Necessary minimum background test

(this should take 10-15 minutes, if you know the material)

1. Multivariate calculus

What is the partial derivative of y with respect to x ?

$$y = x \sin(z)\, e^{-x}$$

2. Vectors and matrices

Consider the matrix **X** and the vector **y** below

$\mathbf{X} = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$ $\mathbf{y} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

What is the product **Xy** ?
Is **X** invertible? If so, give the inverse, if not explain why not.
What is the rank of **X** ?

3. Probability and statistics

Consider a sample of data S obtained by flipping a coin *x*, where 0 denotes the coin turned up heads, and 1 denotes that it turned up tails.

S = {1, 1, 0, 1, 0}

What is the sample mean for this data ?

What is the sample variance ?

What is the probability of observing this data assuming that a coin with an equal probability of heads and tails was used (i.e., by the probability distribution p(*x*=1)=0.5, p(x=0)=0.5).

Note the probability of this data sample would be greater if the value of p(x=1) was not 0.5, but some other value. What is the value that maximizes the probability of sample S? [optional: can you prove your answer is correct?]

Given the following joint distribution between x and y, what is P(x=T |y=b)?

P(x,y) y x	a	b	c
T	0.2	0.1	0.2
F	0.05	0.15	0.3