Sep 14, 2020 (Due: 08:00 Sep 21, 2020)

1. Suppose that $A \in \mathbb{R}^{m\times n}$ and $x \in \mathbb{R}^n$ are matrices already stored in floating-point format. There exists $E \in \mathbb{R}^{m\times n}$ such that

$$\mathrm{fl}(Ax) = (A + E)x,$$

under the assumption no overflow or (gradual) underflow occurs in the calculation. Try to give a tight estimate on E, i.e., find a "small" constant θ such that

$$|e_{ij}| \le \theta |a_{ij}| \boldsymbol{u}$$

holds for all indices $1 \le i \le m$ and $1 \le j \le n$.

2. Let $A \in \mathbb{C}^{n\times n}$ be nonsingular. Show that there exist nonsingular lower triangular matrices L_1 and L_2 such that $A = L_1 L_2^\top$ if and only if all leading principal minors of A are nonzero.

3. Let $A \in \mathbb{C}^{n\times n}$ be strictly diagonally dominant, i.e.,

$$|a_{ii}| > \sum_{\substack{1\le j\le n\\ j\ne i}} |a_{ij}|$$

for $1 \le i \le n$. Show that after one step of Gaussian elimination, the $(n-1)\times(n-1)$ trailing principal submatrix is still strictly diagonally dominant.

4. Suppose that you have a BLAS library with an inefficient `SSYMM` implementation. Try to design an efficient `SSYMM` subroutine by making use of `SGEMM`. You do not have to really implement it. Just describe your algorithm.

5. (optional) Try to evaluate the infinite series $\sum_{n=1}^{\infty} 1/n$ in single precision floating-point arithmetic. What do you observe?