Codebook

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Contents

1 Setup

1.1 Template

```cpp
#include<bits/stdc++.h>
#include<bits/extc++.h>
#define F first
#define S second
#define pb push_back
#define pob pop_back
#define pf push_front
#define pof pop_front
#define mp make_pair
#define mt make_tuple
#define all(x) (x).begin(),(x).end()
#define mem(x,i) memset((x),(i),sizeof((x)))
using namespace std;
//using namespace __gnu_pbds;
using pii = pair<long long,long long>;
using ld = long double;
using ll = long long;
mt19937 mtrd(chrono::steady_clock::now() \
.time_since_epoch().count());
const int mod = 1000000007;
const int mod2 = 998244353;
const ld PI = acos(-1);
#define Bint __int128
#define int long long
namespace DEBUG{
  template <typename T, typename T2>
  ostream& operator<<(ostream& os, const pair<T,
  ↪  T2>& pr) {
    os << "( " << pr.first << ", " << pr.second <<
  ↪  ")";
    return os;
  }
  template <typename T>
  inline void _printv(T l, T r){
    cerr << "DEBUG: [ ";
    for(; l != r; l++)
      cerr << *l << ", ";
    cerr << "]" << endl;
  }
  template <typename T>
  inline void _debug(const char* format, T t) {
    cerr << format << '=' << t << endl;
```

```
  }
  template <class First, class... Rest>
  inline void _debug(const char* format, First first, Rest... rest) {
    while (*format != ',')
      cerr << *format++;
    cerr << '=' << first << ",";
    _debug(format + 1, rest...);
  }
  #define TEST
  #ifdef TEST
  #define printv(x, y) _printv(x, y)
  #define debug(...) cerr << "DEBUG: ",_debug(#__VA_ARGS__, __VA_ARGS__)
  #else
  #define debug(...) void(0)
  #define printv(x, y) void(0)
  #endif
} // namespace DEBUG
using namespace DEBUG;
/* ---------------------------------------- */
void solve(){
}
signed main(){
  ios::sync_with_stdio(0);
  cin.tie(0);
  int t = 1;
  // cin >> t;
  while(t--)
    solve();
}
```

1.2 vimrc

```
syntax on
set mouse=a
set nu
set ts=4
set sw=4
set smartindent
set cursorline
set hlsearch
set incsearch
set t_Co=256
nnoremap y ggyG
colorscheme afterglow
au BufNewFile *.cpp 0r ~/default_code/default.cpp | let IndentStyle = "cpp"
```

2 Data-structure

2.1 PBDS

```
gp_hash_table<T, T> h;
tree<T, null_type, less<T>, rb_tree_tag, tree_order_statistics_node_update> tr;
tr.order_of_key(x); // find x's ranking
tr.find_by_order(k); // find k-th minimum, return iterator
```

2.2 SparseTable

```
template <class T, T (*op)(T, T)> struct SparseTable{
  // idx: [0, n - 1]
  int n;
  T id;
  vector<vector<T>>tbl;
  T query(int l, int r){
    int lg = __lg(r - l + 1);
    return op(tbl[lg][l], tbl[lg][r - (1 << lg) + 1]);
  }
  SparseTable (): n(0) {}
  SparseTable (int _n, vector<T>&arrr, T _id) {
    n = _n;
    id = _id;
    int lg = __lg(n) + 2;
    tbl.resize(lg, vector<T>(n + 5, id));
    for(int i = 0; i < n; i++)
      tbl[0][i] = arrr[i];
    for(int i = 1; i <= lg; i++)
      for(int j = 0; j + (1 << (i - 1)) < n; j++)
        tbl[i][j] = op(tbl[i - 1][j], tbl[i - 1][j + (1 << (i - 1))]);
  }
  SparseTable (int _n, T *arrr, T _id) {
    n = _n;
    id = _id;
    int lg = __lg(n) + 2;
    tbl.resize(lg, vector<T>(n + 5, id));
    for(int i = 0; i < n; i++)
      tbl[0][i] = arrr[i];
    for(int i = 1; i <= lg; i++)
      for(int j = 0; j + (1 << (i - 1)) < n; j++)
        tbl[i][j] = op(tbl[i - 1][j], tbl[i - 1][j + (1 << (i - 1))]);
  }
};
```

2.3 SegmentTree

```
template <class T, T (*op)(T, T)> struct Segment_tree{
  int L, R;
  T id;
  vector<T>seg;
  void _modify(int p, T v, int l, int r, int idx = 1){
    assert(p <= r && p >= l);
    if(l == r){
      seg[idx] = v;
      return;
    }
    int mid = (l + r) >> 1;
    if(p <= mid)
      _modify(p, v, l, mid, idx << 1);
    else
      _modify(p, v, mid + 1, r, idx << 1 | 1);
    seg[idx] = op(seg[idx << 1], seg[idx << 1 | 1]);
  }
```

```
  T _query(int ql, int qr, int l, int r, int idx = 1){
    if(ql == l && qr == r)
      return seg[idx];
    int mid = (l + r) >> 1;
    if(qr <= mid)
      return _query(ql, qr, l, mid, idx << 1);
    else if(ql > mid)
      return _query(ql, qr, mid + 1, r, idx << 1 | 1);
    return op(_query(ql, mid, l, mid, idx << 1), _query(mid + 1, qr, mid + 1, r, idx << 1 | 1));
  }
  void modify(int p, T v){ _modify(p, v, L, R, 1); }
  T query(int l, int r){ return _query(l, r, L, R, 1); }
  Segment_tree(): Segment_tree(0, 0, 0) {}
  Segment_tree(int l, int r, T _id): L(l), R(r) {
    id = _id;
    seg.resize(4 * (r - l + 10));
    fill(seg.begin(), seg.end(), id);
  }
};
```

