Solutions for Problem Set II.2

1

for $\forall \boldsymbol{\alpha} \in \boldsymbol{C}(\boldsymbol{A})$ and $\forall \boldsymbol{\beta} \in \boldsymbol{N}(\boldsymbol{A^T})$, $\boldsymbol{\alpha} = \sum \alpha_i \boldsymbol{A_i}$, $\boldsymbol{A^T \beta} = 0$
so, $\boldsymbol{A_i^T \beta} = 0$, $\boldsymbol{\alpha_T beta} = \sum \alpha_i \boldsymbol{A_i^T \beta} = 0$

2

rank$(A^+) \leq$ rank(A) ,rank$(A) \leq$ rank(A^+), so rank$(A^+) =$ rank(A)

When A is square, A and A^+ have same eigenvectors. The eigenvalues of A^+ is the multiplicative inverse of the eigenvalues of A.

3

pass

4

diagonal matrix, and $A_{ii} = 1$ or 0.

5

5.1

$m \times n - n$

5.2

$\Sigma^T\Sigma$ is diagonal, and $\sigma_{ii} \neq 0$, so $(\Sigma^T\Sigma)^{-1} = \text{diag}(\frac{1}{\sigma_{ii}})$

5.3

obviously.

5.4

$$
\begin{aligned}
A^+ &= (A^TA)^{-1}A^T \\
&= (V\Sigma^TU^TU\Sigma V^T)^{-1}V\Sigma^TU^T \\
&= (V^T)^{-1}\Sigma^{-1}U^T
\end{aligned}
$$

6

$$
\begin{aligned}
Ha &= a - 2\frac{(a-r)(a-r)^T}{(a-r)^T(a-r)}a \\
&= a - 2\frac{(aa^T + rr^T - ra^Ta - ar^Ta)a}{a^Ta + r^Tr - a^Tr - r^Ta} \\
&= a - 2\frac{(a-r)(a^Ta - r^Ta)}{2(a^Ta - r^Ta)} \\
&= a - (a-r) \\
&= r
\end{aligned}
$$

7

pass

8

$$
q_1 = \frac{a}{\|a\|}
$$

$$
\begin{aligned}
A_2 &= b - (b^T q_1) q_1 \\
&= b - 2a
\end{aligned}
$$

9

$$
\begin{aligned}
q_1 &= \frac{a}{\|a\|} \\
A_2 &= b - (b^T q_1) q_1 \\
&= b - 2a \\
&= (2, -2)^T \\
q_2 = A_2 / \|A_2\| &= \frac{1}{2\sqrt{2}} (1, -1)^T
\end{aligned}
$$

$$
\begin{bmatrix} 1 & 4 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} q_1 & q_2 \end{bmatrix} \begin{bmatrix} \sqrt{2} & \sqrt{2} \\ 0 & 2\sqrt{2} \end{bmatrix}
$$

10

lower, upper

11

obvious.

12

$E = \|b\|^2 - \| \frac{b \cdot t}{\|t\|^2} \|^2$

13

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} C \\ D \end{bmatrix} \begin{bmatrix} 1 \\ 5 \\ 13 \\ 17 \end{bmatrix}$$

so, $C = 1,\ D = 4$

14

$e \cdot A_1 = 0$
$e \cdot A_2 = 0$

15

$$Ax - b \begin{bmatrix} C \\ C + D - 8 \\ C + 3D - 8 \\ C + 4D - 20 \end{bmatrix}$$

$$E = \|Ax - b\|^2 = C^2 + (C + D - 8)^2 + (C + 3D - 8)^2 + (C + 4D - 20)^2$$

$\frac{\partial E}{\partial C} = 0,\ \frac{\partial E}{\partial D} = 0,$
so, $C = 1,\ D = 4$

16

question 16 to 22, to be continued