LIN241

Introduction to Semantics

Week 7

Quantifiers

Variables and quantifiers

- Predicate Logic has two quantifiers:
  - ∀ (the universal quantifier)
  - ∃ (the existential quantifier)
- Here is an example of universal quantification:
  - ∀x[LINGUIST(x)]
  - 'Every x is a linguist'

Variables and quantifiers

- Predicate Logic has two quantifiers:
  - ∀ (the universal quantifier)
  - ∃ (the existential quantifier)
- Here is an example of existential quantification:
  - ∃x[LINGUIST(x)]
  - 'There is some x such that x is a linguist'

Variables and quantifiers

- In the following statements, 'x' is a variable:
  - $\forall x[\text{LINGUIST}(x)]$
  - $\exists x[\text{LINGUIST}(x)]$
- In these statements, 'x' does not denote any individual

Variables and quantifiers

- The quantifiers tell us to consider different values for the variable:
  - ∀x[LINGUIST(x)]
    - True in a model M iff every entity in the universe U of M is a linguist
  - ∃x[LINGUIST(x)]
    - True in a model M iff at least one entity in the universe U of M is a linguist

Statements with multiple quantifiers

- Formulas of Predicate Logic can contain several quantifiers:
  - ∀x[∃y[LOVE(x,y)]]
  - ∃y[∀x[LOVE(x,y)]]
- We use the following symbols for variables:
  - x, x', x'', ...
  - y, y', y'', ...
  - z, z', z'', ...

Statements with multiple quantifiers

- With multiple quantifiers, order matters.
  - $\forall x[\exists y[LOVE(x,y)]]$

    For every x, there is a y such that x loves y
  - $\exists y[\forall x[LOVE(x,y)]]$

    There is a y such that for every x, x loves y

Translating sentences with quantifiers

- English sentences with quantifiers have two logical parts:
  - a restriction
  - a nuclear scope
- The restriction is the noun phrase that is determined by the quantifier.
- The nuclear scope is the rest.
- These two parts must appear in the translation into Predicate Logic.

Translating sentences with quantifiers

- Some examples:
  - **Every student is happy.**

    Restriction: **student**

    Nuclear scope: **__ is happy**
  - **John met every student.**

    Restriction: **student**

    Nuclear scope: **John met __**

Existential quantifiers in subject position

- Informal Guidelines (assuming active voice):
    - Translate the predicate and use a variable as its first argument.
    - Translate the restriction and use the same variable as its argument.
    - Conjoin the two formulas with '&'
    - Put the existential quantifier in front of the formula, with the aforementioned variable.

Existential quantifiers in subject position

- Translate the following into Predicate Logic:
    - **Some student is happy.**

Existential quantifiers in subject position

- Translate the following into Predicate Logic:
    - **Some student is happy.**

      $\exists$x[STUDENT(x) & HAPPY(x)]

Existential quantifiers in subject position

- Translate the following into Predicate Logic:
  - **Some student is happy.**

    ∃x[STUDENT(x) & HAPPY(x)]
  - **A policeman arrested Pia**.

Existential quantifiers in subject position

- Translate the following into Predicate Logic:
  - **Some student is happy.**

    $\exists x[\mathrm{STUDENT}(x) \,\&\, \mathrm{HAPPY}(x)]$

  - **A policeman arrested Pia**.

    $\exists x[\mathrm{POLICEMAN}(x) \,\&\, \mathrm{ARREST}(x,p)]$

Existential quantifiers in subject position

- **Something** and **someone** have built in restrictions:
  - **Something exploded**.

    $\exists x[\text{THING}(x)\ \&\ \text{EXPLODE}(x)]$

  - **Someone arrived**.

    $\exists x[\text{PERSON}(x)\ \&\ \text{ARRIVE}(x)]$

- The same holds for **everything** and **everyone**.

Existential quantifiers in object position

- Informal Guidelines (assuming active voice):
  - Translate the predicate and use a variable as its second argument.
  - Translate the restriction and use the same variable as its argument.
  - Conjoin the two formulas with '&'
  - Put the existential quantifier in front of the formula, with the aforementioned variable.

