① 2572865 Assignment 13

13.1

	E:	B:
$2x+5y \le -17$	$z_1 = 2x+5y$	$z_1 \le -17$
$3x+7y \le -24$	$z_2 = 2x+5y$	$z_2 \ge -17$
$2x+5y \ge -17$	$z_3 = 3x+7y$	$z_3 \le -24$
$3x+7y \ge -24$	$z_4 = 3x+7y$	$z_4 \ge -24$

(E, B, β, ϕ, T)

$\xRightarrow{EB}$ $(E; B \uplus \{z_1 \le -17, z_2 \ge -17, z_3 \le -24, z_4 \ge -24\}; \beta; S; T)$

$\Rightarrow (E; B \uplus \{z_1 \le -17, z_2 \ge -17, z_4 \ge -24\}; \beta; S \uplus \{z_3 \le -24\}_{IV})$

$\xRightarrow{FixDV}$ $(E; B \uplus \{z_1 \le -17, z_2 \ge -17, z_4 \ge -24\}; \beta; S \uplus \{z_3 \le -24\}_{DV})$

$\Rightarrow (E'; B; upd(\beta, x, c, E'); S \uplus \{z_3 \le -24\}_{DV})$

pivot on z_3 β' $z_1 \mapsto$ $z_3 \mapsto -24$

$z_2 \mapsto$ $z_4 \mapsto$

$z_3 = 3x+7y \Rightarrow x = \frac{z_3 - 7y}{3}$

E': Replace all x in all eq^N

$z_2 = 2\left(\frac{z_3-7y}{3}\right) + 5y = \frac{2}{3} z_3 + \frac{y}{3} = z_1$

$z_4 = z_3 - 7y + 7y = z_3$

update, β', $z_3 \mapsto -24$ and $y = 0$

[we didn't update y for β']

④

$z_2 \mapsto -16$
$z_1 \mapsto -16$
$z_3 \mapsto -24$
$z_4 \mapsto -24$
$x \mapsto -8$
$y \mapsto 0$

$\overset{\text{FixDV}}{\Longrightarrow} (E'; B \oplus \{z_1 \le -17, z_2 \ge -17, z_4 \ge -24\}, \beta; S \oplus \{z_3 \le -24\}; DV)$

$\overset{\text{ACK}}{\Longrightarrow} (E'; B \oplus \{z_1 \le -17, z_2 \ge -17, z_4 \ge -24\}, \beta; S \oplus \{z_3 \le -24\}; T)$

$\overset{\text{EB}}{\Longrightarrow} (E'; B \oplus \{z_2 \ge -17, z_4 \ge -24\}, \beta'; S \oplus \{z_3 \le -24, z_1 \le -17; IV)$

$\overset{\text{FixDV}}{\Longrightarrow} (E''; B; upd(\beta', X, C, E''); S \oplus \{z_3 \le -24, z_1 \le -17\}; IV)$

pivot on z_1 $\quad \beta'': z_1 \mapsto -17$

$E'': \quad z_1 = \frac{2}{3} z_3 + y/3$

$\Rightarrow y = 3z_1 - 2z_3$

$z_2 = \frac{2}{3} z_3 + z_1 - \frac{3}{z_3} = z_1$

$z_3 = z_4$

$y = -51 + 48 = -3$

$x = \frac{z_3}{3} - \frac{7y}{3}$

$= -8 + 7 = -1$

③

β'' $z_1 \longmapsto -17$

$z_1 \longmapsto -17$

$z_2 \longmapsto -17$

$z_3 \longmapsto -29$

$z_4 \longmapsto -29$

$\overset{ACR}{\Longrightarrow}$ $(E'';\ B \uplus \{z_2 \gg -17,\ z_4 \gg -29\},\ \beta'';\ S \uplus \{z_3, z_1\};\ T)$

$\overset{(EB,\ AB)\times 2}{\Longrightarrow}$ $(E'';\ B;\ \beta'';\ S \uplus \{z_1, z_2, z_3, z_4\};\ T)$

13.2

$x \geq 0$
$y \geq 0$
$z \geq 0$
$2x + y + z \leq 14$
$4x + 2y + 3z \leq 28$
$2x + 5y + 5z \leq 30$

