LIN241

Introduction to Semantics

Lecture 8

Modality

Modal Displacement

- Displacement: the ability to talk about situations that are not here, now and actual.
  - Displacement in space:

    Thomas is in Hamburg.
  - Displacement in time:

    Yesterday was a Wednesday.
  - Modal Displacement:

    If kangaroos had no tail, they would topple over.

Sources and forms of modality

- Modal auxiliaries: **Jess might arrive late.**
- Modal adverbs: **Possibly, Jess will arrive late.**
- Verbs of propositional attitude:

  Chris believes that Jess will arrive late.
- Habitual statements: **Jane smokes.**
- Generic statements: **Bears like honey.**

Modal force

- Possibility modals:

  Jess might be at home.

- Necessity modals:

  Jess must be at home.

Modal flavour

- Epistemic modality:

  As far as we know, Julia might be the murderer.

- Deontic modality:

  According to these rules, you can opt out of the plan.

- Goal oriented:

  If you want to get a good seat, you have to arrive early.

- Dynamic modality

  Fir trees can grow in this climate.

Modal auxiliaries

- Basic notion: world in which a proposition is true

    1. **Galactus exiled the Silver Surfer.**

    2. **In the Marvel Universe, Galactus exiled the Silver Surfer.**

Modal auxiliaries

- Basic notion: world in which a proposition is true

  1. **Galactus exiled the Silver Surfer.**

  2. **In the Marvel Universe, Galactus exiled the Silver Surfer.**

- When we interpret 1, we implicitly evaluate its truth with respect to a fantasy world.

- This is explicit in 2.

- We can judge whether sentences are true or false:

  - in the real, actual world

  - in other possibilities

Modal auxiliaries

- One way to spell out these intuitions:
  - full clauses express propositions
  - a proposition is something that is true or false in a world
- For convenience, we assume that:
  - a proposition is identical with the set of worlds in which is true

Modal auxiliaries

- Ingredients of modal statements:
  - **Jess might be at home.**
  - prejacent: **Jess be at home**
  - modal operator: **might**
  - structure: **might(Jess be at home)**
- The modal operator tells us that the proposition expressed by the prejacent is true in certain worlds.

Modal auxiliaries

- Let w* represent the actual world.
  - **Jess might be at home.**
  - there is a world w' that is compatible with the information that the speaker has in w*, such that the proposition that Jess is at home is true in w'

Modal auxiliaries

- What does it mean for a world w' to be compatible with the information that the speaker has in w*?
  - we can think of this information as a set of propositions:
    - **We haven't seen Jess at work today.**
    - **Jess sometimes works from home.**
    - ...
- We select the worlds in which all those propositions are true.

Modal auxiliaries

- Alternative formulation of the truth conditions:
  - **Jess might be at home.**
  - The proposition that Jess is at home is compatible with the information that the speaker has in w*

Modal auxiliaries

- Epistemic necessity:
  - **Jess must be at home.**
  - for every world w' that is compatible with the information that the speaker has in w*, the proposition that Jess is at home is true in w'
- Alternative formulation:
  - the proposition that Jess is at home follows from the information that the speaker has in w*

Modal auxiliaries

- Deontic possibility:
  - **Jess can submit her assignment tomorrow.**
  - prejacent: **Jess submit her assignment tomorrow**
  - operator: **can**
  - structure: **can(Jess submit her assignment tomorrow)**

Modal auxiliaries

- Truth-conditions:
  - **Jess can submit her assignment tomorrow.**
  - there is a world w' that is compatible with the course regulations in w*, such that the proposition that Jess will submit her assignment tomorrow is true in w'
- The course regulations in w* can also be described as a set of propositions (of the sort that are spelled out in a syllabus)

Modal auxiliaries

- Deontic necessity
  - **Jess must submit her assignment tomorrow.**
  - For every world w' that is compatible with the course regulations in w*, the proposition that Jess will submit her assignment tomorrow is true in w'

Epistemic and deontic modality

- Consider the following statements:

  1. **Jess might be at home.**

  2. **Jess can submit her assignment tomorrow.**

- If you know that Jess is at home, you won't say (1).

- But you can say (2) even if you know that Jess will submit her assignment tomorrow.

Epistemic and deontic modality

- If you know that 3 is true, then it follows from the information you have access to that 1 is true:

  1. **Jess might be at home.**
  2. **Jess must be at home.**
  3. **Jess is at home.**

- It would be more informative to say 2 or 3 rather than 1.

