Problem Set II.4

1. $\frac{\partial L}{\partial p_i} = -\frac{a_i^2}{p_i^2} - \lambda = 0$

$p_i = \frac{a_i}{\sqrt{-\lambda}}$

$\sum p_i = 1 \Rightarrow \sqrt{-\lambda} = a_1 + \cdots + a_n$

$p_i = \frac{a_i}{a_1 + \cdots + a_n}$

2. $\int_0^1 \frac{(a(x))^2}{p(x)} dx \cdot \int_0^1 p(x) dx$

$\geq \left(\int_0^1 a(x) dx \right)^2$

取等时 $\frac{(a(x))^2}{p(x)} = c\,p(x)$

$p(x) = c\,a(x)$

$\int_0^1 p(x) dx = 1$

$\Rightarrow p(x) = \frac{a(x)}{\int_0^1 a(x) dx}$

3. $\sum \frac{a_i^2}{\frac{1}{n}} = \sum n a_i^2$

$= n \sum a_i^2$

$\geq \sum \frac{a_i^2}{\frac{a_i}{\sum a_i}}$

$= (\sum a_i)^2$

4. $x^T (nI - M) x$

$= n x^T I x - x^T M x$

$= n \cdot x^T x - x^T 1 \cdot 1^T x$

$= n(x_1^2 + \cdots + x_n^2) - (x_1 + \cdots + x_n)^2$

$\geq 0.$

$nI - M$ is positive semidefinite

$\det(\lambda I - nI + M) = 0$

$\Rightarrow \lambda = 0, n.$

5. $C = \sum \|a_j\| \|b_j\|$

$= \sum \|a_j\| \|a_j\|$

$= \sum a_j^T a_j$

$= \|A\|_F^2$

7. $L = \frac{a^2}{p} + \frac{b^2}{1-p}$

$\frac{\partial L}{\partial p} = 0. \Rightarrow p = \frac{a}{a+b} \quad (1-p) = \frac{b}{a+b}$

8. $Q^T A = U D V^T$

$QQ^T A = Q U D V^T$

$Y \approx QR$

$\Rightarrow QQ^T \approx YY^+$

$QUDV^T = QQ^T A$

$\approx YY^+ A$

$\approx A$