Existential quantifiers in object position

- Translate the following into Predicate Logic:
  - **Conan interviewed some politician.**

Existential quantifiers in object position

- Translate the following into Predicate Logic:
    - **Conan interviewed some politician.**

      $\exists x[\text{POLITICIAN}(x) \ \& \ \text{INTERVIEW}(c,x)]$

Existential quantifiers in object position

- Translate the following into Predicate Logic:
  - **Conan interviewed some politician.**

    $\exists x[POLITICIAN(x) \,\&\, INTERVIEW(c,x)]$
  - **Conan interviewed someone.**

Existential quantifiers in object position

- Translate the following into Predicate Logic:
  - **Conan interviewed some politician.**

    ∃x[POLITICIAN(x) & INTERVIEW(c,x)]

  - **Conan interviewed someone.**

    ∃x[PERSON(x) & INTERVIEW(c,x)]

Universal quantifiers in subject position

- Informal Guidelines (assuming active voice):
  - Translate the predicate and use a variable as its first argument.
  - Translate the restriction and use the same variable as its argument.
  - Relate the two formulas with '→'
  - Put the universal quantifier in front of the formula, with the aforementioned variable.

Universal quantifiers in subject position

- Translate the following into Predicate Logic:
    - **Every student is happy**.

Universal quantifiers in subject position

- Translate the following into Predicate Logic:
  - **Every student is happy**.

    $\forall x[\text{STUDENT}(x) \rightarrow \text{HAPPY}(x)]$

Universal quantifiers in subject position

- Translate the following into Predicate Logic:
  - **Every student is happy**.

    $\forall x[\text{STUDENT}(x) \rightarrow \text{HAPPY}(x)]$

  - **Every policeman knows Pia**.

Universal quantifiers in subject position

- Translate the following into Predicate Logic:
  - **Every student is happy**.

    $\forall x[\mathrm{STUDENT}(x) \rightarrow \mathrm{HAPPY}(x)]$
  - **Every policeman knows Pia**.

    $\forall x[\mathrm{POLICEMAN}(x) \rightarrow \mathrm{KNOW}(x,p)]$

Universal quantifiers in subject position

- Translate the following into Predicate Logic:
  - **Every student is happy**.

    $\forall x[\text{STUDENT}(x) \rightarrow \text{HAPPY}(x)]$
  - **Every policeman knows Pia**.

    $\forall x[\text{POLICEMAN}(x) \rightarrow \text{KNOW}(x,p)]$
  - **Everyone knows Pia**.

Universal quantifiers in subject position

- Translate the following into Predicate Logic:
  - **Every student is happy**.

    $\forall x[\mathrm{STUDENT}(x) \rightarrow \mathrm{HAPPY}(x)]$

  - **Every policeman knows Pia**.

    $\forall x[\mathrm{POLICEMAN}(x) \rightarrow \mathrm{KNOW}(x,p)]$

  - **Everyone knows Pia**.

    $\forall x[\mathrm{PERSON}(x) \rightarrow \mathrm{KNOW}(x,p)]$

Universal quantifiers in subject position

- Note that 3 is not a correct translation of 1, only 2 is:

1. **Every student is happy**.
2. $\forall x[\text{STUDENT}(x) \rightarrow \text{HAPPY}(x)]$
3. $\forall x[\text{STUDENT}(x) \,\&\, \text{HAPPY}(x)]$

- 3 means 'every entity is a student and is happy'
  - 3 entails that every entity is a student and that every entity is happy.
  - 1 does not entail this

Universal quantifiers in object position

- Informal Guidelines (assuming active voice):
  - Translate the predicate and use a variable as its second argument.
  - Translate the restriction and use the same variable as its argument.
  - Relate the two formulas with →
  - Put the universal quantifier in front of the formula, with the aforementioned variable.

Universal quantifiers in object position

- Universal:
  - Conan interviewed every politician.

