LIN241 - Winter 2021
Lecture 8 summary:modality

1 Modality and Possible World Semantics

(1) Modality is the expression of possibility, necessity and related notions. In English, different categories of expressions can be used to express modality:

Modal Auxiliaries:	**It *may* be snowing in Toronto.**
Modal Adverbs:	***Possibly*, it will snow in Toronto tomorrow.**
Modal Adjectives:	**This is far from *necessary*.**
Modal Nouns:	**This is a *necessity*.**
Verbs of Propositional Attitudes:	**Chris *wants* to play baseball.**

(2) In our semantic theory, we analyze modality as a way to talk about states of affairs that might be different from the ways things actually are. We call these states of affairs "possible worlds."

Consider for instance the modal auxiliaries **might** and **must**, which express possibility and necessity, respectively. Simplifying a bit, we can paraphrase their contribution to the meaning of sentences as follows:

(3) **It might rain tomorrow.**

'There is a possible state of affairs in which it rains tomorrow.'

or: 'There is a possible world in which it rains tomorrow.'

(4) **It has to rain tomorrow.**

'In all possible states of affairs, it rains tomorrow.'

or: 'In all possible worlds, it rains tomorrow.'

(5) You see that the modal auxiliaries **might** and **has to** are analyzed as quantifiers. More precisely, they quantify over possible worlds.

(6) In order to flesh out this intuition, we must assume that the truth of sentences is now relative to possible worlds.

A sentence without modal operator is always evaluated as true or false in the actual world. This is illustrated in (7).

(7) **It is raining in Toronto.**

True iff it is raining in Toronto at the speech time, in the actual world.

(8) The truth-conditions of sentences with modal operators are more complex. We will discuss them step by step in this summary.

2 Quantificational Force and Modal Flavor

(9) Modal auxiliaries like **might** and **must** differ in quantificational force. **Might** is interpreted as an existential quantifier ('there is a possible world in which...'), while **must** is interpreted as a universal quantifier ('in all possible worlds...').

(10) In addition, modal auxiliaries can express different modal flavors, e.g. they can express that something is possible in the sense of being compatible with the information that we have access to (epistemic modality), or in the sense of being compatible with the laws (deontic modality), or in the sense of being compatible with somebody's physical abilities, etc.

The main modal flavors you should remember are:

- Epistemic Modality: possibility or necessity that can be inferred from some body of information (generally: what the speaker knows).

  I must have left my keys in the car!

- Deontic Modality: possibility or necessity given a set of rules or laws.

  You must be home by midnight!

- Goal oriented Modality: possibility or necessity given some agent's goals.

  If you want to get a PhD, you have to write a dissertation.

- Dynamic modality: possibility given the abilities of an agent, or the laws of nature.

  Chris can run 100 meters in 10 seconds.

3 Modal operators as quantifiers

(11) Modal operators (like modal auxiliaries) are interpreted as expressions that quantify over possible worlds ('there is world such that...' or 'for every world such that...').

(12) Different modal flavors correspond to different sets of possible worlds, that relate in a certain way to the actual world. Modal operators that have a certain modal flavor use the relevant set of worlds as a restriction. For instance:

- Epistemic modality: worlds that are compatible with the evidence available to the speaker in the actual world.

  I must have left my keys in the car.

  'In all worlds that are compatible with the evidence that is actually available to the speaker, the speaker has left their keys in the car.'

- Deontic modality: worlds that are compatible with some set of rules or laws that are in force in the actual world.

  Visitors have to leave by 6pm.

  'In all worlds compatible with the hospital's actual regulations, visitors leave by 6 pm.'

(13) Logical properties of modal operators

Possibility modals have been analyzed as existential quantifiers, and necessity modals have been analyzed as universal quantifiers. This analysis captures the logical properties that these modal operators have in common with determiner quantifiers like 'some' or 'every.'

(14) Sets of statements can be consistent or inconsistent. Modal quantifiers and regular quantifiers behave the same with respect to consistency:

Consistent statements (modal):

You may stay, but you don't have to stay.

Consistent statements (non-modal):

Some students stayed, but not every student stayed.

Inconsistent statements (modal):

#**You may stay, but also, you must leave.**

Inconsistent statements (non-modal):

#**Some students stayed, but every student left.**

(15) Existential and universal quantifiers are duals. That is to say "some x is P" is equivalent to "not every x is not P" and "every x is P" is equivalent to "not some x is not P". These equivalences are attested both with modal quantifiers and with non-modal quantifiers.

Equivalences with modal quantifiers:

You must stay ⇔ You aren't allowed to leave

Equivalences with non-modal quantifiers:

Every student stayed ⇔ It is not the case that some student left.

4 Conversational backgrounds

(16) The account of modal flavor we sketched in (12) is useful as a first approximation, but it is too simple. It runs into problems with some examples, like (17)

(17) Context: Albert was caught speeding through a school zone. He got a ticket.

Albert must pay a fine.

For every world w' such that the Ontario rules of the road are respected in w', the proposition that Albert will pay a fine is true in w'

(18) The issue with (17) is that since the restriction of the modal quantifier is a set of worlds in which the Ontario rules of the road are respected, it can't be the case that Albert was caught speeding through a school zone in these worlds, unlike in the actual world. Therefore, it shouldn't be true that Albert will pay a fine in these worlds. Our analysis doesn't seem to work for this example.