Epistemic and deontic modality

- Even if you know that 2 is true, it does not follow from this that 1 is true.

  1. **Jess is allowed to submit her assignment tomorrow.**

  2. **Jess will submit her assignment tomorrow.**

A complication

- Consider the following example and its analysis:
  - Albert was caught speeding through a school zone. He got a ticket.

    Albert must pay a fine.

    For every world w' such that the Ontario rules of the road are respected in w', the proposition that Albert will pay a fine is true in w'.
- Problem:
  - In these worlds, Albert didn't drive over the speed limit through a school zone, because he respected the rules of the road.

A complication

- Solution:
  - Consider worlds in which Albert behaved as he did in w*
  - Among these worlds, keep the ones in which the Ontario rules of the road as they are stated in w* are observed as completely as possible.
  - In all these worlds, Albert will pay a fine.

A complication

- We are dealing with two sets of worlds here:
  - A: worlds in which Albert behaved as he did in w*
  - B: the subset of A in which the Ontario rules of the road as they are stated in w* are observed as completely as possible

A complication

- Both sets can be captured with sets of propositions:
  - for A, a set of proposition m(w*) that describes how Albert behaved in w*
  - for B, the set of propositions o(w*) that describe the Ontario rules of the road as they are stated in w*
- we call the first set a Modal Base
- the second set of propositions is an Ordering Source

A complication

- How do we get A and B from these sets of proposition?
  - A is the set of worlds in which all propositions in m(w*) are true
  - B is the subset of A in which as many propositions from o(w*) as possible are true
- We can write down B more succinctely as BEST(m(w*), o(w*))

A complication

- Revised analysis:

  Albert was caught speeding through a school zone. He got a ticket.

  Albert must pay a fine.

  For every world w' in BEST(m(w*), o(w*)), the proposition that Albert will pay a fine is true in w'

  where:

  - m(w*) describes how Albert behaved in w*
  - o(w*) describes the Ontario rules of the road as they are stated in w*

A complication

- Alternative formulation:

  Albert was caught speeding through a school zone. He got a ticket.

  Albert must pay a fine.

  For every world w', if w' is in BEST(m(w*), o(w*)) then the proposition that Albert will pay a fine is true in w'

  where:

  - m(w*) describes how Albert behaved in w*
  - o(w*) describes the Ontario rules of the road as they are stated in w*

A complication

- The same analysis can be applied to possibility deontic modals:

  Jess can submit her homework assignment tomorrow.

  There is a world w' such that w' is in BEST(m(w*), o(w*)) and the proposition that Jess will submit her homework assignment tomorrow is true in w'

  where:

  - m(w*) describes Jess's situation in w* (for instance: that she is a UofT undergraduate student enrolled in LIN241)
  - o(w*) describes the course regulations in w*

A complication

- We can also apply this analysis to epistemic modality:

  Jess must be at home.

  For every world w', if w' is in BEST(m(w*), o(w*)) then the proposition that Jess is at home is true in w'

  Where:

  - m(w*) describes the information that is available to the speaker in w*
  - o(w*) describes the normal course of events in w*

A complication

- Note that with epistemic modals, the modal base m(w*) captures the core epistemic flavour of the modal operator:

  Jess must be at home.

- The ordering source o(w*) just rules out 'crazy possibilities', such as cases where Jess has been kidnapped by French gangsters.
  - maybe such 'crazy possibilities' are, strictly speaking, compatible with the information we have access to
  - we still want to rule them out, so it's good to include o(w*).

Terminology

- Modal bases and ordering sources are conversational backgrounds.
- Some types of conversational backgrounds:
  - epistemic: contain propositions that describe some information that is available to the speaker
  - deontic: contain propositions that describe rules, laws, or other types of regulations,
  - circumstantial: contain propositions that describe facts or circumstances relevant to the modal statement, for instance how Albert behaved in w*
  - stereotypical: describe the normal course of events

Logical relations between modal statements

- Under the assumption that natural language quantifiers are never used with empty restrictions, (a) entails (b):

  (a) $\forall$ w' [if w' $\in$ BEST(m(w*),o(w*)), then p is true in w']

  (b) $\exists$ w' [w' $\in$ BEST(m(w*),o(w*)) and p is true in w']