Universal quantifiers in object position

- Universal:
    - Conan interviewed every politician.

      $\forall x[\mathrm{POLITICIAN}(x) \rightarrow \mathrm{INTERVIEW}(c,x)]$

Universal quantifiers in object position

- Universal:
  - Conan interviewed every politician.

    $\forall x[\mathrm{POLITICIAN}(x) \rightarrow \mathrm{INTERVIEW}(c,x)]$
  - Conan interviewed everyone.

Universal quantifiers in object position

- Universal:
  - Conan interviewed every politician.

    $\forall x[\mathrm{POLITICIAN}(x) \rightarrow \mathrm{INTERVIEW}(c,x)]$

  - Conan interviewed everyone.

    $\forall x[\mathrm{PERSON}(x) \rightarrow \mathrm{INTERVIEW}(c,x)]$

Negative quantifiers

- We can express the quantifier **no** in terms of ∀ and ∃.
- The following sentence has two good translations into Predicate Logic:
  - **No student complained**.

    ~∃x[STUDENT(x) & COMPLAINED(x)]

    ∀x[STUDENT(x) → ~ COMPLAINED(x)]

Negative quantifiers

- We can also translate **not every** in terms of ∀ and ∃.
- The following sentence has two good translations into Predicate Logic:
    - **Not every student complained**.

      $\exists x[\mathrm{STUDENT}(x)\ \&\ {\sim}\mathrm{COMPLAINED}(x)]$

      ${\sim}\forall x[\mathrm{STUDENT}(x) \rightarrow \mathrm{COMPLAINED}(x)]$

Generalized Quantifiers

Generalized Quantifiers

- When studying semantics, it is useful to know how to translate English sentences with quantifiers into Predicate Logic.
- However, not all quantifiers of English or other natural languages can be translated in Predicate Logic.
- A more general view of quantifiers is that they express relations between sets.

Generalized Quantifiers

- Here is an example of the interpretation of quantifiers as relations between sets:

  $[\![\textbf{every cat is cute}]\!]^M = T$ iff $[\![\textbf{cat}]\!]^M \subseteq [\![\textbf{cute}]\!]^M$

  $[\![\textbf{some cat is cute}]\!]^M = T$ iff $[\![\textbf{cat}]\!]^M \cap [\![\textbf{cute}]\!]^M \neq \varnothing$

- These formulas use the inclusion relation between sets ($\subseteq$) and the operation of set intersection ($\cap$):
  - $A \subseteq B$ is true iff every member of A is a member of B
  - $A \cap B$ is the set of all elements that are both members of A and members of B

Generalized Quantifiers

- A quantifier that cannot be analyzed in Predicate Logic but that can be analyzed as a relation between sets is **most**:

  $[\![$ **most cats are cute** $]\!]^M$ = T iff
  $|[\![$ **cats** $]\!]^M \cap [\![$ **cute** $]\!]^M| > |[\![$ **cats** $]\!]^M|/2$

  $[\![$ **most cats are cute** $]\!]^M$ = T iff over half of the cats are cute.

