SAARLAND UNIVERSITY   ARTIFICIAL INTELLIGENCE

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Exercise Sheet 5.

Solutions due Tuesday, **June 5**, 16:00 – 16:15, in the lecture hall.[1]

Exercise 18.   (3.5 Points)

(a) Perform DPLL with clause learning as explained in slides 44-46 of Chapter 11 on the following clause sets (give the full DPLL trace). Start by using the splitting rule and assign the value T **to the proposition** A. For the next splitting rule assign **E to** T. If you encounter a case where 2 or more different unit propagation rules are applicable choose the one which gets assigned to T (**remember: if a new unit clause is learned start assigning this clause**). Whenever you encounter a conflict, draw the corresponding implication graph as well as its conflict graph, and mention which clause can be learned with the clause learning method. Highlight vertices of choice and implied literals with different colors. Then use this information and continue with the DPLL procedure, backtracking the last choice as specified in slide 44 of Chapter 11. Do this until the clause set is proven to be satisfiable or unsatisfiable.

$\Delta_a = \{\{A, B, C\}, \{D, E, F\}, \{\neg C, E\}, \{\neg A, \neg D\}, \{\neg B, \neg E\}, \{C, \neg F\}, \{\neg E, B\}\}$

(b) Here, just draw the conflict and implication graph for the first conflict you encounter. What is the learned clause? Start by using the splitting rule and assign the value F **to the proposition** A. For the next splitting rule assign **B to** F.

$\Delta_b = \{\{B, D\}, \{\neg B, \neg D\}, \{A, \neg B, D\}, \{\neg A, \neg C, \neg D\}, \{A, \neg C\}, \{A, B, \neg D\}, \{C, \neg E\}\}$

Note: You do not need to continue with the DPLL with clause learning procedure.

Exercise 19. (2 Points)

Prove that an arbitrary formula ϕ in CNF is valid if and only if each clause contains an atom P and its negation $\neg P$.

Exercise 20. (2 Points)

(a) For each of the following sentences, write down a **predicate logic formula that has the same meaning**.

   - Every student who solves all exercise sheets passes at least one exam.
   - At most one AI exam grade can be used.

(b) For each of the following predicate logic formulas, write down an **English sentence with the same meaning.**

   - $\forall x[\text{Parent}(x) \rightarrow \exists y(\text{Person}(y) \wedge \text{Child}(y, x))]$
   - $\exists x[Student(x) \wedge \exists y[Book(y) \wedge knows(x, y) \wedge \forall z[Scientist(z) \rightarrow read(z, y)]]]$

Exercise 21. (2.5 Points)

Transform all of the following predicate logic formulas into clausal normal form. Write down the results of all intermediate steps (cf. Chapter 12, slides 38–39), specifying which steps are you applying and giving the intermediate results.

   Note: Simplify the resulting formulas where possible.

(a) $\varphi_1 = \neg\forall x[\neg A(x) \leftrightarrow \exists y[B(y)]]$

(b) $\varphi_2 = \exists x \forall y \exists z[A(x, y) \leftrightarrow (B(x) \vee B(z))]$

(c) $\varphi_3 = \forall x[(\neg C(x) \wedge (\exists y[A(y) \wedge B(y, x)])) \rightarrow \neg D(x)]$