- Example:
  - **Jess must be home.**
  - **Jess might be home.**

Logical relations between modal statements

- But can we also see that (c) is equivalent to (d):

  (c) ~ (∃ w' [w' ∈ BEST(m(w*), o(w*)) and p is true in w'])

  (d) ∀ w' [if w' ∈ BEST(m(w*), o(w*)), then ~ (p is true in w')]

- We can observe this equivalence in English:
  - **Jess is not allowed to leave.**
  - **Jess has to stay.**

Logical relations between modal statements

- Likewise, (e) is equivalent to (f):

  (e) ~ (∀ w' [if w' ∈ BEST(m(w*), o(w*)), then p is true in w'])

  (f) ∃ w' [w' ∈ BEST(m(w*), o(w*)) and ~ (p is true in w')]

- We can observe this equivalence in English:
  - **Jess does not have to stay.**
  - **Jess is allowed to leave.**

Modality in predicate logic

- The crucial step is to spell out the notion of "truth in a world"
- One strategy is to add a world argument to all predicates.
- Instead of translating **Jess is happy** as 1, we translate it as 2:

  1. HAPPY(j)

  2. HAPPY(w*, j)

- Here, we assume that w* is an expressions that refers to the actual world.

Modality in predicate logic

- What what changes do we have to do to our models to interpret these new statements?
- We can just add worlds to sequences of objects in the denotation of predicates:
  - $[\![\text{HAPPY}(w^*, j)]\!]^M = T$ iff $\langle [\![w^*]\!]^M, [\![j]\!]^M \rangle$ in $[\![\text{HAPPY}]\!]^M$

Modality in predicate logic

- An example model that takes this into account; let W be the set of all possible worlds:

  U = {Jess, Kelly, Marc, Bruno, Chris, ...}

  W = {w_1, w_2, w_3, ...}

  $[\![\text{HAPPY}]\!]^M$ = {〈w_1, Jess〉, 〈w_1, Chris〉, 〈w_2, Bruno〉, 〈w_2, Chris〉, ...}

  $[\![\text{LIKES}]\!]^M$ = {〈w_1, Bruno, Jess〉, 〈w_1, Jess, Chris〉, 〈w_2, Chris, Chris〉, 〈w_2, Chris, Jess〉, ...}

Modality in predicate logic

- Now that we know how to express the notion of "truth in a world" in predicate logic, we can translate modal statements into Predicate Logic:
  - **Jess must be happy.**

    $\forall w' [w' \in BEST(o(w^*),m(w^*)) \rightarrow HAPPY(w', j)]$

  - Using generalized quantifiers:

    [all w': BEST(o(w*),m(w*))] HAPPY(w', j)

Modality in predicate logic

- Now that we know how to express the notion of "truth in a world" in predicate logic, we can write down logical forms of modal statement easily:
  - **Jess might be happy.**

    ∃w' [w' ∈ BEST(o(w*),m(w*)) & HAPPY(w', j)]

  - Using generalized quantifiers:

    [some w': BEST(o(w*),m(w*))] HAPPY(w', j)

Where does modal flavour come from?

- English modal auxiliaries encode modal force:
  - Existential (possibility): **may**, **might**, **can**, …
  - Universal (necessity): **must**, **should**, **ought to**, …
- By contrast, modal flavour often depends on the context.

Where does modal flavour come from?

- Epistemic modality:

  Julia must be the murderer.

- Deontic modality:

  You must pay a fine.

- Goal oriented:

  If you want to get a good seat, you must arrive early.

Where does modal flavour come from?

- Adverbial phrases can be used to make conversational background explicit:
  - ***In view of what their tribal duties are***, **the Maori children must learn the names of their ancestors.**
  - ***In view of what is known***, **the ancestors of the Maoris must have arrived from Tahiti.**
  - **If—*in view of what your dispositions are*—you must sneeze, at least use your handkerchief.**

Examples from Kratzer, A. (1977). What 'Must' and 'Can' Must and Can Mean. Linguistics and Philosophy 1, 337–55.

Where does modal flavour come from?

- There are also lexical and morphosyntactic restrictions on modal flavour.
- For instance, **can** resists epistemic interpretation in non-negative sentences:
  - **As far as I know, the keys might be in the car.**
  - ***As far as I know, the keys can be in the car.**
  - **As far as I know, the keys cannot be in the car.**