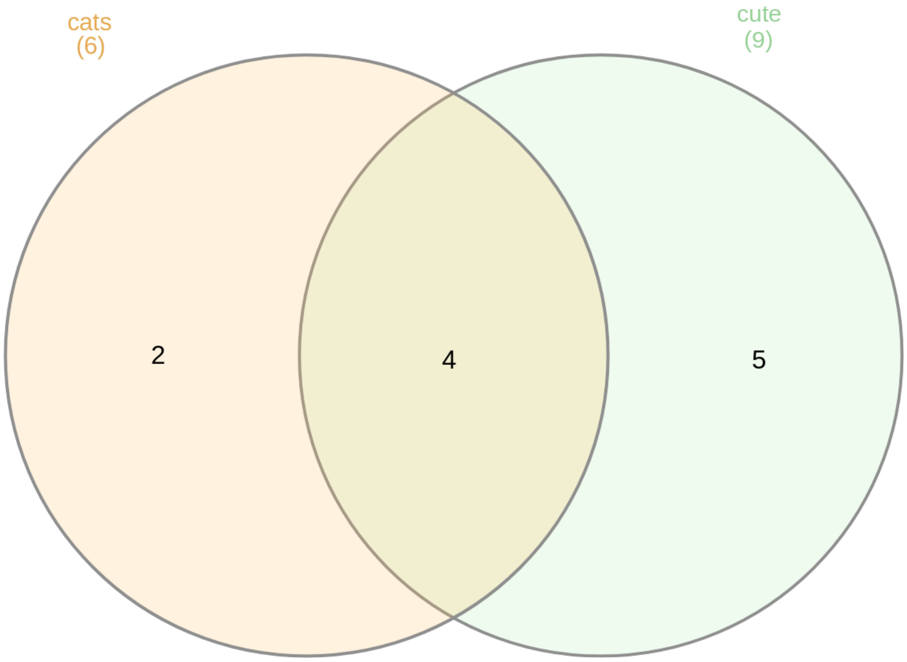
- Note that here, $|[\![$ **cats** $]\!]^M|$ is the cardinality of the set of cats.

- For any set S, |S| is the number of members of S.

Generalized Quantifiers

- **Most cats are cute**.

- Example of a situation in which the sentence is true:

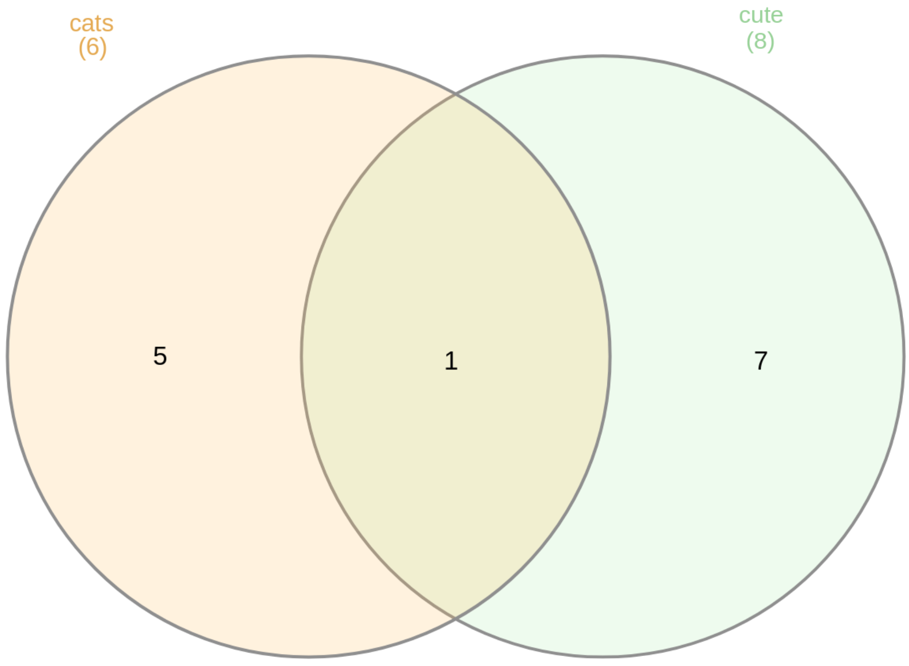


Some common generalized quantifiers

- **Every cat is cute**/**All the cats are cute**.
- True iff the set of cats is included in the set of cute entities.

Some common generalized quantifiers

- Some cat is cute/Some cats are cute.
- True iff the intersection of the set of cats with the set of cute entities is not empty.

