RGR MS

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Analyzing growth of individuals is fundamental in many areas of ecology and biology. A common situation is the need to intercompare multiple individuals across genotypes or species in experimental or observational settings where variations in initial sizes and environmental factors both contribute to observed variations in size at any point in time. In this setting, a common default practice is to re-express growth as a relative measure, dividing the growth increment by the initial size. In the limit as the time period goes to zero, this can be represented as

Without explicit specification of a time-varying dynamic, e.g. some kind of non-linear growth function, this representation corresponds to exponential growth. That is, the quantity obtained by integration of

over some time period, and given some initial size

is simply the familiar exponential equation

The quantity in equation 1 is often referred to as relative growth rate (RGR), and the usual method of quantification, hereafter the “log measure” corresponds to the solution in 2, as is readily checked. The log measure is, simply

The log measure is very frequently utilized as a default in place of taking the simply difference . XXX et al. (2012) summarized several flaws of the log measure and recommended instead to fit non-linear growth functions. I wholeheartedly concur with this advice. However, ecologists are often confronted with datasets where only 2 or 3 time periods are available, thus precluding effective fitting of non-linear functions.

In this note, I demonstrate that the log measure should also not be used in the data limited setting of only 2 or 3 observation times. Instead, the simply linear difference is to be preferred for three reasons: 1) simplicity of interpretation, 2) ecological/biological validity, and 3) ease of use.

First, let us consider a simple theoretical framework for growth: the sigmoidal curve. We will compare the linear difference and the log measure to the a sigmoidal curve, presumed to represent a far superior approximation to the underlying biological/ecological reality.

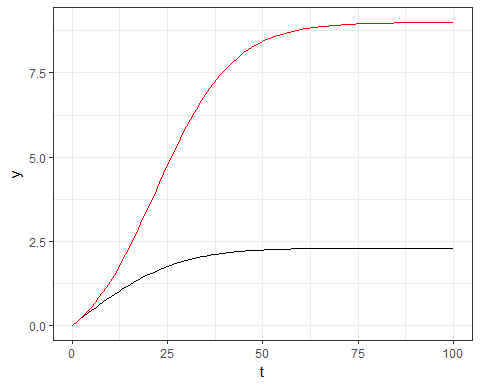
The logistic equation, a reasonable option for representing the universal sigmoidal growth of organisms:

All well and good. What happends if we utilize the log measure , as practiced when calculating RGR using the canonical approach? We can analytically express the result as:

If we have size observations and from two times, and ,the difference between them is:

If we simplify, and set , re-expressing , we have:

Thus, if we theoretically make error-free observations over time of a plant, or any other organism for that matter, following sigmoidal growth function above, and re-express our growth increment as the log of the size at time minus log of initial size, our observations will follow equation X above. Graphically, this curve:



Thus, a good deal of the potential for detecting **growth** is washed out of the measure, particularly of course where the sigmoidal curve flips from convex to concave. This is not theoretically surprising, and, as noted above, we concur with the general recommendation to always fit a more suitable non-linear growth model where a time series of data is available.

From a theoretical point of view, in fact, it makes just as much sense to exponentiate the observed size values and , . On the face of it, this seems absurd. Certainly, unlike the log measure, this does not correspond to any kind of potential growth dynamic. But the effect here is to linearize the concave portion of the sigmoidal curve, exactly analogous to linearizing the convex portion of the curve with the log measure.

The practical point here is that use of the log-transformed measure should *not* be routine in analyses of ecological growth data, outside of the context where exponential growth actually makes sense *a priori*.

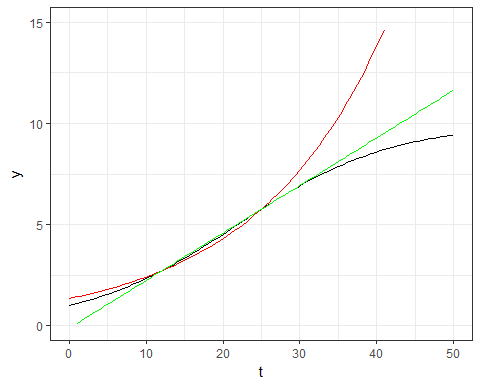
### Approximation error of linear differencing versus log measure

Now, use of the linear difference corresponds to assumption of a static linear growth rate dynamic, just as use of corresponds to assuming a constant exponential growth rate dynamic. In the latter case, the log-measure has the nice property of representing an ergodic observable (sense Peters and Gell-man 2016). While widely (and rightly) dismissed as unrealistic, the linear dynamic may in fact be a generally superior measure for ecological analysis where no time series of size/biomass data is available.

As can be seen, the linear model is a far better approximation where data are taken from within the center part of the growth cycle.

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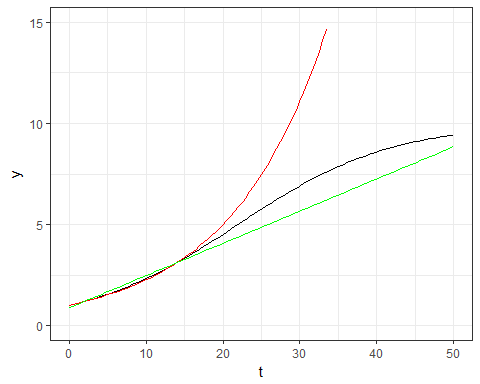
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Even where collection of data is exclusively from the convex portion of the growth curve, the superior fit of the exponential model provides only a marginal gain in the convex portion of the curve, and then in absolute value, quickly diverges thereafter.

t2 <- 14  
t1 <- 2  
  
  
ggplot(data = data.frame(t=c(0,50)),aes(x=t)) + stat\_function(fun = sigmoid2, args = list(Po,K,r)) + stat\_function(fun = sigmoid\_exp\_approx, args = list(t1,t2,Po,K,r), color = "red") +  
 stat\_function(fun = sigmoid\_lin\_approx, args = list(t1,t2,Po,K,r),color = "green") +  
 theme\_bw() + scale\_y\_continuous(limits = c(0,15))

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In summary, the chief virtue of the measure is that it effectively linearizes the differences in size from the convex portion of biological growth curve. Thus, it arguably might increase the ability to discern subtle but consequential differences in growth rates in experiments or observations. However, this strength is also a liability - given process and measurement error, I suspect it inflates the odds of infering patterns that do not exist.

The much maligned linear measure is a superior default on two grounds therefore. First, it corresponds far more directly with current ecological reality. It is a measure with an interpretable biological dimension (usually mass or length) that helps us understand and describe our system. Interpreted as a dynamic, it is obviously flawed, but is demonstrably better in e.g. MSE than the exponential dynamic.

The widespread use of “RGR” should be abandoned. Where only two or three time points are available, fitting a linear growth trend is just as good if not better than estimating an exponential growth rate. The ideal scenario is to collect a proper time series (5-7+) and fit a proper growth model. Where data are at all limiting, we recommend careful incorporation of literature values and other external information as priors in a fully Bayesian analysis in order to regularize inferences.