2.4 LazyTagSegtree

```
struct segment_tree{
  int seg[N << 2];
  int tag1[N << 2], tag2[N << 2];
  void down(int l, int r, int idx, int pidx){
    int v = tag1[pidx], vv = tag2[pidx];
    if(v)
      tag1[idx] = v, seg[idx] = v * (r - l + 1), tag2[idx] = 0;
    if(vv)
      tag2[idx] += vv, seg[idx] += vv * (r - l + 1);
  }
  void Set(int l, int r, int ql, int qr, int v, int idx = 1){
    if(ql == l && qr == r){
      tag1[idx] = v;
      tag2[idx] = 0;
      seg[idx] = v * (r - l + 1);
      return;
    }
    int mid = (l + r) >> 1;
    down(l, mid, idx << 1, idx);
    down(mid + 1, r, idx << 1 | 1, idx);
    tag1[idx] = tag2[idx] = 0;
    if(qr <= mid)
      Set(l, mid, ql, qr, v, idx << 1);
    else if(ql > mid)
      Set(mid + 1, r, ql, qr, v, idx << 1 | 1);
    else{
      Set(l, mid, ql, mid, v, idx << 1);
      Set(mid + 1, r, mid + 1, qr, v, idx << 1 | 1);
    }
    seg[idx] = seg[idx << 1] + seg[idx << 1 | 1];
  }
  void Increase(int l, int r, int ql, int qr, int v, int idx = 1){
    if(ql ==l && qr == r){
      tag2[idx] += v;
      seg[idx] +=  v * (r - l + 1);
      return;
    }
    int mid = (l + r) >> 1;
    down(l, mid, idx << 1, idx);
    down(mid + 1, r, idx << 1 | 1, idx);
    tag1[idx] = tag2[idx] = 0;
    if(qr <= mid)
      Increase(l, mid, ql, qr, v, idx << 1);
    else if(ql > mid)
      Increase(mid + 1, r, ql, qr, v, idx << 1 | 1);
    else{
      Increase(l, mid, ql, mid, v, idx << 1);
      Increase(mid + 1, r, mid + 1, qr, v, idx << 1 | 1);
    }
    seg[idx] = seg[idx << 1] + seg[idx << 1 | 1];
  }
  int query(int l, int r, int ql, int qr, int idx = 1){
    if(ql ==l && qr == r)
      return seg[idx];
    int mid = (l + r) >> 1;
    down(l, mid, idx << 1, idx);
    down(mid + 1, r, idx << 1 | 1, idx);
    tag1[idx] = tag2[idx] = 0;
    if(qr <= mid)
      return query(l, mid, ql, qr, idx << 1);
    else if(ql > mid)
      return query(mid + 1, r, ql, qr, idx << 1 | 1);
    return query(l, mid, ql, mid, idx << 1) + query(mid + 1, r, mid + 1, qr, idx << 1 | 1);
  }
  void modify(int l, int r, int ql, int qr, int v, int type){
    // type 1: increasement, type 2: set
    if(type == 2)
      Set(l, r, ql, qr, v);
    else
      Increase(l, r, ql, qr, v);
  }
```

2.5 LiChaoTree

```
struct line{
  int m, c;
  int val(int x){
    return m * x + c;
  }
  line(){}
  line(int _m, int _c){
    m = _m, c = _c;
  }
};
struct Li_Chao_Tree{
  line seg[N << 2];
  void ins(int l, int r, int idx, line x){
```

```
    if(l == r){
      if(x.val(l) > seg[idx].val(l))
        seg[idx] = x;
      return;
    }
    int mid = (l + r) >> 1;
    if(x.m < seg[idx].m)
      swap(x, seg[idx]);
    // ensure x.m > seg[idx].m
    if(seg[idx].val(mid) <= x.val(mid)){
      swap(x, seg[idx]);
      ins(l, mid, idx << 1, x);
    }
    else
      ins(mid + 1, r, idx << 1 | 1, x);
  }
  int query(int l, int r, int p, int idx){
    if(l == r)
      return seg[idx].val(l);
    int mid = (l + r) >> 1;
    if(p <= mid)
      return max(seg[idx].val(p), query(l, mid, p,
  ↪  idx << 1));
    else
      return max(seg[idx].val(p), query(mid + 1, r,
  ↪  p, idx << 1 | 1));
  }
```

2.6 Treap

```
struct Treap{
  Treap *l, *r;
  int pri, key, sz;
  Treap(){}
  Treap(int _v){
    l = r = NULL;
    pri = mtrd();
    key = _v;
    sz = 1;
  }
  ~Treap(){
       if ( l )
           delete l;
       if ( r )
           delete r;
    }
  void push(){
    for(auto ch : {l, r}){
      if(ch){
        // do something
      }
    }
  }
};
int getSize(Treap *t){
  return t ? t->sz : 0;
}
void pull(Treap *t){
  t->sz = getSize(t->l) + getSize(t->r) + 1;
}
Treap* merge(Treap* a, Treap* b){
  if(!a || !b)
    return a ? a : b;
  if(a->pri > b->pri){
    a->push();
    a->r = merge(a->r, b);
    pull(a);
    return a;
  }
  else{
    b->push();
    b->l = merge(a, b->l);
    pull(b);
    return b;
  }
}
void splitBySize(Treap *t, Treap *&a, Treap *&b,
  ↪  int k){
  if(!t)
    a = b = NULL;
  else if(getSize(t->l) + 1 <= k){
    a = t;
    a->push();
    splitBySize(t->r, a->r, b, k - getSize(t->l) -
  ↪  1);
    pull(a);
  }
  else{
    b = t;
    b->push();
    splitBySize(t->l, a, b->l, k);
    pull(b);
  }
}
void splitByKey(Treap *t, Treap *&a, Treap *&b, int
  ↪  k){
    if(!t)
        a = b = NULL;
    else if(t->key <= k){
        a = t;
        a->push();
        splitByKey(t->r, a->r, b, k);
        pull(a);
    }
    else{
        b = t;
        b->push();
        splitByKey(t->l, a, b->l, k);
        pull(b);
    }
}
// O(n) build treap with sorted key nodes
void traverse(Treap *t){
  if(t->l)
    traverse(t->l);
  if(t->r)
    traverse(t->r);
  pull(t);
}
Treap *build(int n){
  vector<Treap*>st(n);
  int tp = 0;
  for(int i = 0, x; i < n; i++){
    cin >> x;
    Treap *nd = new Treap(x);
    while(tp && st[tp - 1]->pri < nd->pri)
      nd->l = st[tp - 1], tp--;
    if(tp)
```

```
      st[tp - 1]->r = nd;
    st[tp++] = nd;
  }
  if(!tp){
    st[0] = NULL;
    return st[0];
  }
  traverse(st[0]);
  return st[0];
}
```