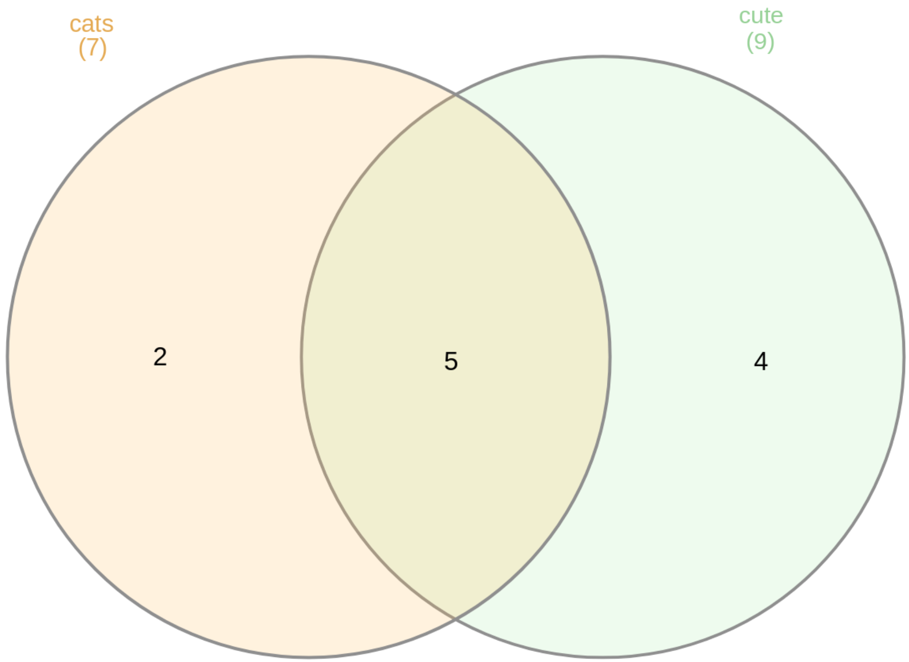


Some common generalized quantifiers

- **No cat is cute**/**No cats are cute**.
- True iff the intersection of the set of cats with the set of cute entities is empty.

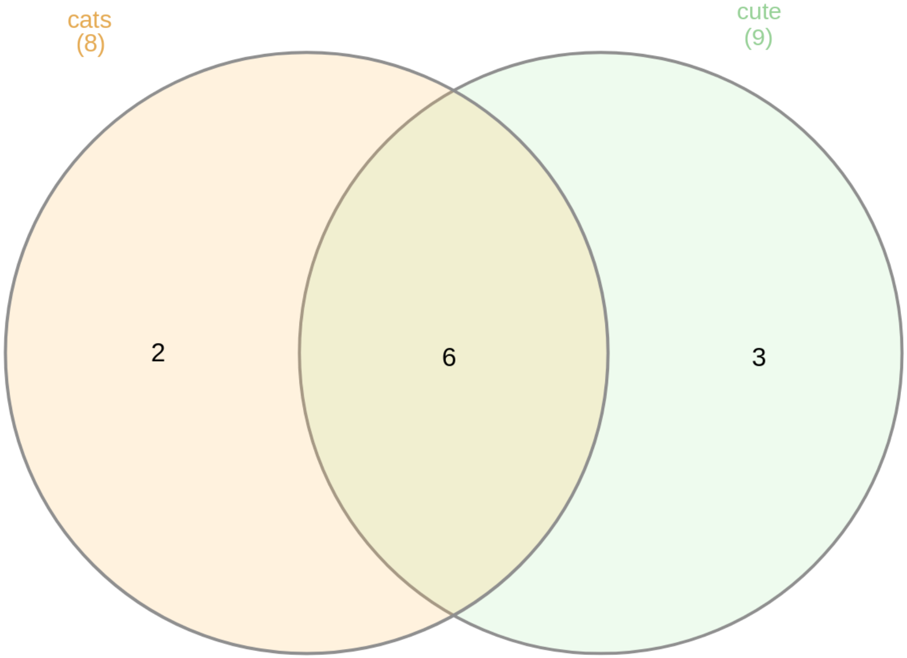
Some new quantifiers

- Exactly five cats are cute.
- True iff the cardinality of the intersection of the set of cats with the set of cute entities is 5.



