Solutions for Problem Set II.1

1

obvious.

2

$$D^{-1} = \begin{bmatrix} 1 & & 0 \\ \vdots & \ddots & \\ 1 & \dots & 1 \end{bmatrix}$$

Obviously D^{-1} is lower triangular "sum matrix" of 1's.
And

$$(DD^T)^{-1} = (D^T)^{-1}D^{-1}$$

3

When $n = 3$,

$$(DD^T)^{-1} = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$zz^T = (DD^T)^{-1} - A^{-1} = \begin{bmatrix} \frac{9}{4} & \frac{3}{2} & \frac{3}{4} \\ \frac{3}{2} & 1 & \frac{1}{2} \\ \frac{3}{4} & \frac{1}{2} & \frac{1}{4} \end{bmatrix}$$

And, $z = (\frac{3}{2}, 1, \frac{1}{2})^T$

4

When $n = 2$,

$$(-S + 2I)^{-1} S^T = \begin{bmatrix} 0 & -\frac{1}{2} \\ 0 & -\frac{1}{4} \end{bmatrix}$$

Then, λ are -0.25 and 0.

When $n = 3$,

$$(-S + 2I)^{-1} S^T = \begin{bmatrix} 0 & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{4} & -\frac{1}{2} \\ 0 & -\frac{1}{8} & -\frac{1}{4} \end{bmatrix}$$

Then, λ are -0.5 and 0.

5

$$q_1 = b = (1, 0, 0)^T$$

For q_2,

$$v = Aq_1 = (2, -1, 0)^T$$

$$h_{11} = q_1^T v = 2$$

$$v = v - h_{11} q_1 = (0, -1, 0)^T$$

$$h_{21} = 1$$

$$q_2 = (0, -1, 0)^T$$

For q_3,

$$v = Aq_2 = (1, -2, 1)^T$$

$$h_{12} = q_1^T v = 1$$

$$v = v - h_{12} q_1 = (0, -2, 1)^T$$

$$h_{22} = q_2^T v = 2$$

$$v = v - h_{22} q_2 = (0, 0, 1)^T$$

$$h_{32} = 1$$

$$q_3 = (0, 0, 1)^T$$

Then,

$$H = \begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 0 & 1 \end{bmatrix}$$

6

$$Q_2^T A Q_2 = H_2 = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Obviously $Q_2^T A Q_2$ is a tridiagonal matrix.

7

$$A_0 = A$$

$$A_0 = QR$$

$$A_1 = RQ = Q^{-1} A_0 Q$$

8

$$s_0 = A_{33}$$

$$A_0 = A$$

$$A_0 - s_0 I = QR$$

$$A_1 = RQ + s_0 I$$