SAARLAND UNIVERSITY ARTIFICIAL INTELLIGENCE
Dr. Álvaro Torralba, Prof. Wolfgang Wahlster
Dr. Cosmina Croitoru, Daniel Gnad, Marcel Steinmetz
Yannick Körber, Michael Barz
Christian Bohnenberger, Sören Bund-Becker, Sophian Guidara,
Alexander Rath, Khansa Rekik, Julia Wichlacz, Anna Wilhelm

Exercise Sheet 4.

Solutions due Tuesday, **May 29**, 16:00 – 16:15, in the lecture hall.[1]

Exercise 14: CNF. (2 Points)

Transform the following formulas to CNF. To do so, follow the steps from the lecture (Chapter 9 slide 21) specifying which steps are you applying and giving the intermediate results.

Note: Simplify the resulting formulas where possible.

(a) $(P \leftrightarrow R) \rightarrow (Q \leftrightarrow R)$

(b) $(P \vee Q) \rightarrow (P \leftrightarrow \neg R)$

Exercise 15: Resolution. (3 Points)

For the two formulas below, use resolution to prove that the formulas are unsatisfiable. To do so, first **give a set of clauses Δ that is equivalent to the formula** and second, **use resolution to prove that it is unsatisfiable**. Write the resolution process in the form of a tree for easier readability.

(a) $\phi_1 = (\neg P \vee Q \vee R) \wedge (\neg P \vee \neg Q) \wedge (\neg P \vee \neg S) \wedge (\neg R \vee S) \wedge P$

(b) $\phi_2 = (P \vee Q \vee \neg R) \wedge (P \vee R \vee S) \wedge (\neg P \vee S) \wedge (R \leftrightarrow S) \wedge (\neg P \vee \neg S) \wedge (\neg Q \vee \neg S) \wedge (\neg Q \vee S)$

[1] Solutions in paper form only, and solution submission only at the stated time at the stated place. At most 3 authors per solution. All authors must be in the same tutorial group. All sheets of your solution must be stapled together. At the top of the first sheet, you must write the names of the authors and the name of your tutor. Your solution must be placed into the correct box for your tutorial group. If you don't comply with these rules, 3 points will be subtracted from your score for this sheet.

Exercise 16: DPLL. (3 Points)

For each of the following formulas, **use the DPLL procedure to determine whether ϕ is satisfiable or unsatisfiable**. Give a complete trace of the algorithm, **showing the simplified formula for each recursive call of the DPLL function**. Assume that DPLL selects variables in alphabetical order (i.e. $A, B, C, D, E, \ldots$), and that the **splitting rule first attempts the value False (F) and then the value True (T)**.

(a) $\phi_1 = (\neg A \vee \neg B) \wedge (B \vee C \vee D) \wedge (B \vee \neg D) \wedge (B \vee \neg C) \wedge (A \vee B \vee D) \wedge (\neg B \vee D)$

(b) $\phi_2 = (A \vee B \vee C) \wedge (A \vee \neg B) \wedge (\neg A \vee \neg B) \wedge (\neg A \vee B) \wedge (\neg C \vee D) \wedge (D \vee \neg E)$
$\wedge (\neg D \vee \neg E) \wedge (\neg D \vee E)$

Exercise 17: Contraposition theorem. (2 Points)

Prove the contraposition theorem: $\mathbf{KB} \cup \{\varphi\} \models \neg\psi$ iff $\mathbf{KB} \cup \{\psi\} \models \neg\varphi$.

Note: You **must prove any other theorem used as part of your proof** unless it appears in the lecture slides.