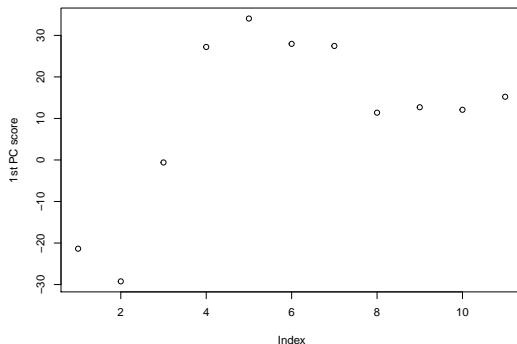


Preprocessing

- Remove outliers by Savitzky-Golay smoothing filter on each band and index.
- Functional normalize three indices: NDVI, MNDWI, SWIR by univariate FPCA estimation.

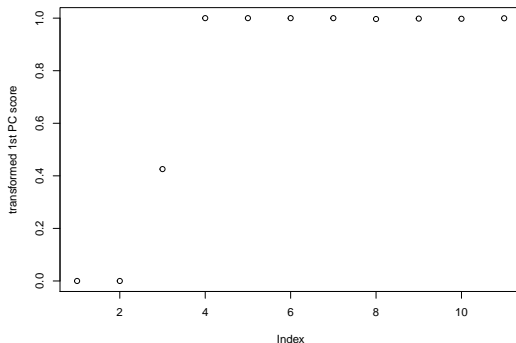
Multivariate FPCA

Consider 30 principle components and Epan kernel. Then select ξ_{ik} only between 2001 and 2011 as the detection set: $\hat{\xi}_{ik}$, $\hat{\lambda}_k$ and $i = 1, 2, \dots, N = 11$. $\text{id} = 7$



Sigmoid function

Apply a sigmoid function $1/\{1 + \exp(-\beta x)\}$ to the estimated $\tilde{\xi}_{ik}$, $\beta = 0.5$.



Detection

For each of the first three principle components ($k = 1, 2, 3$):

- Compute

$$T_N^k(x) = \frac{1}{N} \left(\sum_{1 \leq i \leq N_x} \tilde{\xi}_{ik} - x \sum_{i=1}^N \tilde{\xi}_{ik} \right)^2$$

for $0 \leq x \leq 1$.

- Then estimate start and end of the urbanization

$$\hat{P}_1^k = \lfloor N \times \min\{x : T_N^k(x) = \max_{0 \leq y \leq 1} T_N^k(y)\} \rfloor$$

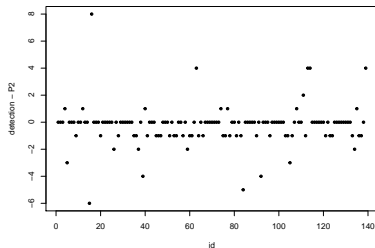
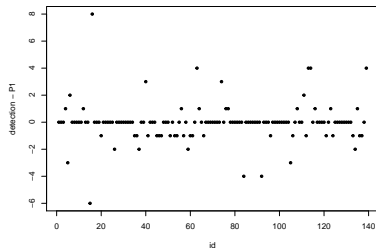
$$\hat{P}_2^k = \lceil N \times \max\{x : T_N^k(x) = \max_{0 \leq y \leq 1} T_N^k(y)\} \rceil$$

Decide the changing type by Δ_k be the $|\hat{\xi}_{\hat{P}_1^k, k} - \max(\hat{\xi}_{\hat{P}_2^k, k}, \hat{\xi}_{\hat{P}_2^k+1, k})|$:

$$\hat{P}_1 = \sum_{k=1}^3 \hat{P}_1^k I\{\Delta_k = \max_{1 \leq k \leq 3} \{\Delta_k\}\}, \hat{P}_2 = \sum_{k=1}^3 \hat{P}_2^k I\{\Delta_k = \max_{1 \leq k \leq 3} \{\Delta_k\}\}.$$

Results

of correct detection for P_1 : 121/139, and # of correct detection for P_2 : 123/139.



pointID = 116

$$P_2 = 2005, \hat{P}_2 = 2009$$

