$\boxed{2\text{-}3}$

$$\text{posterior} = \frac{\text{likelihood} \times \text{Prior}}{\text{margin}}$$

likelihood : Binomial

Prior : Beta

$$\because \text{margin} = \int_0^1 \beta(\theta, m+a-1, N-m+b-1)\, d\theta$$

$$= \int_0^1 \theta^{m+a-1} (1-\theta)^{N-m+b-1} \frac{\Gamma(a+N+b)}{\Gamma(m+a)\Gamma(N-m+b)} d\theta$$

$$= \frac{\Gamma(a+N+b)}{\Gamma(m+a)\Gamma(N-m+b)} \cdot \int_0^1 \theta^{m+a-1} (1-\theta)^{N-m+b-1} d\theta = 1$$

推出 $\Rightarrow \int_0^1 \theta^{m+a-1} \cdot (1-\theta)^{N-m+b-1} d\theta = \frac{\Gamma(m+a)\Gamma(N-m+b)}{\Gamma(a+N+b)}$

$$p(\theta, \text{event}) = \frac{\cancel{C^N_m}\, p^m (1-p)^{N-m}\, p^{a-1} (1-p)^{b-1} \cancel{\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}}}{\int_0^1 \cancel{C^N_m}\, \theta^m (1-\theta)^{N-m}\, \theta^{a-1} (1-\theta)^{b-1} \cancel{\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}} d\theta}$$

$$= \frac{p^{m+a-1} (1-p)^{N-m+b-1}}{\int_0^1 \theta^{m+a-1} (1-\theta)^{N-m+b-1} d\theta} = \frac{p^{m+a-1} (1-p)^{N-m+b-1}}{\frac{\Gamma(m+a)\Gamma(N-m+b)}{\Gamma(a+N+b)}}$$

轉換

$$= \frac{\Gamma(a+N+b)}{\Gamma(m+a)\Gamma(N-m+b)} \cdot p^{m+a-1} \cdot (1-p)^{N-m+b-1}$$

證明 posterior 也是 beta distribution

$$= \beta(m+a, N-m+b)$$

故求posterior 只需使 $\begin{cases} a+m-1 \\ b+N-m-1 \end{cases}$