2.7 DSU

```
struct Disjoint_set{
  int n;
  vector<int>sz, p;
  int fp(int x){
    return (p[x] == -1 ? x : p[x] = fp(p[x]));
  }
  bool U(int x, int y){
    x = fp(x), y = fp(y);
    if(x == y)
      return false;
    if(sz[x] > sz[y])
      swap(x, y);
    p[x] = y;
    sz[y] += sz[x];
    return true;
  }
  Disjoint_set() {}
  Disjoint_set(int _n){
    n = _n;
    sz.resize(n, 1);
    p.resize(n, -1);
  }
};
```

2.8 RollbackDSU

```
struct Rollback_DSU{
  vector<int>p, sz;
  vector<pair<int, int>>history;
  int fp(int x){
    while(p[x] != -1)
      x = p[x];
    return x;
  }
  bool U(int x, int y){
    x = fp(x), y = fp(y);
    if(x == y){
      history.push_back(make_pair(-1, -1));
      return false;
    }
    if(sz[x] > sz[y])
      swap(x, y);
    p[x] = y;
    sz[y] += sz[x];
    history.push_back(make_pair(x, y));
    return true;
  }
  void undo(){
    if(history.empty() || history.back().first ==
    ↪ -1){
      if(!history.empty())
        history.pop_back();
      return;
    }
    auto [x, y] = history.back();
    history.pop_back();
    p[x] = -1;
    sz[y] -= sz[x];
  }
  Rollback_DSU(): Rollback_DSU(0) {}
  Rollback_DSU(int n): p(n), sz(n) {
    fill(p.begin(), p.end(), -1);
    fill(sz.begin(), sz.end(), 1);
  }
};
```

3 Graph

3.1 RoundSquareTree

```
int cnt;
int dep[N], low[N]; // dep == -1 -> unvisited
vector<int>G[N], rstree[2 * N]; // 1 ~ n: round, n
↪ + 1 ~ 2n: square
vector<int>stk;
void init(){
    cnt = n;
    for(int i = 1; i <= n; i++){
        G[i].clear();
        rstree[i].clear();
        rstree[i + n].clear();
        dep[i] = low[i] = -1;
    }
    dep[1] = low[1] = 0;
}
void tarjan(int x, int px){
    stk.push_back(x);
    for(auto i : G[x]){
        if(dep[i] == -1){
            dep[i] = low[i] = dep[x] + 1;
            tarjan(i, x);
            low[x] = min(low[x], low[i]);
            if(dep[x] <= low[i]){
                int z;
        cnt++;
                do{
                    z = stk.back();
                    rstree[cnt].push_back(z);
                    rstree[z].push_back(cnt);
                    stk.pop_back();
                }while(z != i);
                rstree[cnt].push_back(x);
                rstree[x].push_back(cnt);
            }
        }
        else if(i != px)
            low[x] = min(low[x], dep[i]);
    }
}
```

3.2 SCC

```cpp
struct SCC{
  int n;
  int cnt;
  vector<vector<int>>G, revG;
  vector<int>stk, sccid;
  vector<bool>vis;
  SCC(): SCC(0) {}
  SCC(int _n): n(_n), G(_n + 1), revG(_n + 1),
  ↪  sccid(_n + 1), vis(_n + 1), cnt(0) {}
  void addEdge(int u, int v){
    // u -> v
    assert(u > 0 && u <= n);
    assert(v > 0 && v <= n);
    G[u].push_back(v);
    revG[v].push_back(u);
  }
  void dfs1(int u){
    vis[u] = 1;
    for(int v : G[u]){
      if(!vis[v])
        dfs1(v);
    }
    stk.push_back(u);
  }
  void dfs2(int u, int k){
    vis[u] = 1;
    sccid[u] = k;
    for(int v : revG[u]){
      if(!vis[v])
        dfs2(v, k);
    }
  }
  void Kosaraju(){
    for(int i = 1; i <= n; i++)
      if(!vis[i])
        dfs1(i);
    fill(vis.begin(), vis.end(), 0);
    while(!stk.empty()){
      if(!vis[stk.back()])
        dfs2(stk.back(), ++cnt);
      stk.pop_back();
    }
  }
};
```

3.3 2SAT

```cpp
struct two_sat{
  int n;
  SCC G; // u: u, u + n: ~u
  vector<int>ans;
  two_sat(): two_sat(0) {}
  two_sat(int _n): n(_n), G(2 * _n), ans(_n + 1) {}
  void disjunction(int a, int b){
    G.addEdge((a > n ? a - n : a + n), b);
    G.addEdge((b > n ? b - n : b + n), a);
  }
  bool solve(){
    G.Kosaraju();
    for(int i = 1; i <= n; i++){
      if(G.sccid[i] == G.sccid[i + n])
        return false;
      ans[i] = (G.sccid[i] > G.sccid[i + n]);
    }
    return true;
  }
};
```

3.4 Bridge

```cpp
int dep[N], low[N];
vector<int>G[N];
vector<pair<int, int>>bridge;
void init(){
  for(int i = 1; i <= n; i++){
    G[i].clear();
    dep[i] = low[i] = -1;
  }
  dep[1] = low[1] = 0;
}
void tarjan(int x, int px){
  for(auto i : G[x]){
    if(dep[i] == -1){
      dep[i] = low[i] = dep[x] + 1;
      tarjan(i, x);
      low[x] = min(low[x], low[i]);
      if(low[i] > dep[x])
        bridge.push_back(make_pair(i, x));
    }
    else if(i != px)
      low[x] = min(low[x], dep[i]);
  }
}
```

3.5 BronKerboschAlgorithm

```cpp
vector<vector<int>>maximal_clique;
int cnt, G[N][N], all[N][N], some[N][N],
  ↪  none[N][N];
void dfs(int d, int an, int sn, int nn)
{
    if(sn == 0 && nn == 0){
    vector<int>v;
    for(int i = 0; i < an; i++)
      v.push_back(all[d][i]);
    maximal_clique.push_back(v);
    cnt++;
    }
  int u = sn > 0 ? some[d][0] : none[d][0];
    for(int i = 0; i < sn; i ++)
    {
        int v = some[d][i];
        if(G[u][v])
      continue;
        int tsn = 0, tnn = 0;
        for(int j = 0; j < an; j ++)
      all[d + 1][j] = all[d][j];
        all[d + 1][an] = v;
        for(int j = 0; j < sn; j ++)
            if(g[v][some[d][j]])
        some[d + 1][tsn ++] = some[d][j];
```

```
        for(int j = 0; j < nn; j ++)
            if(g[v][none[d][j]])
        none[d + 1][tnn ++] = none[d][j];
        dfs(d + 1, an + 1, tsn, tnn);
        some[d][i] = 0, none[d][nn ++] = v;
    }
}
void process(){
    cnt = 0;
    for(int i = 0; i < n; i ++)
    some[0][i] = i + 1;
    dfs(0, 0, n, 0);
}
```

