SAARLAND UNIVERSITY ARTIFICIAL INTELLIGENCE
Dr. Álvaro Torralba, Prof. Wolfgang Wahlster
Dr. Cosmina Croitoru, Daniel Gnad, Marcel Steinmetz
Yannick Körber, Michael Barz
Christian Bohnenberger, Sören Bund-Becker, Sophian Guidara,
Alexander Rath, Khansa Rekik, Julia Wichlacz, Anna Wilhelm

Exercise Sheet 3.
Solutions due Tuesday, **May 22**, 16:00 – 16:15, in the lecture hall.[1]

Exercise 10. (3 Points)

Consider the following constraint network $\gamma = (V, D, C)$. There are 7 aliens that want to fly to their favourite planet. – Et, An, Flo, We, Eve, Git, Fee. Every alien can only fly using an airplane, that during the day is used by humans, while during the night he becomes invisible. Hence our 7 aliens have to schedule themselfs on the magical airplane. The time slots under which they can use it are Mondays (M), Tuesdays (Tu), Wednesdays (W), Thursdays (Th), Fridays (F). To define the constraints we will use the + operation on days of the weeks. To do that, we interpret every day as a number from Monday (1) to Friday (5). For example, $F = Th + 1$, and $Th = M + 3$ but $M \neq F + 1$. Formally, the network is defined as follows:

- Variables: $V = \{Et, An, Flo, Wale, Eve, Git, Fee\}$.
- Domains: For all $v \in V$: $D_v = \{\mathrm{M}, \mathrm{Tu}, \mathrm{W}, \mathrm{Th}, \mathrm{F}\}$.
- Constraints :
  - Flo and Et are best friends, and always want to share the airplane. (Flo=Et)
  - An finds it cool how the airplanes smells after Eve flew with it. So, they have to be in a 1 day distance. (An=Eve+1)
  - Git and Eve are in love so they like to travel together. (Git=Eve)

[1] Solutions in paper form only, and solution submission only at the stated time at the stated place. At most 3 authors per solution. All authors must be in the same tutorial group. All sheets of your solution must be stapled together. At the top of the first sheet, you must write the names of the authors and the name of your tutor. Your solution must be placed into the correct box for your tutorial group. If you don't comply with these rules, 3 points will be subtracted from your score for this sheet.

- Wale and Fee are good friends. As Wale knows how Fee is leaving good fresh invisible food in the fridge and loves it. So, Wale wants to fly one day after Fee, such that he still can have some of the food. (Wale=Fee+1)
- Flo and Wale hate each over so much, that Flo can fly only 3 days after Wale. (Flo=Wale+3)
- Git loves that Wale always leaves nice surprises on the airplane. Hence, Wale will always travel one day after him. (Git=Wale+1)

Run the AC-3(γ) algorithm, as specified in the lecture, on the given constraint network. For each iteration of the while-loop, give the content of M at the start of the iteration, give the pair (u, v) removed from M, give the domain D_u of u after the call to Revise(γ, u, v), and give the pairs (w, u) added into M.

Note: Initialize M as a lexicographically ordered list (i.e., (a, b) would be before (a, c), both before (b, a) etc., if any of those exist). Furthermore, use M as a FIFO queue, i.e., always remove the element at the front and add new elements to the back.

Exercise 11. (2,5 Points)

Run the AcyclicCG algorithm on the problem defined in Exercise 10, as specified in the lecture. More precisely, execute its 4 steps as follows:

(a) Draw the constraint graph of γ. Pick "Flo" as the root and draw the directed tree obtained by step 1 (see Chapter 8 slides 36 and 37 for examples). Give the resulting variable order obtained by step 2.

(b) List the calls to Revise($\gamma, v_{parent(i)}, v_i$) in the order executed by step 3, and for each of them give the resulting domain of $v_{parent(i)}$.

(c) For each recursive call to BacktrackingWithInference during step 4, give the domain D'_{v_i} of the selected variable v_i after Forward Checking, and give the value $d \in D'_{v_i}$ assigned to v_i.

Note: Step 4 runs BacktrackingWithInference with variable order $v_1, \ldots, v_n$. This means that, at the ith recursion level, "select some variable v for which a is not defined" will select v_i.

Exercise 12. (2,5 Points)

Consider the following constraint network $\gamma = (V, D, C)$:

- Variables: $V = \{a, b, c, d, e, f, g, h, i, j, k\}$.
- Domains: For all $v \in V$: $D_v = \{20, 30, 40, 50, 60\}$.
- Constraints: $k < j$; $k < i$; $j < i$; $i < h$; $i < g$; $f < h$; $f < g$; $f < e$; $f < d$; $e < c$; $d < c$; $c < b$; $c < a$; $b < a$; $a < d$; $e < k$;

(a) Draw the constraint graph of γ.

(b) What is the optimal (minimal) cutset V_0 for γ?

(c) If the CutsetConditioning algorithm from the lecture is called with such a minimal cutset V_0, how many calls to AcyclicCG will be performed in the worst case? Justify your answer.

Exercise 13. (2 Points)

Assume a solvable constraint network γ with acyclic constraint graph. Assume that the AcyclicCG algorithm, as specified on Chapter 8 slide 35, reaches step 4 (i.e., AcyclicCG(γ) does not return "inconsistent" at some point during step 3). Assume that Backtracking-WithInference, as specified on Chapter 8 slide 14, uses the variable order $v_1, \dots, v_n$, i.e., at the i-th recursion level, "select some variable v for which a is not defined" will select v_i.

For each variable v_i, denote by $D^3_{v_i}$ the domain of v_i after the completion of AcyclicCG(γ) step 3 (i.e., the domain as in the input to backtracking in step 4). Denote by D_{v_i} the domain of v_i at the beginning of the call of BacktrackingWithInference at the ith recursion level, and denote by D'_{v_i} the domain of v_i after forward checking, i.e., after "Inference(γ', a)" has been run.

Prove that: (i) $a \cup \{v_i = d\}$ is consistent for every $d \in D'_{v_i}$, and (ii) $D'_{v_i} \neq \emptyset$.

Note: (i) and (ii) together imply that step 4 finds a solution without ever having to backtrack.