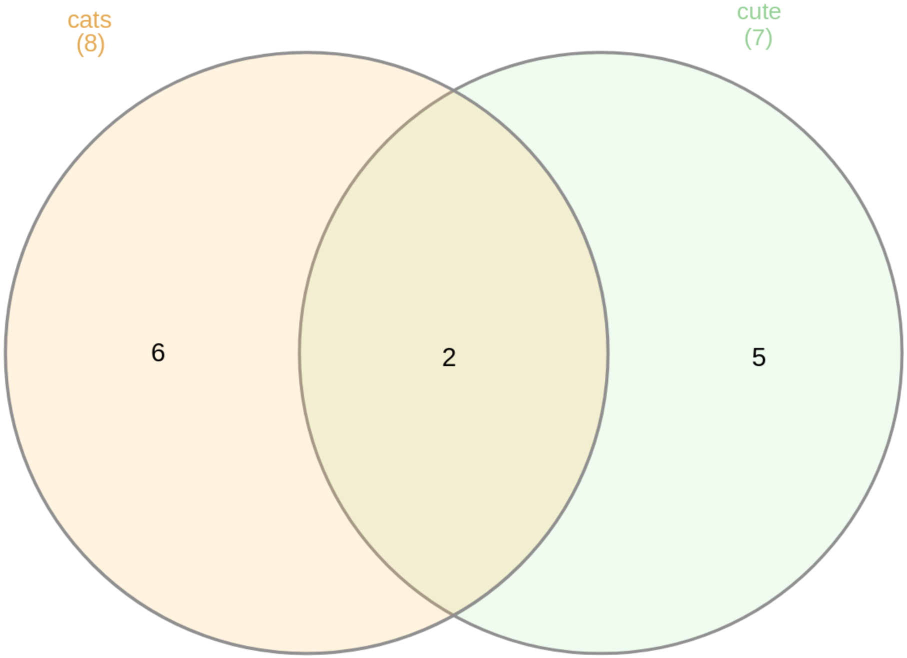
Some new quantifiers

- **At least five cats are cute**.
- True iff the cardinality of the intersection of the set of cats with the set of cute entities is at least 5.



Some new quantifiers

- Several cats are cute.
- True iff the cardinality of the intersection of the set of cats with the set of cute entities is more than 1.



Quantifiers Scope

Sentences with multiple quantifiers

- When a sentence has a verb with several quantifier arguments, several translations are possible.
- It matters which quantifier is placed in the scope of the other in the translation.
- It's often easier to think about scope ambiguities in Predicate Logic.

Sentences with multiple quantifiers

- Someone loves everyone.
  - $\exists x[\text{PERSON}(x) \;\&\; \forall y[\text{PERSON}(y) \rightarrow \text{LOVE}(x,y)]]$
  - $\forall y[\text{PERSON}(y) \rightarrow \exists x[\text{PERSON}(x) \;\&\; \text{LOVE}(x,y)]]$
- Everyone hides something.
  - $\forall x[\text{PERSON}(x) \rightarrow \exists y[\text{THING}(y) \;\&\; \text{HIDE}(x,y)]]$
  - $\exists y[\text{THING}(y) \;\&\; \forall x[\text{PERSON}(x) \rightarrow \text{HIDE}(x,y)]]$

Quantifier scope in Predicate Logic

- The notion of scope can be made precise in Predicate Logic.
- Let f be a formula of Predicate Logic:
  - In the expressions ∀x[f], f is called the scope of ∀x
  - In the expressions ∃x[f], f is called the scope of ∃x
- What is the scope of each quantifier in these formulas?
  - ∀x[PERSON(x) → ∃y[THING(y) & HIDE(x,y)]]
  - ∃y[THING(y) & ∀x[PERSON(x) → HIDE(x,y)]]