3.6 Theorem

- Kosaraju's algorithm visit the strong connected components in topolocical order at second dfs.
- Euler's formula on planar graph: $V - E + F = C + 1$
- Kuratowski's theorem: A simple graph G is a planar graph iff G doesn't has a subgraph H such that H is homeomorphic to K_5 or $K_{3,3}$
- A complement set of every vertex cover correspond to a independent set. $\Rightarrow$ Number of vertex of maximum independent set + Number of vertex of minimum vertex cover $= V$
- Maximum independent set of G = Maximum clique of the complement graph of G .
- A planar graph G colored with three colors iff there exist a maximal clique I such that $G - I$ is a bipartite.

4 Tree

4.1 HLD

```
struct Heavy_light_decomposition{
  int n;
  int cnt;
  vector<int>dep, sz, mx_son, fa, top;
  vector<int>id, inv_id;
  vector<vector<pii>>G;
  void addEdge(int u, int v, int c){
    G[u].push_back(make_pair(v, c));
    G[v].push_back(make_pair(u, c));
  }
  void dfs1(int x, int px){
    dep[x] = dep[px] + 1;
    sz[x] = 1;
    fa[x] = px;
    for(auto [i, c] : G[x])if(i != px){
      dfs1(i, x);
      sz[x] += sz[i];
      mx_son[x] = (sz[i] > sz[mx_son[x]] ? i :
↪   mx_son[x]);
    }
  }
  void dfs2(int x, int root){
    top[x] = root;
    id[x] = ++cnt;
    inv_id[cnt] = x;
    if(mx_son[x])
      dfs2(mx_son[x], root);
    for(auto [i, c] : G[x]){
      if(i != fa[x] && i != mx_son[x])
        dfs2(i, i);
    }
  }
  void decompose(){
    dfs1(1, 0);
    dfs2(1, 1);
    // initialize data structure
  }
  int lca(int u, int v){
    int mx = 0;
    while(top[u] != top[v]){
      if(dep[top[u]] < dep[top[v]])
        swap(u, v);
      u = fa[top[u]];
    }
    if(dep[u]>dep[v])
      swap(u, v);
    return u;
  }
  Heavy_light_decomposition():
↪   Heavy_light_decomposition(0) {}
  Heavy_light_decomposition(int _n): n(_n), cnt(0)
↪   {
    dep.resize(_n + 1, 0);
    sz.resize(_n + 1, 0);
    mx_son.resize(_n + 1, 0);
    fa.resize(_n + 1);
    top.resize(_n + 1);
    id.resize(_n + 1);
    inv_id.resize(_n + 1);
    G.resize(_n + 1, vector<pii>(0));
  }
};
```

5 Geometry

5.1 Point

```
template<class T> struct Point {
  T x, y;
  Point(): x(0), y(0) {};
  Point(T a, T b): x(a), y(b) {};
  Point(pair<T, T>p): x(p.first), y(p.second) {};
  Point operator + (const Point& rhs){ return
↪   Point(x + rhs.x, y + rhs.y); }
  Point operator - (const Point& rhs){ return
↪   Point(x - rhs.x, y - rhs.y); }
  Point operator * (const int& rhs){ return Point(x
↪   * rhs, y * rhs); }
  Point operator / (const int& rhs){ return Point(x
↪   / rhs, y / rhs); }
  T cross(Point rhs){ return x * rhs.y - y * rhs.x;
↪   }
  T dot(Point rhs){ return x * rhs.x + y * rhs.y; }
  T cross2(Point a, Point b){ // (a - this) cross
↪   (b - this)
    return (a - *this).cross(b - *this);
```

```
  }
  T dot2(Point a, Point b){ // (a - this) dot (b - this)
    return (a - *this).dot(b - *this);
  }
};
```

5.2 Geometry

```
template<class T> int ori(Point<T>a, Point<T>b, Point<T>c){
  // sign of (b - a) cross(c - a)
  auto res = a.cross2(b, c);
  // if type if double
  // if(abs(res) <= eps)
  if(res == 0)
    return 0;
  return res > 0 ? 1 : -1;
}
template<class T> bool collinearity(Point<T>a, Point<T>b, Point<T>c){
  // if type is double
  // return abs(c.cross2(a,b)) <= eps;
  return c.cross2(a, b) == 0;
}
template<class T> bool between(Point<T>a, Point<T>b, Point<T>c){
  // check if c is between a, b
  return collinearity(a, b, c) && c.dot2(a, b) <= 0;
}
template<class T> bool seg_intersect(Point<T>p1, Point<T>p2, Point<T>p3, Point<T>p4){
  // seg (p1, p2), seg(p3, p4)
  int a123 = ori(p1, p2, p3);
  int a124 = ori(p1, p2, p4);
  int a341 = ori(p3, p4, p1);
  int a342 = ori(p3, p4, p2);
  if(a123 == 0 && a124 == 0)
    return between(p1, p2, p3) || between(p1, p2, p4) || between(p3, p4, p1) || between(p3, p4, p2);
  return a123 * a124 <= 0 && a341 * a342 <= 0;
}
template<class T> Point<T> intersect_at(Point<T> a, Point<T> b, Point<T> c, Point<T> d) {
  // line(a, b), line(c, d)
  T a123 = a.cross(b, c);
  T a124 = a.cross(b, d);
  return (d * a123 - c * a124) / (a123 - a124);
}
template<class T> int
  point_in_convex_polygon(vector<Point<T>>& a, Point<T>p){
  // 1: IN
  // 0: OUT
  // -1: ON
  // the points of convex polygon must sort in counter-clockwise order
  int n = a.size();
  if(between(a[0], a[1], p) || between(a[0], a[n - 1], p))
    return -1;
  int l = 0, r = n - 1;
  while(l <= r){
    int mid = (l + r) >> 1;
    auto a1 = a[0].cross2(a[mid], p);
    auto a2 = a[0].cross2(a[(mid + 1) % n], p);
    if(a1 >= 0 && a2 <= 0){
      auto res = a[mid].cross2(a[(mid + 1) % n], p);
      return res > 0 ? 1 : (res >= 0 ? -1 : 0);
    }
    else if(a1 < 0)
      r = mid - 1;
    else
      l = mid + 1;
  }
  return 0;
}
template<class T> int
  point_in_simple_polygon(vector<Point<T>>&a, Point<T>p, Point<T>INF_point){
  // 1: IN
  // 0: ON
  // -1: OUT
  // a[i] must adjacent to a[(i + 1) % n] for all i
  // collinearity(a[i], p, INF_point) must be false for all i
  // we can let the slope of line(p, INF_point) be irrational (e.g. PI)
  int ans = -1;
  for(auto l = prev(a.end()), r = a.begin(); r != a.end(); l = r++){
    if(between(*l, *r, p))
      return 0;
    if(seg_intersect(*l, *r, p, INF_point)){
      ans *= -1;
      if(collinearity(*l, p, INF_point))
        assert(0);
    }
  }
  return ans;
}
template<class T> T area(vector<Point<T>>&a){
  // remember to divide 2 after calling this function
  if(a.size() <= 1)
    return 0;
  T ans = 0;
  for(auto  l = prev(a.end()), r = a.begin(); r != a.end(); l = r++)
    ans += l->cross(*r);
  return abs(ans);
}
```

