

Sep 21, 2020 (Due: 08:00 Sep 28, 2020)

1. Let $v \in \mathbb{C}^n$ be a unit vector. Find all eigenvalues of $I - 2vv^*$.
2. Let $A \in \mathbb{C}^{n \times n}$ be nonsingular. Show that there exists a unitary matrix Q such that $R = AQ$ is upper triangular. Design an algorithm to compute such a factorization.
3. Assume that $b, \delta b, x, \delta x$ satisfy

$$Ax = b, \quad A(x + \delta x) = b + \delta b,$$

where

$$A = \begin{bmatrix} 375 & 374 \\ 752 & 750 \end{bmatrix}.$$

Construct examples such that

- (1) $\|\delta b\|_\infty / \|b\|_\infty$ is very small while $\|\delta x\|_\infty / \|x\|_\infty$ is very large;
 - (2) $\|\delta b\|_\infty / \|b\|_\infty$ is very large while $\|\delta x\|_\infty / \|x\|_\infty$ is very small.
4. Implement a linear-time (i.e., $O(n)$ complexity) linear system solver for solving a diagonally dominant tridiagonal linear system.
 5. (optional) Implement LU factorization with
 - (1) partial pivoting;
 - (2) complete pivoting.Compare the performance and numerical stability for some randomly generated 1000×1000 real matrices.
 6. (optional) Implement Householder triangularization for a general matrix $A \in \mathbb{C}^{m \times n}$ with $m \geq n$.
 7. (optional) Suppose that you are given a strange linear algebra package with no subroutines for matrix–matrix multiplications. But it contains a linear system solver that is capable for solving multiple right-hand sides. Can you make use of the linear system solver to implement GEMM?