(19) As a solution, we can build the restriction of the modal operator in the previous example in two steps:

1. First, we define a set of worlds A in which Albert behaved just as he did in the actual world, up to the moment when the sentence was uttered. In particular, Albert was caught speeding through a school zone in all of these worlds.

2. Then, we take the subset B of these worlds in which the Ontario rules of the road are as respected as can be. In this subset, since Albert was caught speeding through a school zone, he will pay a fine, in accordance with the Ontario rules of the road.

(20) The two sets A and B discussed in (17) can be built from two sets of propositions, which we call the modal base and the ordering source.

In our example, the modal base is a set of proposition m(w*) that describe how Albert behaved in the actual world (w*) up to the moment when the sentence was uttered.

The ordering source is a set of proposition o(w*) which describe the Ontario rules of road, as they apply in the actual world (w*).

The set A is the set of worlds in which all the propositions in m(w*) are true. These worlds may differ from the actual world in many ways, but in all of them Albert behaved as he did in w* up to the moment when sentence (17) was uttered.

The set B is the subset of A where as many propositions in o(w*) as possible are true. This is the set of worlds where Albert behaved as he did in the actual world, but in which the Ontario rules of the road are otherwise respected. In these worlds, Albert will pay a fine, since he was caught speeding.

In general, we represent the set B as BEST(m(w*), o(w*)). This set is obtained by forming the set A of all possible worlds in which every proposition in m(w*) is true, and then forming the subset of A where the greatest number of propositions in o(w*) are true.

(21) The modal base and the ordering source are *conversational backgrounds*. These are sets of propositions that are used in the interpretation of modal operators. Different types of modal backgrounds have different types of propositions in them:

Epistemic conversational backgrounds: contain propositions that describe some information that is available to the speaker.

Deontic conversational backgrounds: contain propositions that describe rules, laws, or other types of regulations.

Circumstantial conversational backgrounds: contain propositions that describe facts or circumstances relevant to the modal statement, for instance how Albert behaved in the actual world.

Stereotypical conversational backgrounds: describe the normal course of events, how things normally go in the actual world.

(22) Deontic modal operators use a circumstantial modal base and a deontic ordering source (as discussed in (20) for example (17)).

Epistemic modal operators use an epistemic modal base and no ordering source, or a stereotypical ordering source:

Jess must be at home.

True iff Jess is at home in the most normal worlds among those that are compatible with the evidence we have access to in the actual world.

That is to say:

True iff for every w′ in BEST(m(w*), o(w*)), Jess is at home in w′.
(where m(w*) is the set of propositions that describe the information we have access to in w* and o(w*) is a stereotypical ordering source).

5. Translating modal statements in predicate logic

(23) In order to translate modal statements into Predicate Logic, we need to add possible world arguments to predicates.

Consider the way we used to translate and a sentence like **Jess is happy**:

HAPPY(j)

When we interpret this formula in a model, the best we can do is say that the sentence is true if and only if $[\![j]\!]^M$ is a member of the set of happy individuals in M.

However, we would like to account for the notion of truth in a world. To do so, we can add an argument to the predicate:

HAPPY(w*, j)

We now think of HAPPY as a binary predicate, which denotes a set of pairs of possible worlds and individuals:

$[\![\text{HAPPY}]\!]^M = \{\langle w_1, \text{Jess}\rangle, \langle w_1, \text{Chris}\rangle, \langle w_2, \text{Bruno}\rangle, \langle w_2, \text{Chris}\rangle, ...\}$

Jess and Chris are happy in world w_1, Bruno and Chris are happy in world w_2, …

This allows us to capture the notion of truth in a world:

$[\![\text{HAPPY}(w^*, j)]\!]^M = T$ iff then $\langle [\![w^*]\!]^M, [\![j]\!]^M\rangle \in [\![\text{HAPPY}]\!]^M$

The formula HAPPY(w*, j) is true in M iff the pair $\langle [\![w^*]\!]^M, [\![j]\!]^M\rangle$ is a member of $[\![\text{HAPPY}]\!]^M$

(24) We must also make adjustments to our models, by adding a set of possible worlds W, in addition to the universe U of the model:

U = {Jess, Kelly, Marc, Bruno, Chris, ...}

$W = \{w_1, w_2, w_3, ...\}$

$[\![\text{HAPPY}]\!]^M = \{\langle w_1, \text{Jess}\rangle, \langle w_1, \text{Chris}\rangle, \langle w_2, \text{Bruno}\rangle, \langle w_2, \text{Chris}\rangle, ...\}$

$[\![\text{LIKES}]\!]^M = \{\langle w_1, \text{Bruno, Jess}\rangle, \langle w_1, \text{Jess, Chris}\rangle, \langle w_2, \text{Chris, Chris}\rangle, \langle w_2, \text{Chris, Jess}\rangle, ...\}$

(25) We can now translate modal statements into Predicate Logic. Necessity modals are universal quantifiers, their restriction is a set of possible worlds that is defined as explained in (20), and their nuclear scope is the proposition that is said to be necessary:

Jess must be happy.

$\forall w' \, [w' \in \text{BEST}(m(w^*), o(w^*)) \rightarrow \text{HAPPY}(w', j)]$

(26) Possibility modals are existential quantifiers, their restriction is a set of possible worlds that is defined as explained in (20), and their nuclear scope is the proposition that is said to be possible:

Jess might be happy.

$\exists w' \, [w' \in \mathrm{BEST}(m(w^*), o(w^*)) \,\&\, \mathrm{HAPPY}(w', j)]$