5.3 ConvexHull

```
template<class T> vector<Point<T>>
  convex_hull(vector<Point<T>>&a){
  int n  = a.size();
  sort(a.begin(), a.end(), [](Point<T>p1, Point<T>p2){
    if(p1.x == p2.x)
      return p1.y < p2.y;
    return p1.x < p2.x;
```

```
  });
  int m = 0, t = 1;
  vector<Point<T>>ans;
  auto addPoint = [&](const Point<T>p) {
    while(m > t && ans[m - 2].cross2(ans[m - 1], p)
↪   <= 0)
      ans.pop_back(), m--;
    ans.push_back(p);
    m++;
  };
  for(int i = 0; i < n; i++)
    addPoint(a[i]);
  t = m;
  for(int i = n - 2; ~i; i--)
    addPoint(a[i]);
  if(a.size() > 1)
    ans.pop_back();
  return ans;
}
```

5.4 Theorem

- Pick's theorem: Suppose that a polygon has integer coordinates for all of its vertices. Let i be the number of integer points interior to the polygon, b be the number of integer points on its boundary (including both vertices and points along the sides). Then the area A of this polygon is:

$$A = i + \frac{b}{2} - 1$$

6 String

6.1 RollingHash

```
struct Rolling_Hash{
  int n;
  const int P[5] = {146672737, 204924373,
↪   585761567, 484547929, 116508269};
  const int M[5] = {922722049, 952311013,
↪   955873937, 901981687, 993179543};
  vector<int>PW[5], pre[5], suf[5];
  Rolling_Hash(): Rolling_Hash("") {}
  Rolling_Hash(string s): n(s.size()){
    for(int i = 0; i < 5; i++){
      PW[i].resize(n), pre[i].resize(n),
↪   suf[i].resize(n);
      PW[i][0] = 1, pre[i][0] = s[0] - 'a';
      suf[i][n - 1] = s[n - 1] - 'a';
    }
    for(int i = 1; i < n; i++){
      for(int j = 0; j < 5; j++){
        PW[j][i] = PW[j][i - 1] * P[j] % M[j];
        pre[j][i] = (pre[j][i - 1] * P[j] + s[i] -
↪   'a') % M[j];
      }
    }
    for(int i = n - 2; i >= 0; i--){
      for(int j = 0; j < 5; j++)
        suf[j][i] = (suf[j][i + 1] * P[j] + s[i] -
↪   'a') % M[j];
    }
  }
  int _substr(int k, int l, int r) {
    int res = pre[k][r];
    if(l > 0)
      res -= 1LL * pre[k][l - 1] * PW[k][r - l + 1]
↪   % M[k];
    if(res < 0)
      res += M[k];
    return res;
  }
  vector<int>substr(int l, int r){
    vector<int>res(5);
    for(int i = 0; i < 5; ++i)
      res[i] = _substr(i, l, r);
    return res;
  }
};
```

6.2 SuffixArray

```
struct Suffix_Array{
  int n, m; // m is the range of s
  string s;
  vector<int>sa, rk, lcp;
  // sa[i]: the i-th smallest suffix
  // rk[i]: the rank of suffix i (i.e. s[i, n - 1])
  // lcp[i]: the longest common prefix of sa[i] and
↪   sa[i - 1]
  Suffix_Array(): Suffix_Array(0, 0, "") {};
  Suffix_Array(int _n, int _m, string _s): n(_n),
↪   m(_m), sa(_n), rk(_n), lcp(_n), s(_s) {}
  void Sort(int k, vector<int>&bucket,
↪   vector<int>&idx, vector<int>&lst){
    for(int i = 0; i < m; i++)
      bucket[i] = 0;
    for(int i = 0; i < n; i++)
      bucket[lst[i]]++;
    for(int i = 1; i < m; i++)
      bucket[i] += bucket[i-1];
    int p = 0;
    // update index
    for(int i = n - k; i < n; i++)
      idx[p++] = i;
    for(int i = 0; i < n; i++)
      if(sa[i] >= k)
        idx[p++] = sa[i] - k;
    for(int i = n - 1; i >= 0; i--)
      sa[--bucket[lst[idx[i]]]] = idx[i];
  }
  void build(){
    vector<int>idx(n), lst(n), bucket(max(n, m));
    for(int i = 0; i < n; i++)
      bucket[lst[i] = (s[i] - 'a')]++; // may
↪   change
    for(int i = 1; i < m; i++)
      bucket[i] += bucket[i - 1];
    for(int i = n - 1; i >= 0; i--)
      sa[--bucket[lst[i]]] = i;
    for(int k = 1; k < n; k <<= 1){
      Sort(k, bucket, idx, lst);
      // update rank
      int p = 0;
      idx[sa[0]] = 0;
      for(int i = 1; i < n; i++){
```

```
        int a = sa[i], b = sa[i - 1];
        if(lst[a] == lst[b] && a + k < n && b + k < n && lst[a + k] == lst[b + k]);
          else
            p++;
        idx[sa[i]] = p;
      }
      if(p == n - 1)
        break;
      for(int i = 0; i < n; i++)
        lst[i] = idx[i];
      m = p + 1;
    }
    for(int i = 0; i < n; i++)
      rk[sa[i]] = i;
    buildLCP();
  }
  void buildLCP(){
    // lcp[rk[i]] >= lcp[rk[i - 1]] - 1
    int v = 0;
    for(int i = 0; i < n; i++){
      if(!rk[i])
        lcp[rk[i]] = 0;
      else{
        if(v)
          v--;
        int p = sa[rk[i] - 1];
        while(i + v < n && p + v < n && s[i + v] == s[p + v])
          v++;
        lcp[rk[i]] = v;
      }
    }
  }
};
```