(1) $x + 2y - z \geq 10$
(2) $x + 2y - z \geq 14$

E:

$2x + y + z = z_1$
$4x + 2y + 3z = z_2$
$2x + 5y + 5z = z_3$
$x + 2y - z = z_4$

B:

$x, y, z \geq 0$
$z_1 \leq 14$
$z_2 \leq 28$
$z_3 \leq 30$

$B = \{ z_4 \geq 10 \}$

$(E; B; B_0; \emptyset; T)$

$\overset{EB}{\Rightarrow} (E; B \setminus \{x \geq 0\}; \beta; SU\{x \geq 0\}; IV)$

$\overset{AB}{\Rightarrow} (E; B; \beta; S; T)$

$\overset{EB}{\Rightarrow} (E; B \setminus \{y \geq 0\}; \beta; SU\{y \geq 0\}; IV)$

$\overset{AB}{\Rightarrow} (E; B; \beta; S; T)$

$\overset{EB}{\Rightarrow} (E; B \setminus \{z \geq 0\}; \beta; SU\{z \geq 0\}; IV)$

$\overset{AB}{\Rightarrow} (E; B; \beta; S; T)$

⑤

$\xRightarrow{EB}$ $(E;\ B \setminus \{z_4 \geq 10\},\ \beta,\ S \cup \{z_4 \geqslant 10\};\ IV\}$

$\Rightarrow$ Act Indep Bound $(E;\ B;\ \beta;\ S;\ DV)$

$\Rightarrow$ FixDV $(E',\ B,\ upd(\beta, E'),\ S,\ DV)$

recrranging the defining eqN

pivot $z_4 \geqslant 10$ for z_4 in terms of x

$$x + 2y - z = z_4 \quad \in E'$$

$$\Rightarrow x = -2y + z + z_4 \quad \in E'$$

replace x in all eqN;

$$z_1 = -4y + 2z + 2z_4 + y + z \quad \in E'$$

$$= -3y + 3z + 2z_4 \in E'$$

$$z_2 = -6y + 7z + 4z_4 \in E'$$

$$z_3 = y + 7z + 2z_4 \in E'$$

β' $z_4 \longmapsto 10$

$x \longmapsto 10$

$y \longmapsto 0$

$z \longmapsto 0$

$z_1 \longmapsto 20$

$z_2 \longmapsto 40$

$z_3 \longmapsto 20$

$z_4 \longmapsto 10$

(6)

Fix DV $\Longrightarrow$ $(E'', B, upd(B', E''), S, DV)$

$E'': z_1 \leq 14$ ✓

pivot on y.

~~$y = 14 - 2x - z$~~ $z_1 = -3y + 3z + 2z_4$

$\Rightarrow y = \frac{3z}{3} + \frac{2z_4}{3} - \frac{z_1}{3} \in E''$

$= z + \frac{2}{3} z_4 - \frac{1}{3} z_1 \in E''$

$x = -z - \frac{z_4}{3} + \frac{2z_1}{3} \in E''$

$z_2 = -6\left[z + \frac{2}{3} z_4 - \frac{1}{3} z_1 \right] + 7z + 4z_4$

$= z + 2z_1 \in E''$

$z_3 = 8z + \frac{8}{3} z_4 - \frac{z_1}{3} \in E''$

update: β'', $z_1 \longmapsto 14$

$y \mapsto \frac{20}{3} - \frac{14}{3} = \frac{6}{3} = 2$

$x \mapsto -\frac{10}{3} + \frac{28}{3} = \frac{+18}{3} = 6$

$z_2 \mapsto 28$

$z_3 \mapsto \frac{~~+12~~ 80}{3} - \frac{14}{3} = \frac{66}{3} = 22$

$z_4 \longmapsto 10$

$z \longmapsto 0$

⑦

$$\overset{EB\times 2}{\Rightarrow} (E''; B \setminus \{z_2, z_3\}; \beta'', s \cup \{z_2, z_3\}; IV)$$

$$\underset{AB\times 2}{\Rightarrow} (E''; B; \beta''; s; T)$$