Homework-Phase

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1 Problem

One popular formulation of the phase retrieval problem is solving a system of quadratic equations in the form

$$y_r = |\langle a_r, z\rangle|^2, r = 1, 2, \ldots, m$$

where $z \in \mathcal{C}^n$ is the decision variable, $a_r \in \mathcal{C}^n$ are known sampling vectors, $\langle a_r, z\rangle$ is the inner product between a_r and z in $\mathcal{C}^n$, $|a|$ is the magnitude of $a \in \mathcal{C}$ and $y_r \in \mathcal{R}$are the observed measurements.

2 Algorithms

2.1 Wirtinger flow

Wirtinger flow 算法分为初始化和迭代两步。其初始化算法如1所示，迭代算法如2所示

Algorithm 1 Wirtinger Flow: Initialization

Require: Observations $\{y_r\} \in \mathbb{R}^m$.

Set

$$\lambda^2 = n\frac{\sum_r y_r}{\sum_r \|\boldsymbol{a_r}\|^2}$$

Set z_0, normalized to $\|z_0\| = \lambda$ to be the eigenvector corresponding to the largest eigenvalue of

$$\boldsymbol{Y} = \frac{1}{m}\sum_{r=1}^{m} y_r \boldsymbol{a_r}\boldsymbol{a_r^*}$$

Ensure: Initial guess z_0.

Algorithm 2 Wirtinger Flow: Iteration

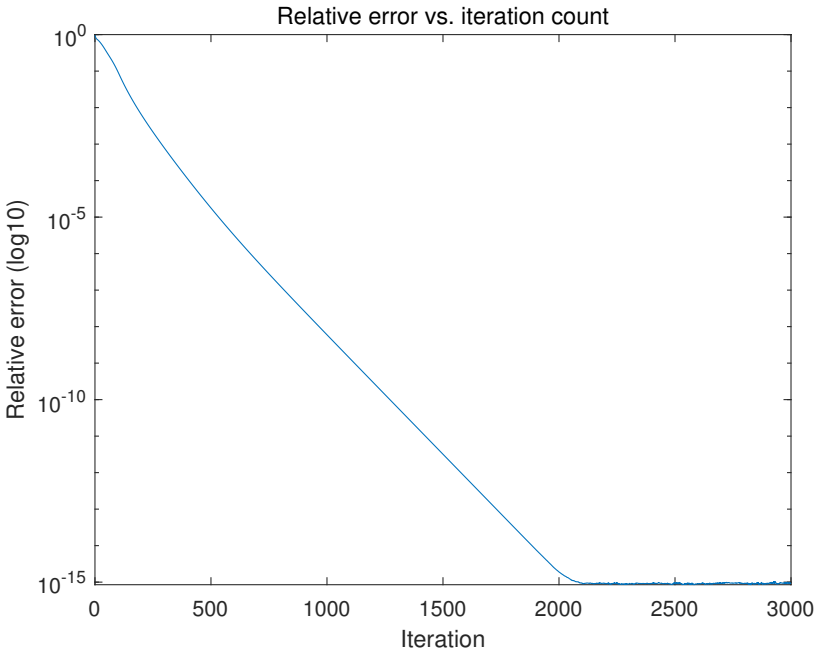
Require: z_0.

$$\begin{aligned} z_{\tau+1} &= z_\tau - \frac{\mu_{\tau+1}}{\|z_0\|^2}\left(\frac{1}{m}\sum_{r=1}^{m}\left(|a_r^* z|^2 - y_r\right)\left(a_r a_r^*\right) z\right) \\ &:= z_\tau - \frac{\mu_{\tau+1}}{\|z_0\|^2}\nabla f\left(z_\tau\right) \\ \mu_\tau &= \min\left(1 - e^{-\tau/\tau_0}, \mu_{\max}\right) \end{aligned}$$

3 Experiment

3.1 Gaussian

如图所示，相对误差随着迭代次数的增加而迅速减小。图中显示，大约 2000 次迭代就可以通过 Wirtinger Flow 算法进行精确恢复。


Relative error vs. iteration count
Relative error (log10)
Iteration
10^0
10^-5
10^-10
10^-15
0
500
1000
1500
2000
2500
3000