6.3 KMP

```
struct KMP {
  int n;
  string s;
  vector<int>fail;
  // s: pattern, t: text => find s in t
  int match(string &t){
    int ans = 0, m = t.size(), j = -1;
    for(int i = 0; i < m; i++){
      while(j != -1 && t[i] != s[j + 1])
        j = fail[j];
      if(t[i] == s[j + 1])
        j++;
      if(j == n - 1){
        ans++;
        j = fail[j];
      }
    }
    return ans;
  }
  KMP(string &_s){
    s = _s;
    n = s.size();
    fail = vector<int>(n, -1);
    int j = -1;
    for(int i = 1; i < n; i++){
      while(j != -1 && s[i] != s[j + 1])
        j = fail[j];
      if(s[i] == s[j + 1])
        j++;
      fail[i] = j;
    }
  }
};
```

6.4 Trie

```
struct Node {
  int hit = 0;
  Node *next[26];
  // 26 is the size of the set of characters
  // a - z
  Node(){
    for(int i = 0; i < 26; i++)
      next[i] = NULL;
  }
};
void insert(string &s, Node *node){
  // node cannot be null
  for(char v : s){
    if(node->next[v - 'a'] == NULL)
      node->next[v - 'a'] = new Node;
    node = node->next[v - 'a'];
  }
  node->hit++;
}
```

6.5 Zvalue

```
struct Zvalue {
  const string inf = "$"; // character that has never used
  vector<int>z;
  // s: pattern, t: text => find s in t
  int match(string &s, string &t){
    string fin = s + inf + t;
    build(fin);
    int n = s.size(), m = t.size();
    int ans = 0;
    for(int i = n + 1; i < n + m + 1; i++)
      if(z[i] == n)
        ans++;
    return ans;
  }
  void build(string &s){
    int n = s.size();
    z = vector<int>(n, 0);
    int l = 0, r = 0;
    for(int i = 0; i < n; i++){
      z[i] = max(min(z[i - l], r - i), 0LL);
      while(i + z[i] < n && s[z[i]] == s[i + z[i]])
        l = i, r = i + z[i], z[i]++;
    }
  }
};
```

7 Flow

7.1 Dinic

```cpp
struct Max_Flow{
  struct Edge{
    int cap, to, rev;
    Edge(){}
    Edge(int _to, int _cap, int _rev){
      to = _to, cap = _cap, rev = _rev;
    }
  };
  const int inf = 1e18+10;
  int s, t; // start node and end node
  vector<vector<Edge>>G;
  vector<int>dep;
  vector<int>iter;
  void addE(int u, int v, int cap){
    G[u].pb(Edge(v, cap, G[v].size()));
    // direct graph
    G[v].pb(Edge(u, 0, G[u].size() - 1));
    // undirect graph
    // G[v].pb(Edge(u, cap, G[u].size() - 1));
  }
  void bfs(){
    queue<int>q;
    q.push(s);
    dep[s] = 0;
    while(!q.empty()){
      int cur = q.front();
      q.pop();
      for(auto i : G[cur]){
        if(i.cap > 0 && dep[i.to] == -1){
          dep[i.to] = dep[cur] + 1;
          q.push(i.to);
        }
      }
    }
  }
  int dfs(int x, int fl){
    if(x == t)
      return fl;
    for(int _ = iter[x] ; _ < G[x].size() ; _++){
      auto &i = G[x][_];
      if(i.cap > 0 && dep[i.to] == dep[x] + 1){
        int res = dfs(i.to, min(fl, i.cap));
        if(res <= 0)
          continue;
        i.cap -= res;
        G[i.to][i.rev].cap += res;
        return res;
      }
      iter[x]++;
    }
    return 0;
  }
  int Dinic(){
    int res = 0;
    while(true){
      fill(all(dep), -1);
      fill(all(iter), 0);
      bfs();
      if(dep[t] == -1)
        break;
      int cur;
      while((cur = dfs(s, INF)) > 0)
        res += cur;
    }
    return res;
  }
  void init(int _n, int _s, int _t){
    s = _s, t = _t;
    G.resize(_n + 5);
    dep.resize(_n + 5);
    iter.resize(_n + 5);
  }
};
```

