| | | | | | |
|---|---|---|---|---|---|
| **X** | **0** | **1** | **2** | **3** | **4** |
| **Y** | **1** | **1.8** | **3.3** | **4.5** | **6.3** |

Ans.

Ans 10

| $x$ | $y$ | $xy$ | $x^2$ | $\hat{y}-2y$ | $y-y$ |
|---|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 11 | -10 |
| 1 | 1.8 | 1.8 | 1 | 13 | -11.2 |
| 2 | 3.3 | 6.6 | 4 | 15 | -11.7 |
| 3 | 4.5 | 13.5 | 9 | 17 | -12.5 |
| 4 | 6.3 | 25.2 | 16 | 19 | -12.7 |
| $\Sigma x = 10$ | 16.0 | 47.1 | 30 | 75 | |

The equation of least sequare line $y = a + bx$

normal equation for 'a' $\Sigma y = \cancel{4a + 10b}$

$na + b\Sigma x = 4a + 10b$ — (1)

Normal equation for 'b' $\Sigma xy$

$= a\Sigma x + b\Sigma x^2$ $30 \Rightarrow 10a + 30b$ — (2)

$a = 9$, $b = 2$

The equation of the least sequre line leanor

$y = 9 + 2$

**Question 11: Compute the approximate derivatives of $f(x) = x^2$ at $x = 0.5$ for the increasing value of h from 0.01 to 0.03 with a step size of 0.005 using :**
**(i) first order forward difference model**
**(ii) first order backward difference model.**

Ans.

(i) First order forward difference model:-

$$\frac{f[x+h - f[x-h]}{2h}$$



$\Rightarrow$ $x = 0.5$ $h = 0.005$

$$\frac{f[0.5+0.005] - f[0.5-0.005]}{2 \times 0.005}$$

$$\Rightarrow \frac{0.505 - 0.495}{0.10} = \frac{0.010}{0.010} = 0.1$$

(ii) Backward Difference model:-

$$\frac{f'(x) - f[x-h]}{0.1 - 0.495} \Rightarrow 0.395$$

**Question 12: Find the root of the equation $x^3 - x - 1 = 0$ lying between 1 and 2 by Bisection method.**

Here, $f(x) = x^3 - x - 1$

Since $f(1.324) = -.00306$ i.e., (–)ve

$f(1.325) = .00120$ i.e., (+)ve

Hence, the root lies between 1.324 and 1.325.

∴ First approximation to the root is

$$x_1 = \frac{1.324 + 1.325}{2} = 1.3245$$

Now $f(x_1) = -.000929$ i.e., (–)ve

Hence, the root lies between 1.3245 and 1.325

∴ Second approximation to the root is

$$x_2 = \frac{1.3245 + 1.325}{2} = 1.32475$$

Now $f(x_2) = .000136$ i.e., (+)ve

Hence, the root lies between 1.3245 and 1.32475.

Third approximation to the root is

$$x_3 = \frac{1.3245 + 1.32475}{2} = 1.324625$$

Now $f(x_3) = -.000396$ i.e., (–)ve

Hence, the root lies between 1.324625 and 1.32475.

∴ Fourth approximation to the root is

$$x_4 = \frac{1.324625 + 1.32475}{2} = 1.3246875$$

Now $f(x_4) = -.0001298$ i.e., (–)ve

Hence, the root lies between 1.3246875 and 1.32475

∴ Fifth approximation to the root is

$$x_5 = \frac{1.3246875 + 1.32475}{2} = 1.32471875$$

Hence, the real root of the given equation is **1.324** correct to three decimal places after computing five iterations.

**Question 13: A problem in statistics is given to the three students A, B and C, whose chances of solving it are ½, ¾, and ¼ respectively. What is the probability that the problem will be solved?**

Ans.

Ans 13 Let A, B, C be the respective event of solving the problem and $\bar{A}, \bar{B}, \bar{C}$ be the respective event of not solving the problem

Then A, B, C. are independent event.

$\therefore \bar{A}, \bar{B}, \bar{C}$ are independent events

Now, $P(\bar{A}) = \frac{1}{2}$

, $P(\bar{B}) = \frac{3}{4}$

, $P(\bar{C}) = \frac{1}{4}$



$\therefore$ P( non solves the problem) = P(not A) and (not B) and (not C)

$= P(\bar{A} \cap \bar{B} \cap \bar{C})$

$= P(\bar{A})\, P(\bar{B})\, P(\bar{C})$ [$\because \bar{A}, \bar{B}, \bar{C}$ and independent]

$= \frac{1}{2} \times \frac{3}{4} \times \frac{1}{4}$

$\Rightarrow \frac{3}{16}$

Hence, P (the problem will be solved)

= 1 - P(non, solves the problem)

$= 1 - \frac{3}{16}$

$= \frac{16-3}{16} \Rightarrow \frac{13}{16}$ solved

**Question 14: a partially destroyed laboratory, the record of an analysis of correlation data, the following results are legible :**
**Variance of X = 9 Regression equations:**
**8X - 10Y + 66 = 0 ; 40X - 18Y - 214 = 0**
**Find: (i) The mean values of X and Y (ii) The correlation coefficient between X and Y (iii) Standard deviation of Y**

Ans.

