# NERS551 Winter, 2022

#### **HOMEWORK #4**

Due Tuesday, March 22

# Numerical Solution of the Point Kinetics Equations w/ Feedback

The objective of this problem is to develop analytic and numerical solutions of the point kinetics equations. The first part of the problem set will be to develop the analytic solution and the second part will be to develop a program to solve the point kinetics equations for several applications.

# Part A. Analytic Solution of Basic Kinetics Problems

This part of the problem involves analytic solutions to the P.K.E. developed in Chapter 8. Develop the solution of the Eq. 8.10 for a general **asymmetric** ramp reactivity (see class notes). This solution will be used to verify your point kinetics program in Part B.

# Part B. Development of Exact P.K. program

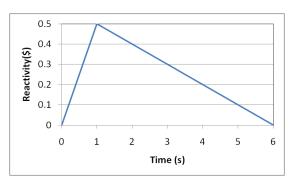
In this part of the problem, you are asked to develop a program to solve numerically the Exact Point Kinetics equations. Appendix A and the Class ppt slides provide a guide to the numerical discretization methods that can be used. However, you should feel free to implement any variation on the solution provided in Appendix A, but then be sure to explain it in your homework solution.

### Part C. Application of the P.K. program

The first problem will be to solve a subprompt critical ramp reactivity insertion problem in which no T/H feedback is considered. The purpose of this part of the problem is to verify your PK code using a simple transient for which you derived an analytic solution in Part A. This problem will also be used to analyze the differences in one group and 6 group delayed neutron solutions for a reactivity insertion event.

# **C.1 One Delayed Neutron Group**

The **asymmetric** ramp reactivity is shown in the Figure. Use one group of delayed neutrons,  $\beta$ =0.76%,  $\lambda$ =0.49405, to solve the ramp problem up to 5.0 sec using your PK code. (Note: to simulate the PJA in your code set  $\Lambda$ =2.6E-15 seconds).



- a) Use  $\Delta t$ =1 ms and compare the power at  $t_{max}$  (time of maximum reactivity) with the analytic solution from part A. Increase the time step size and plot/analyze the dependence of error on the time step size. You should observe second order convergence for C-N and first order convergence for explicit/implicit time discretization. Based on this analysis, determine a proper time step size for C-N that gives an error of ~0.01% which you can use throughout the problem set.
- b) Consider the more realistic case with  $\Lambda$ =2.6E-5 sec. Run the case with the time step size determined in a). Plot the reactivity, power, and precursor concentration on one page. Discuss the results.
- c) By comparing the analytic solution with the PK solution from your program up to 1.0 sec, draw a conclusion on the validity of the PJA.

## **C.2 Six Delayed Neutron Groups**

Using six delayed neutron groups, solve the same ramp problem given in Part C.1

Group	1	2	3	4	5	6
λ	0.0128	0.0318	0.119	0.3181	1.4027	3.9286
β(%)	0.02584	0.152	0.13908	0.30704	0.1102	0.02584

- a) Plot and compare the power and precursor changes with those of the one group case. How can you explain the difference in power change behaviors between the six and one group?
- b) Locate the maximum power point and using Eq. (8.10), evaluate one group decay constant and compare this value with the values obtained with two methods for determining one group of delayed neutrons, Eqs. (6-33) and (6-34). Discuss which of the three values would be most meaningful to use in a one precursor group approximation? Develop a fourth way to compute a time dependent one-group precursor concentration utilizing the time-dependent precursor concentration.

#### Part D. Point Kinetics with T/H Feedback

In this part, you will use the PK code for the analysis of superprompt reactivity insertion corresponding to a control rod ejection accident in which modeling of Doppler feedback effect is essential.

# **D.1 Precomputed Reactivity**

Use the reactivity variation obtained from the PARCS spatial kinetics calculation for the **OECD/NEACRP Control Rod Ejection case A1** as input to your PK program (This file is provided on the course CTOOLS website and a sample is shown in Appendix C). Run your program with  $\Delta t = 0.1$ ms. Plot the core power and reactivity.

#### **D.2** Linear Feedback Model

- a) Run your code with a linear feedback model with a time delay using the following values for the feedback model (which were determined for Case B1 of the NEACRP problem):  $\rho_{max}=1.078\$$ ,  $\gamma=-1.2\$/fp$ -sec and  $\lambda_{H}=1.0$  /sec. Compare the reactivity and power with those of D.1.
- b) Try to find different values for  $\gamma$  and  $\lambda_H$  which give better agreement with B. Does using different values for  $\gamma$  and  $\lambda_H$  for different transients of the same core make physical sense? Note: An analytic solution of the time constant is shown in Appendix B.

#### **D.3** Use of Prompt Kinetics Approximation

This part of the problem will examine the applicability of the PKA to the rod ejection event. Use the PKA prescriptions developed in Chapter 10 to compute the maximum power, the pulse width, the energy deposition, and the asymptotic power for Case A1. Use  $\gamma$  determined in D.2.b and one group decay constant from C.2.b. Discuss the results.