7.2 MCMF

```cpp
struct MCMF{
  struct Edge{
    int from, to;
    int cap, cost;
    Edge(int f, int t, int ca, int co): from(f),
    ↪  to(t), cap(ca), cost(co) {}
  };
  int n, s, t;
  vector<Edge>edges;
  vector<vector<int>>G;
  vector<int>d;
  vector<int>in_queue, prev_edge;
  MCMF(){}
  MCMF(int _n, int _s, int _t): n(_n), G(_n + 1),
  ↪  d(_n + 1), in_queue(_n + 1), prev_edge(_n + 1),
  ↪  s(_s), t(_t) {}
  void addEdge(int u, int v, int cap, int cost){
    G[u].push_back(edges.size());
    edges.push_back(Edge(u, v, cap, cost));
    G[v].push_back(edges.size());
    edges.push_back(Edge(v, u, 0, -cost));
  }
  bool bfs(){
    bool found = false;
    fill(d.begin(), d.end(), (int)1e18+10);
    fill(in_queue.begin(), in_queue.end(), false);
    d[s] = 0;
    in_queue[s] = true;
    queue<int>q;
    q.push(s);
    while(!q.empty()){
      int u = q.front();
      q.pop();
      if(u == t)
        found = true;
      in_queue[u] = false;
      for(auto &id : G[u]){
        Edge e = edges[id];
        if(e.cap > 0 && d[u] + e.cost < d[e.to]){
          d[e.to] = d[u] + e.cost;
          prev_edge[e.to] = id;
          if(!in_queue[e.to]){
            in_queue[e.to] = true;
            q.push(e.to);
          }
        }
      }
```

```
    }
    return found;
  }
  pair<int, int>flow(){
    // return (cap, cost)
    int cap = 0, cost = 0;
    while(bfs()){
      int send = (int)1e18 + 10;
      int u = t;
      while(u != s){
        Edge e = edges[prev_edge[u]];
        send = min(send, e.cap);
        u = e.from;
      }
      u = t;
      while(u != s){
        Edge &e = edges[prev_edge[u]];
        e.cap -= send;
        Edge &e2 = edges[prev_edge[u] ^ 1];
        e2.cap += send;
        u = e.from;
      }
      cap += send;
      cost += send * d[t];
    }
    return make_pair(cap, cost);
  }
};
```

8 Math

8.1 FastPow

```
long long qpow(long long x, long long powcnt, long long tomod){
  long long res = 1;
  for(; powcnt ; powcnt >>= 1 , x = (x * x) % tomod)
    if(1 & powcnt)
      res = (res * x) % tomod;
  return (res % tomod);
}
```

8.2 EXGCD

```
// ax + by = c
// return (gcd(a, b), x, y)
tuple<long long, long long, long long>exgcd(long long a, long long b){
  if(b == 0)
    return make_tuple(a, 1, 0);
  auto[g, x, y] = exgcd(b, a % b);
  return make_tuple(g, y, x - (a / b) * y);
}
```

8.3 EXCRT

```
long long inv(long long x){ return qpow(x, mod - 2, mod); }
long long mul(long long x, long long y, long long m){
   x = ((x % m) + m) % m, y = ((y % m) + m) % m;
  long long ans = 0;
  while(y){
    if(y & 1)
      ans = (ans + x) % m;
    x = x * 2 % m;
    y >>= 1;
  }
  return ans;
}
pii ExCRT(long long r1, long long m1, long long r2, long long m2){
  long long g, x, y;
  tie(g, x, y) = exgcd(m1, m2);
  if((r1 - r2) % g)
    return {-1, -1};
  long long lcm = (m1 / g) * m2;
  long long res = (mul(mul(m1, x, lcm), ((r2 - r1) / g), lcm) + r1) % lcm;
  res = (res + lcm) % lcm;
  return {res, lcm};
}
void solve(){
  long long n, r, m;
  cin >> n;
  cin >> m >> r; // x == r (mod m)
  for(long long i = 1 ; i < n ; i++){
    long long r1, m1;
    cin >> m1 >> r1;
    if(r != -1 && m != -1)
      tie(r, m) = ExCRT(r m, r1, m1);
  }
  if(r == -1 && m == -1)
    cout << "no solution\n";
  else
    cout << r << '\n';
}
```

8.4 FFT

```
struct Polynomial{
  int deg;
  vector<int>x;
  void FFT(vector<complex<double>>&a, bool invert){
    int a_sz = a.size();
    for(int len = 1; len < a_sz; len <<= 1){
      for(int st = 0; st < a_sz; st += 2 * len){
        double angle = PI / len * (invert ? -1 : 1);
        complex<double>wnow(1), w(cos(angle), sin(angle));
        for(int i = 0; i < len; i++){
          auto a0 = a[st + i], a1 = a[st + len + i];
          a[st + i] = a0 + wnow * a1;
          a[st + i + len] = a0 - wnow * a1;
```

```
          wnow *= w;
        }
      }
    }
    if(invert)
      for(auto &i : a)
        i /= a_sz;
  }
  void change(vector<complex<double>>&a){
    int a_sz = a.size();
    vector<int>rev(a_sz);
    for(int i = 1; i < a_sz; i++){
      rev[i] = rev[i / 2] / 2;
      if(i & 1)
        rev[i] += a_sz / 2;
    }
    for(int i = 0; i < a_sz; i++)
      if(i < rev[i])
        swap(a[i], a[rev[i]]);
  }
  Polynomial multiply(Polynomial const&b){
    vector<complex<double>>A(x.begin(), x.end()),
    B(b.x.begin(), b.x.end());
    int mx_sz = 1;
    while(mx_sz < A.size() + B.size())
      mx_sz <<= 1;
    A.resize(mx_sz);
    B.resize(mx_sz);
    change(A);
    change(B);
    FFT(A, 0);
    FFT(B, 0);
    for(int i = 0; i < mx_sz; i++)
      A[i] *= B[i];
    change(A);
    FFT(A, 1);
    Polynomial res(mx_sz);
    for(int i = 0; i < mx_sz; i++)
      res.x[i] = round(A[i].real());
    while(!res.x.empty() && res.x.back() == 0)
      res.x.pop_back();
    res.deg = res.x.size();
    return res;
  }
  Polynomial(): Polynomial(0) {}
  Polynomial(int Size): x(Size), deg(Size) {}
};
```

