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Assignment 12
Exercise 12.1

12.1

$x + y \geq 16$
$9x + 7y \leq 28$
$2x - 7y \leq 20$
$2x - 3y \geq -9$

transform N st. x is on LHS

i) $x \geq 16 - y$
ii) $x \leq 7 - \frac{7}{4}y$
iii) $x \leq 10 + 7/2\ y$
iv) $x \geq -\frac{9}{2} + \frac{3}{2}y$

$\phi \cup \{(ii), (iii)\} \cup \{(i), (iv)\}$

$\Rightarrow$ (ii, i) $7 - \frac{7}{4}y \geq 16 - y$

(ii, iv) $7 - \frac{7}{4}y \geq -\frac{9}{2} + \frac{3}{2}y$

(iii, i) $10 + \frac{7}{2} \geq 16 - y$

(iii, iv) $10 + 7/2 \geq -\frac{9}{2} + \frac{3}{2}y$

1. $y \leq -12$
2. $y \leq 46/13$
3. $y \geq 4/3$
4. $y \geq 11/9$

1, 3 $\Rightarrow$ $-12 \geq 4/3$

1, 4 $\Rightarrow$ $-12 \geq 11/4$ $\Rightarrow$ N is unsat

2, 3 $\Rightarrow$ $\frac{46}{13} \geq 4/3$

2, 4 $\Rightarrow$ $46/13 \geq 11/4$

12.2

1. $\forall x \cdot \exists y \; (2x + y > 7 \wedge x + y < 6)$

- with width the inner innermost quantifier
- if it is $\exists x_{n+1}$, then we do $\Rightarrow$ FM "Eliminate" on x_n

$\rightsquigarrow \forall x \cdot \exists y \; (y > 7 - 2x \wedge y < 6 - x)$

$\rightsquigarrow \forall x. \; (6 - x > 7 - 2x)$

$\Rightarrow \neg \exists x \quad \neg (6 - x > 7 - 2x)$

$\Rightarrow \neg \exists x \quad (6 - x \leq 7 - 2x)$

$\Rightarrow \neg \quad \emptyset \quad \Rightarrow \bot$

2. $\exists x \forall y \ (2x - y > 7 \wedge 2x + y > 7)$

$\Rightarrow \exists x . \neg \exists y \quad \neg (2x - y > 7 \wedge 2x + y > 7)$

$\Rightarrow \exists x . \neg \exists y \quad (2x - y \leq 7 \vee 2x + y \leq 7)$ two DNF clause

$\Rightarrow \exists x \quad \neg (\emptyset \vee \emptyset)$ - empty conjunction

12.3 $\Sigma_{LA} = \{\{s\}, \{0, 1, +/2, -/1\}\cup$
$\mathbb{Q}, \{<, \leq, \neq, \geq, >\}\}$

$\mathcal{H} := \{(x, n) \mid$ the turing machine with global number x halts after n step $\}$

Then asking,

$\forall x . \exists n, \ x \neq n$

is asking general halting problem which is undecidable, Σ_{LA} can not decidable.