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Exercise Sheet 6.

Solutions due Tuesday, **June 12**, 16:00 – 16:15, in the lecture hall.[1]

Exercise 22. (2 Points)

Run the unification algorithm from the lecture on each of the given sets. Assume v, w, x, y, z to be *variables*, and a, b to be *constants*. **For each step, write down the set to unify, the disagreement set, and the updated set of substitutions, i.e.,** $T_0, D_0, s_1, T_1, D_1, \ldots$ **until the algorithm terminates** (cf. Chapter 13, slide 26). In particular, if the algorithm does not output a failure, list the *entire* content of the substitutions $s_1, \ldots, s_n$.

(a) $\{P(x, f(y), a), P(x, f(b), z)\}$

(b) $\{P(a, b), P(x, f(x))\}$

(c) $\{P(z, x), P(f(z), a)\}$

(d) $\{P(f(a, x), y), P(w, f(a, a)), P(f(z, a), v)\}$

[1] Solutions in paper form only, and solution submission only at the stated time at the stated place. At most 3 authors per solution. All authors must be in the same tutorial group. All sheets of your solution must be stapled together. At the top of the first sheet, you must write the names of the authors and the name of your tutor. Your solution must be placed into the correct box for your tutorial group. If you don't comply with these rules, 3 points will be subtracted from your score for this sheet.

Exercise 23. (3 Points)

In this exercise you will see whether the princess Elsa is saved by the prince, or eaten by the dragon. For this consider the following statements and the set $\theta^* = \{\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5\}$ of corresponding predicate logic formulas in Skolem normal form:

1. Elsa lives in the tower of big old castle.
   $\varphi_1 = lives(Elsa, castle)$
2. If Olaf does not save Elsa, then she can't escape the castle.
   $\varphi_2 = saves(Olaf, Elsa) \vee \neg escapes(Elsa, castle)$
3. Everyone who lives in the castle and does not escape will be eaten by the dragon.
   $\varphi_3 = \forall x[\neg lives(x, castle) \vee escapes(x, castle) \vee eat(dragon, Elsa)]$
4. Olaf does not save anyone that lives in a castle.
   $\varphi_4 = \forall x[\neg saves(Olaf, x) \vee \neg lives(x, castle)]$
5. The dragon does not eat Elsa.
   $\varphi_5 = \neg eats(dragon, Elsa)$

(a) Write down $CF(\theta^*)$ (compare Chapter 13 slide 9) and the resulting Herbrand universe $HU(\theta^*)$.

(b) Write down the Herbrand expansion $HE(\theta^*)$. Bring each of the formulas in $HE(\theta^*)$ into CNF, resulting in a set of clauses Δ.

(c) Use **propositional resolution** to prove that Δ is unsatisfiable. Assuming that statements 1.–4. are true, will Olaf save Elsa from the bad dragon?

Exercise 24. (3 Points)

Consider now the following set of predicate logic formulas in Skolem normal form:

1. Everyone who drinks coffee is awesome.
   $\varphi_1 = \forall x[\neg Drinks(x, coffee) \vee Awesome(x)]$

2. Olaf or Sven drink coffee.
   $\varphi_2 = Drinks(Olaf, coffee) \vee Drinks(Sven, coffee)$

3. All people that are awesome have a bike and go to Uni by it. Note: $f(x)$ comes from removing an existential quantification ($\exists y[Bike(y) \wedge Goes(x, Uni, y)]$) and represents the bike with which x goes to Uni.
   $\varphi_3 = \forall x[\neg Awesome(x) \vee (Bike(f(x)) \wedge Goes(x, Uni, f(x)))]$

4. Everyone who is awesome plays Tennis.
   $\varphi_4 = \forall x[\neg Awesome(x) \vee Plays(x, tennis)]$

5. Olaf does not go anywhere by bike.
   $\varphi_5 = \forall x[\neg Goes(Olaf, Uni, x), \neg Bike(x)]$

We want to prove by contradiction that Sven plays Tennis. Therefore, we add to our set of clauses:

6. Sven does not play Tennis.
   $\varphi_6 = \neg plays(Sven, tennis)$

Perform the following operations:

(a) Write down the **set of clauses Δ corresponding to the clausal normal forms** of these formulas.

(b) Use **binary PL1 resolution** to show that Δ is unsatisfiable.
Note: If you need to use a clause more than once, make a copy of the clause and rename the variables to avoid name collisions.

Exercise 25. (2 Points)

Let φ be a predicate logic formula taking the form $\varphi = \forall x_1 \ldots \forall x_i \exists y \psi$ where all quantified variables are pairwise distinct. Say that f is a function symbol that does not occur in φ and a is a constant that does not occur in φ. Denote $\varphi^* = \forall x_1 \cdots \forall x_i \psi \frac{y}{f(x_1,\ldots,x_i)}$ and $\varphi' = \forall x_1 \cdots \forall x_i \psi \frac{y}{a}$.

(a) Prove that, if φ is satisfiable, then φ^* is satisfiable as well.

(b) Prove that the same is not true for φ', i.e., there are cases where φ is satisfiable but φ' is not. (Tip: Remember here that predicate and function symbols can be interpreted in arbitrary ways, and that our language does not include "=" to directly compare constants.)