8.5 NTT

```
/*
p = r * 2^k + 1
p          r   k   root
998244353      119  23  3
2013265921      15  27  31
2061584302081    15  37  7
*/
template<int MOD, int RT>
struct NTT {
    #define OP(op) static int op(int x, int y)
    OP(add) { return (x += y) >= MOD ? x - MOD : x; }
    OP(sub) { return (x -= y) < 0 ? x + MOD : x; }
    OP(mul) { return ll(x) * y % MOD; } // multiply by bit if p * p > 9e18
    static int mpow(int a, int n) {
        int r = 1;
        while (n) {
            if (n % 2) r = mul(r, a);
            n /= 2, a = mul(a, a);
        }
        return r;
    }
  static const int MAXN = 1 << 21;
    static int minv(int a) { return mpow(a, MOD - 2); }
    int w[MAXN];
    NTT() {
        int s = MAXN / 2, dw = mpow(RT, (MOD - 1) / MAXN);
        for (; s; s >>= 1, dw = mul(dw, dw)) {
            w[s] = 1;
            for (int j = 1; j < s; ++j)
                w[s + j] = mul(w[s + j - 1], dw);
        }
    }
    void apply(vector<int>&a, int n, bool inv = 0) {
        for (int i = 0, j = 1; j < n - 1; ++j) {
            for (int k = n >> 1; (i ^= k) < k; k >>= 1);
            if (j < i) swap(a[i], a[j]);
        }
        for (int s = 1; s < n; s <<= 1) {
            for (int i = 0; i < n; i += s * 2) {
                for (int j = 0; j < s; ++j) {
                    int tmp = mul(a[i + s + j], w[s + j]);
                    a[i + s + j] = sub(a[i + j], tmp);
                    a[i + j] = add(a[i + j], tmp);
                }
            }
        }
        if(!inv)
      return;
        int iv = minv(n);
    if(n > 1)
      reverse(next(a.begin()), a.end());
        for (int i = 0; i < n; ++i)
      a[i] = mul(a[i], iv);
    }
  vector<int>convolution(vector<int>&a, vector<int>&b){
    int sz = a.size() + b.size() - 1, n = 1;
    while(n <= sz)
      n <<= 1; // check n <= MAXN
    vector<int>res(n);
    a.resize(n), b.resize(n);
    apply(a, n);
    apply(b, n);
    for(int i = 0; i < n; i++)
      res[i] = mul(a[i], b[i]);
    apply(res, n, 1);
    return res;
  }
};
```

8.6 GeneratingFunctions

- Ordinary Generating Function $A(x) = \sum_{i \geq 0} a_i x^i$
  - $A(rx) \Rightarrow r^n a_n$
  - $A(x) + B(x) \Rightarrow a_n + b_n$
  - $A(x)B(x) \Rightarrow \sum_{i=0}^{n} a_i b_{n-i}$
  - $A(x)^k \Rightarrow \sum_{i_1+i_2+\cdots+i_k=n} a_{i_1} a_{i_2} \ldots a_{i_k}$
  - $xA(x)' \Rightarrow na_n$
  - $\frac{A(x)}{1-x} \Rightarrow \sum_{i=0}^{n} a_i$
- Exponential Generating Function $A(x) = \sum_{i \geq 0} \frac{a_i}{i!} x_i$
  - $A(x) + B(x) \Rightarrow a_n + b_n$
  - $A^{(k)}(x) \Rightarrow a_{n+k}$
  - $A(x)B(x) \Rightarrow \sum_{i=0}^{n} nia_i b_{n-i}$
  - $A(x)^k \Rightarrow \sum_{i_1+i_2+\cdots+i_k=n} ni_1, i_2, \ldots, i_k a_{i_1} a_{i_2} \ldots a_{i_k}$
  - $xA(x) \Rightarrow na_n$
- Special Generating Function
  - $(1+x)^n = \sum_{i \geq 0} nix^i$
  - $\frac{1}{(1-x)^n} = \sum_{i \geq 0} in-1x^i$

8.7 Numbers

- Stirling numbers of the second kind Partitions of n distinct elements into exactly k groups. $S(n,k) = S(n-1,k-1) + kS(n-1,k), S(n,1) = S(n,n) = 1$ $S(n,k) = \frac{1}{k!}\sum_{i=0}^{k}(-1)^{k-i}\binom{k}{i}i^n$ $x^n = \sum_{i=0}^{n} S(n,i)(x)_i$
- Catalan numbers $C_n = \frac{1}{n+1}2nn = 2nn - 2nn + 1$, $\forall n \geq 0$ $C_{n+1} = \sum_{i=0}^{n} C_i C_{n-i} = \frac{2(2n+1)}{n+2} C_n,\ C_0 = 1$

8.8 Theorem

- Cayley's Formula
  - Given a degree sequence $d_1, d_2, \ldots, d_n$ for each *labeled* vertices, there are $\frac{(n-2)!}{(d_1-1)!(d_2-1)!\cdots(d_n-1)!}$ spanning trees.
  - Let $T_{n,k}$ be the number of *labeled* forests on n vertices with k components, such that vertex $1, 2, \ldots, k$ belong to different components. Then $T_{n,k} = kn^{n-k-1}$.
- Erdős–Gallai theorem A sequence of nonnegative integers $d_1 \geq \cdots \geq d_n$ can be represented as the degree sequence of a finite simple graph on n vertices if and only if $d_1 + \cdots + d_n$ is even and $\sum_{i-1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k)$ holds for every $1 \leq k \leq n$.
- Gale–Ryser theorem A pair of sequences of nonnegative integers $a_1 \geq \cdots \geq a_n$ and $b_1, \ldots, b_n$ is bigraphic if and only if $\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} b_i$ and $\sum_{i=1}^{k} a_i \leq \sum_{i=1}^{n} \min(b_i, k)$ holds for every $1 \leq k \leq n$.
- Flooring and Ceiling function identity
  - $\lfloor \frac{\lfloor \frac{a}{b} \rfloor}{c} \rfloor = \lfloor \frac{a}{bc} \rfloor$
  - $\lceil \frac{\lceil \frac{a}{b} \rceil}{c} \rceil = \lceil \frac{a}{bc} \rceil$
  - $\lceil \frac{a}{b} \rceil \leq \frac{a+b-1}{b}$
  - $\lfloor \frac{a}{b} \rfloor \leq \frac{a-b+1}{b}$
- Möbius inversion formula
  - $f(n) = \sum_{d|n} g(d) \Leftrightarrow g(n) = \sum_{d|n} \mu(d) f(\frac{n}{d})$
  - $f(n) = \sum_{n|d} g(d) \Leftrightarrow g(n) = \sum_{n|d} \mu(\frac{d}{n}) f(d)$
  - $\sum_{d|n}^{n=1} \mu(d) = 1$
  - $\sum_{d|n}^{n\neq 1} \mu(d) = 0$
- Spherical cap
  - A portion of a sphere cut off by a plane.
  - r: sphere radius, a: radius of the base of the cap, h: height of the cap, θ: $\arcsin(a/r)$.
  - Volume $= \pi h^2(3r-h)/3 = \pi h(3a^2+h^2)/6 = \pi r^3(2+\cos\theta)(1-\cos\theta)^2/3$.
  - Area $= 2\pi rh = \pi(a^2+h^2) = 2\pi r^2(1-\cos\theta)$.