1) Since both the regression lines pass through the point ($\bar{x}$, $\bar{y}$), we have

$$8\bar{x} - 10\bar{y} + 66 = 0$$
$$40\bar{x} - 18\bar{y} - 214 = 0$$

Solving we get, $\bar{x} = 13$
$\bar{y} = 17.$

Let $8x - 10y + 66 = 0$ and $40x - 18y - 214 = 0$

Be the lines of regression of $y$ and $x$ and $x$ on $y$ respectively. Now, we put them in the following form.

$$= \frac{8}{10}x + \frac{66}{10} \text{ and } x = \frac{18}{40}y + \frac{214}{40} \qquad (4)$$

$\therefore$ byx = regression coeff of $y$ on $x = \frac{8}{10} = \frac{4}{5}$

bxy = regression coeff of $x$ on $y = \frac{18}{40} = \frac{9}{20}$

Hence, $r^2$ = bxy. by$x = \frac{4}{5} \; \frac{9}{20} = \frac{9}{25}$

So $r = \pm \frac{3}{5} = \pm 0.6$ **www.ignousite.blogspot.com**

Since, both the regression coeff are +ve, we take $r = +0.6$

3) We have, byx $= r\frac{\sigma_y}{\sigma_x} \Rightarrow \frac{4}{5} = \frac{3}{5} \; x \; \frac{\sigma_y}{3}$

$\therefore \; \sigma_y = 4$

Remarks (i) had we taken $8x - 10y + 66 = 0$ as regression equation of $x$ on $y$ and $40x - 18y = 214$, as regression equation of $y$ on $x$.

Then bxy $= \frac{10}{8}$ and byx $= \frac{40}{18}$

or $r^2$ = bxy byx $= \frac{10}{8} \times \frac{40}{18} = 2.78$

so $r = \pm 1.66$

Which is wrong as r lies between $\pm$ 1.

**Question15: An individual's IQ score has a Normal distribution N (100,152). Find the probability that an individual IQ score is between 91 and121.**

Ans.

**Solution**: We require P[91 < X < 121]. Standardising gives

$$P\left[\frac{91-100}{15} < \frac{X-100}{15} < \frac{121-100}{15}\right]$$

The middle term is standardised normal random variable and so we have,

$$P\left[\frac{-9}{15} < Z < \frac{21}{15}\right] = P[-0.6 < Z < 1.4] = 0.9192 - 0.2743 = 0.6449 .$$

**Question16: What do you mean by term "Goodness to fit test"? What for the said test is required?**

Ans. We have seen in the previous subsection that the regression line provides estimates of the dependent variable for a given value of the independent variable. The regression line is called the best fitted line in the sense of minimizing the sum of squared errors. The best fitted line shows the relationship between the independent (x) and dependent (y) variables better than any other line. Naturally the question arises "How good is our best fitted line?". We want a measure of this goodness of fit. More precisely we want to have a numerical value which measures this goodness of fit.For developing a measure of goodness of fit, we first examine the variation in *y*. Let us first try the variation in the response *y*. Since *y* depends on *x*, if we change *x*, then *y* also changes. In other words, a part of variation in *y*'s is accounted by the variation in *x*'s. Actually, we can mathematically show that the total variation in y's can be split up as follows:

$$S_{yy} = \sum_{i=1}^{n}(y_i - \bar{y})^2 = \frac{S^2{}_{xy}}{S_{xx}} + \sum_{i=1}^{n}(y_i - \hat{y}_i)^2,$$

where

$$S_{xx} = \sum_{i=1}^{n}(x_i - \bar{x})^2; S_{xy} = \sum_{i=1}^{n}(x_i - \bar{x})(y_i - \hat{y}_i)$$

Now if we divide (12) by $S_{yy}$ on both sides, we get

$$1 = \frac{S^2{}_{xy}}{S_{xx}S_{yy}} + \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{S_{yy}}$$

Since the quantities on the right hand side are both non-negative, none of them can exceed one. Also if one of them is closer to zero the other one has to be closer to one. Thus if we denote

$$R = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$$



then

$$R^2 = \frac{S^2{}_{xy}}{S_{xx}S_{yy}}$$

Since $R^2$ must be between 0 and 1, R must be between – 1 and 1. It is clear that if $R^2 = 1$, then

$$\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{S_{yy}} = 0 \text{ or } \sum_{i=1}^{n}(y_i - \hat{y}_i)^2 = 0 \quad \text{ or } \quad y_i = \hat{y}_i \quad \text{ for all } i.$$

Again when $R^2$ is close to 1, $\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$ is close to zero. When $R$ is negative, it means that $y$ decreases as $x$ increase and when $R$ is positive $y$ increases when $x$ increases. Thus R gives a measure of strength of the relationship between the variables $x$ and $y$.

Now let us compute the value of $R$ for Example 1. For calculating the numerical value of $R$, the following formula can be used;

$$R = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{\sum_{i=1}^{n}(x_i - x)(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}} = \frac{\sum_{i=1}^{n}x_i y_i - n\bar{x}\,\bar{y}}{\sqrt{\sum_{i=1}^{n}x_i^2 - n\bar{x}^2}\sqrt{\sum_{i=1}^{n}y_i^2 - n\bar{y}^2}}$$

Therefore, for Example 1, the value of R becomes;

$$\text{R} = \frac{101,570 - 10 \times 145 \times 67.3}{\sqrt{218,500 - 10 \times 145^2}\sqrt{47225 - 10 \times 67.3^2}} = \frac{3985}{\sqrt{8250}\sqrt{1932.1}} = 0.9981$$

and $\quad R^2 = 0.9963.$

Therefore, it is clear from the value of $R$ or from $R^2$ that both of them are very close to one. From the figure also it is clear that the predicted line fits the data very well.

Moreover $R$ is positive means, there is a positive relation between the temperature and yield. As the temperature increases the yield also increases.

Now the natural question is how large this $R$ or $R^2$ will be to say that the fit is very good. There is a formal statistical test based on $F$-distribution which can be used to test whether $R^2$ is significantly large or not. We are not going into that details. But as a thumb rule we can say that if $R^2$ is greater that 0.9, the fit is very good, if it is between 0.6 to 0.8, the fit is moderate and if it is less than 0.5 it is not good.