Problem Set III.3

1. $AK = [Ab \ A^2b \ \cdots \ A^nb]$

$KB = [Ab \ A^2b \ \cdots A^{n-1}b \ . -b]$

$AK - KB = [0 . \ 0 . \ \text{—} \ A^nb + b] = C$

2. A has positive eigenvalues.

B has negative eigenvalues.

$H^T = H . \quad A^T = A$

$AH - HB = AH + HA \triangleq M .$

$M_{ij} = \frac{\alpha_i + \alpha_j}{2} H_{ij}$

$= \frac{2i-1+2j-1}{2} \cdot \frac{1}{i+j-1} = 1$

$M = \text{ones}(n) = C$

$\text{rank}(C) = 1$

so. Sylvester test is passed.

5. $M \triangleq AP + PA$

$M_{ij} = (\alpha_i + \alpha_j) \cdot \frac{s_j + s_i}{\alpha_i + \alpha_j}$

$= s_i + s_j$

so. $M_{ij} = s1^T + 1s^T$

7. $A = Q \Lambda \bar{Q}^T$

$\bar{A}^T = Q \bar{\Lambda}^T \bar{Q}^T$

$\bar{A}^T A = Q \bar{\Lambda}^T \bar{Q}^T Q \Lambda \bar{Q}^T = Q \bar{\Lambda}^T \Lambda \bar{Q}^T$

$A \bar{A}^T = Q \Lambda \bar{Q}^T Q \bar{\Lambda}^T \bar{Q}^T = Q \Lambda \bar{\Lambda}^T \bar{Q}^T$

$\bar{\Lambda}^T \Lambda = \Lambda \bar{\Lambda}^T$

$\bar{A}^T A = A \bar{A}^T$