# **Appendix A** Derivation of the Numerical PKE

#### A. Exact PKE:

$$\dot{P}(t) = \frac{(\rho(t) - \beta(t))}{\Lambda(t)} P(t) + \frac{1}{\Lambda_0} \sum_{k} \lambda_k \zeta_k(t) + \frac{1}{\Lambda(t)} s(t)$$
 (1)

$$\dot{\zeta}_{k}(t) = -\lambda_{k} \zeta_{k}(t) + \frac{\Lambda_{0}}{\Lambda(t)} \beta_{k}(t) P(t) \qquad k = 1: K$$
 (2)

Where s(t) is external source which should be zero for this homework. With initial condition:

$$\zeta_k(0) = \frac{\beta_k^{eff}(0)}{\lambda_k} p(0), \qquad (3)$$

Apply Exponential Transformation to Exact PKE,

$$p(t) = \tilde{p}(\tilde{t})e^{\alpha_{n}\tilde{t}}, \quad \tilde{t} = t - t_{n-1} \in [-\Delta t_{n-1}, \Delta t_{n}], \quad (4)$$

$$\tilde{p}^{n-1} = p^{n-1}$$

$$\frac{d\tilde{p}(\tilde{t})}{d\tilde{t}} = \left(\frac{\rho(\tilde{t}) - \beta^{eff}(\tilde{t})}{\Lambda(\tilde{t})} - \alpha^{n}\right)\tilde{p}(\tilde{t}) + \frac{e^{-\alpha^{n}\tilde{t}}}{\Lambda_{0}} \sum_{k} \lambda_{k} \zeta_{k}(\tilde{t}), \quad (5)$$

$$\frac{d}{d\tilde{t}} \left(\zeta_{k}(\tilde{t})e^{\lambda_{k}\tilde{t}}\right) = e^{\lambda_{k}\tilde{t}} \frac{d\zeta_{k}(\tilde{t})}{d\tilde{t}} + \lambda_{k}e^{\lambda_{k}\tilde{t}} \zeta_{k}(\tilde{t}) = \frac{\Lambda_{0}}{\Lambda(\tilde{t})} \beta_{k}^{eff}(\tilde{t})p(\tilde{t})e^{\lambda_{k}\tilde{t}} \quad (6)$$

Finite differencing power equation (5) with  $\theta$  method:

$$e^{-\alpha^{n}\Delta t_{n}}p^{n}-p^{n-1}=\theta\Delta t_{n}e^{-\alpha^{n}\Delta t_{n}}\left(\left(\frac{\rho^{n}-\beta^{eff,n}}{\Lambda^{n}}-\alpha^{n}\right)p^{n}+\frac{1}{\Lambda_{0}}\sum_{k}\lambda_{k}^{n}\zeta_{k}^{n}\right)$$
$$+\left(1-\theta\right)\Delta t_{n}\left(\left(\frac{\rho^{n-1}-\beta^{eff,n-1}}{\Lambda^{n-1}}-\alpha^{n}\right)p^{n-1}+\frac{1}{\Lambda_{0}}\sum_{k}\lambda_{k}^{n-1}\zeta_{k}^{n-1}\right)$$
(7)

Where  $\theta$  is user input parameter.

If  $\theta$ =0, the finite differencing is explicit,

If  $\theta$ =1, the finite differencing is implicit,

If  $\theta$ =0.5, the finite differencing is called Crank-Nicolson scheme, which has high accuracy (  $2^{nd}$  order)

If same method is apply to precursor equations (6), then we get K+1 equations for K+1 unknowns.

When we advance time from step n-1 to step n, we already know all quantities of step n-1, are given, and  $\Lambda_0, \lambda_k$  are constants.

So, only  $P^n$ ,  $\zeta_k^n$  are K+1 unknowns which can be solved from a system of K+1 finite difference equations.

As the precursors depend only on power, and are independent from each other, we can get solution without introducing finite different error to precursor equations.

# B. Analytic integration of precursor equations with linear approximation

$$G(\tilde{t}) \equiv \frac{\Lambda_0}{\Lambda(\tilde{t})} \beta_k^{eff}(\tilde{t}) p(\tilde{t}) e^{-\alpha \tilde{t}} \approx G^n w + G^{n-1} (1 - w)$$
(8)  
$$w = \tilde{t} / \Delta t_n$$

where

Integrate (6) over step

$$\zeta_k^n = e^{-\tilde{\lambda}_k \Delta t_n} \zeta_k^{n-1} + e^{\alpha \Delta t_n} \Delta t_n \left\{ G^n \kappa_1 (\tilde{\lambda}_k^n) + G^{n-1} \left[ \kappa_0 (\tilde{\lambda}_k^n) - \kappa_1 (\tilde{\lambda}_k^n) \right] \right\}$$
(9)
$$\tilde{\lambda}_k^n = (\lambda_k^n + \alpha) \Delta t_n$$

