Oct 26, 2020 (Due: 08:00 Nov 2, 2020)

1. Pick some smooth bivariate functions $z = f(x, y)$ over $[0, 1] \times [0, 1]$. By discretizing the functions (i.e., sampling over uniform mesh), we obtain matrices whose entries are $f(x_i, y_j)$'s. Visualize the singular values of these matrices.

2. What happens if GMRES breaks down (i.e., the bottom-right entry of H_k becomes zero)? Justify your claim.

3. Suppose that we are using the GMRES method to solve the linear system $Ax = b$, where

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 6 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 0 \end{bmatrix}, \qquad b = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

If a zero initial guess is used, what can you say about the convergence?

4. Implement the Arnoldi procedure. Test it with some matrices and starting vectors. Visualize the orthogonality of the Arnoldi vectors.

5. Implement your own GMRES solver. Test it with at least two systems of linear equations (for symmetric and nonsymmetric coefficient matrices) with 1000 unknowns and plot the residual history.

6. (optional) Test GMRES and FOM with some examples and compare the convergence rate.

7. (optional) Use truncated SVD to compress some grayscale images. If you only have colored images, you can convert them to grayscale using

$$\text{gray} = \alpha \cdot \text{red} + \beta \cdot \text{green} + \gamma \cdot \text{blue}.$$

Common choices of the constants are $(\alpha, \beta, \gamma) = (0.299, 0.587, 0.114)$ and $(\alpha, \beta, \gamma) = (0.2126, 0.7152, 0.0722)$.