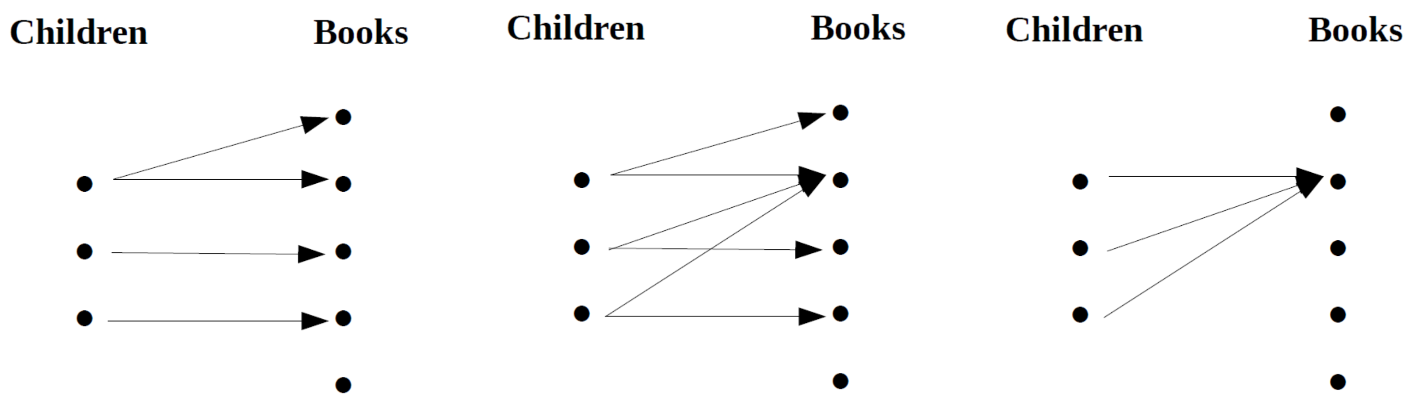
Quantifier scope in Predicate Logic

- In order to think about scope ambiguities, it can be convenient to draw diagrams that represent situations in which one or the other interpretation is true.

- Consider for instance:
    - **Every child read a book**.
    - $\forall x[\mathrm{CHILD}(x) \rightarrow \exists y[\mathrm{BOOK}(y) \,\&\, \mathrm{READ}(x,y)]]$
    - $\exists y[\mathrm{BOOK}(y) \,\&\, \forall x[\mathrm{CHILD}(x) \rightarrow \mathrm{READ}(x,y)]]$

Quantifier scope in Predicate Logic

- Imagine that there are 3 children and 5 books.



situation a	situation b	situation c

- (1) is true in (a), (b) and (c). (2) Is true only in (b) and (c).

1. ∀x[CHILD(x) → ∃y[BOOK(y) & READ(x,y)]]

2. ∃y[BOOK(y) & ∀x[CHILD(x) → READ(x,y)]]

Surface scope and Inverse Scope

- Pay attention to the order of quantifiers in the English sentence and its translation into Predicate Logic:
    - **Every child read some book**.

1. $\forall x[CHILD(x) \rightarrow \exists y[BOOK(y) \& READ(x,y)]]$
2. $\exists y[BOOK(y) \& \forall x[CHILD(x) \rightarrow READ(x,y)]]$

- What do you notice?

Surface scope and Inverse Scope

- In 1, the order of quantifiers match their order in the English sentence, but in 2 it doesn't:
    - Every child read some book.

1. $\forall x[\text{CHILD}(x) \rightarrow \exists y[\text{BOOK}(y) \,\&\, \text{READ}(x,y)]]$

2. $\exists y[\text{BOOK}(y) \,\&\, \forall x[\text{CHILD}(x) \rightarrow \text{READ}(x,y)]]$

- In interpretation 1, the quantifiers have **surface scope**.
- In interpretation 2, the quantifiers have **inverse scope**.

Surface scope and Inverse Scope

- Note that in the interpretation in 3, sentence 1 has inverse scope, but sentence 2 has direct scope:

  1. **Every child read some book**.
  2. **Some book was read by every child**.
  3. ∃y[BOOK(y) & ∀x[CHILD(x) → READ(x,y)]]

- Having direct scope or inverse scope is not a property of quantifiers in statements of Predicate Logic in isolation.
- Having direct scope or inverse scope is a property of quantifiers in the logical forms of specific English sentences.