

①

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Assignment 11

11.1

①  $f(f(x)) = g(x)$

②  $f(a) = b$

orient ③  $f(f(x)) \rightarrow g(x)$

orient ④  $f(a) \rightarrow b$

deduce ⑤  $g(a) = f(b)$

orient ⑥  $g(a) \rightarrow f(b)$

deduce ⑦  $g(f(x)) = f(g(x))$

orient ⑧  $g(f(x)) \rightarrow f(g(x))$

deduce ⑨  $f(g(f(x))) = g(g(x))$

simplify ⑩  $f(f(g(x))) = g(g(x))$   
[because of ⑦]

simplify & delete ⑪  $g(g(x)) = g(g(x))$   
[because of ③]

deduce ⑫  $f(g(a)) = g(b)$

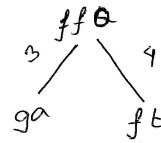
simplify ⑬  $f(f(b)) = g(b)$  [from ⑤]

simplify ⑭  $g(b) = g(b)$  [from ③]  
delete

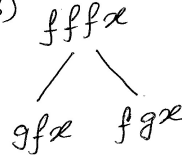
all critical pair check.

 $R = \{3, 4, 6, 8\}$  is the result.

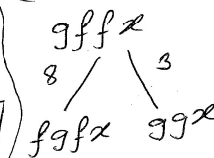
(3,4)



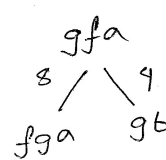
(3,3)



(8,3)



(8,4)



②

11.2

$$① f(g(x), x) = h(x)$$

$$② f(g(x), h(y)) = f(x, y)$$

$$③ h(a) = a$$

orient ④  $f(g(x), x) \Rightarrow h(x)$

orient ⑤  $f(g(x), h(y)) \rightarrow f(x, y)$

orient ⑥  $h(a) \rightarrow a$

deduce ⑦  $f(g(x), a) = f(x, a)$

orient ⑧  $f(g(x), a) \rightarrow f(x, a)$

deduce ⑨  $f(h(h(y)), y) = f(h(y), y)$

orient ⑩  $f(h(y), y) \rightarrow h(h(y))$

deduce ⑪  $h(a) = f(a, a)$

orient ⑫  $f(a, a) \rightarrow h(a)$

deduce ⑬  $f(a, a) = h(h(a))$

orient ⑭  ~~$f(a, a) \rightarrow h(h(a))$~~

R Simplify ⑭  $f(a, a) = h(a)$  [from 6]

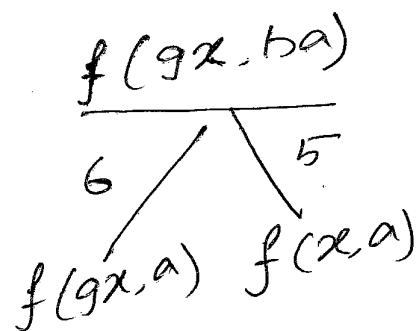
L Simplify ⑮  $h(a) = h(a)$

delete

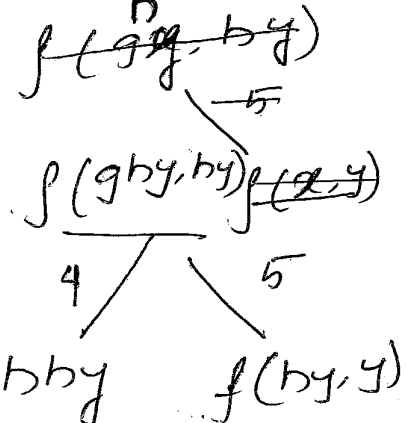
All CPs are checked

$R = \{4, 5, 6, 8, 10, 12\}$  is the result.

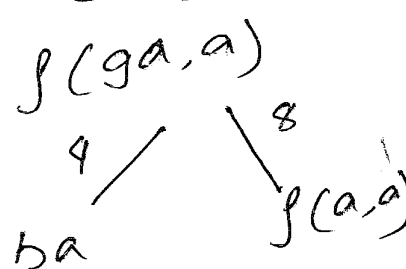
(5, 6)



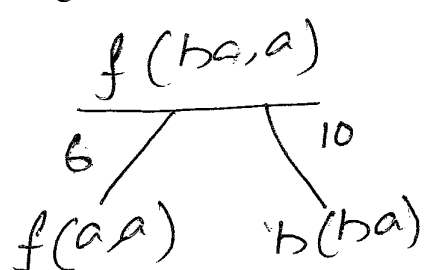
(9, 5)



(8, 4)



(10, 6)



12

11.3

$$(1) f(a, g(a)) = f(b, g(b))$$

$$(2) g(a) = h(c)$$

$$(3) h(d) = g(b)$$

$$(4) d = c$$

$$(5) f(a, h(d)) = f(h(d), a)$$

orient (6)  $d \rightarrow c$

L Simplify (7)  $h(c) = g(b)$

Simplify (8)  $f(a, h(c)) = f(h(c), a)$

orient (9)  $h(g(a)) \rightarrow h(c) \quad h(c) \rightarrow g(a)$

Simplify (10)  $f(a, g(a)) = f(g(a), a)$

orient (11)  $g(a) = g(b) \quad h(c) \rightarrow g(b)$

R Simplify (12)  $g(a) = g(b)$

orient (13)  $g(a) \rightarrow g(b)$

L Simplify (14)  $f(a, g(b)) = f(g(b), a) \quad f(h(c), a)$

orient (15)  $f(a, g(a)) \Rightarrow f(b, g(b))$

Simplify (16)  $f(b, g(b)) = f(h(c), a)$

unsatisfiable.