where:

$$\kappa_m(x) = e^{-x} \int_0^1 w^m e^{xw} dw = \frac{1 - m\kappa_{m-1}(x)}{x}$$
 (10)  

$$\kappa_m(x) = 1 - \frac{x}{2} + \frac{x^2}{6} - \frac{x^3}{24} + \frac{x^4}{120} - \frac{x^5}{720} + \frac{x^6}{5040} + O[x]^7 x < 0.001,$$
 (11)

$$\kappa_{0}(x) = \frac{1 - e^{-x}}{x}, \kappa_{0}(x) = 1 - \frac{x}{2} + \frac{x^{2}}{6} - \frac{x^{3}}{24} + \frac{x^{4}}{120} - \frac{x^{5}}{720} + \frac{x^{6}}{5040} + O[x]^{7} x < 0.001, \quad (11)$$

$$\kappa_{1}(x) = \frac{[1 - \kappa_{0}(x)]}{x}, \kappa_{1}(x) = \frac{1}{2} - \frac{x}{6} + \frac{x^{2}}{24} - \frac{x^{3}}{120} + \frac{x^{4}}{720} - \frac{x^{5}}{5040} + \frac{x^{6}}{40320} + O[x]^{7}, x < 0.001, \quad (12)$$

$$\kappa_{0}(x) - \kappa_{1}(x) = \frac{1}{2} - \frac{x}{3} + \frac{x^{2}}{8} - \frac{x^{3}}{30} + \frac{x^{4}}{144} - \frac{x^{5}}{840} + \frac{x^{6}}{5760} + O[x]^{7}, \quad (13)$$

Rearrange (9) as,

$$\zeta_k^n = p^n \Omega_k^n + \hat{\zeta}_k^n, \tag{14}$$

Where

$$\Omega_k^n = \frac{\Lambda^0}{\Lambda^n} \beta_k^{eff,n} \Delta t_n \kappa_1(\tilde{\lambda}_k^n) \quad (15)$$

$$\hat{\zeta}_k^n = e^{-\lambda_k \Delta t_n} \zeta_k^{n-1} + e^{\alpha \Delta t_n} \Delta t_n G^{n-1} \left[ \kappa_0(\tilde{\lambda}_k^n) - \kappa_1(\tilde{\lambda}_k^n) \right] \quad (16)$$

Sum up eq (14) for all group precursors with weight  $\lambda_k^n$ 

$$\sum_k \lambda_k \, \zeta_k^n = p^n \sum_k \lambda_k \, \Omega_k^n + \sum_k \lambda_k \, \hat{\zeta}_k^{n-1} = \tau^n p^n + \hat{S}_d^n \quad (17)$$

Where:

where:  

$$\tau^{n} = \sum_{k} \lambda_{k}^{n} \Omega_{k}^{n}, \qquad \hat{S}_{d}^{n} = \sum_{k} \lambda_{k}^{n} \hat{\zeta}_{k}^{n}, \quad (18)$$

$$S_{d}^{n-1} = \sum_{k} \lambda_{k}^{n-1} \zeta_{k}^{n-1}, \quad (19)$$

## C. Flux level solution

Substitute (18) (19)into (6)

$$e^{-\alpha^{n}\Delta t_{n}} p^{n} - p^{n-1} = \theta \Delta t_{n} e^{-\alpha^{n}\Delta t_{n}} \left( \left( \frac{\rho^{n} - \beta^{eff,n}}{\Lambda^{n}} - \alpha^{n} \right) p^{n} + \frac{\tau^{n}}{\Lambda_{0}} p^{n} + \frac{1}{\Lambda_{0}} \hat{S}_{d}^{n} \right)$$

$$+ (1 - \theta) \Delta t_{n} \left( \left( \frac{\rho^{n-1} - \beta^{eff,n-1}}{\Lambda^{n-1}} - \alpha^{n} \right) p^{n-1} + \frac{S_{d}^{n-1}}{\Lambda_{0}} \right)$$

$$(20)$$

Rearranging:

$$p^{n} = \frac{e^{\alpha^{n}\Delta t_{n}} \left(p^{n-1} + (1-\theta)\Delta t_{n} \left(\left(\frac{\rho^{n-1} - \beta^{eff, n-1}}{\Lambda^{n-1}} - \alpha^{n}\right)p^{n-1} + \frac{S_{d}^{n-1}}{\Lambda_{0}}\right)\right) + \theta \Delta t_{n} \frac{\hat{S}_{d}^{n}}{\Lambda_{0}}}{1 - \theta \Delta t_{n} \left(\frac{\rho^{n} - \beta^{eff, n}}{\Lambda^{n}} - \alpha^{n} + \frac{\tau^{n}}{\Lambda_{0}}\right)}$$
(21)

## **D.** Power

$$H^n = f_{f_0} p^n, \tag{22}$$

where  $f_{fp}$  is power normalization factor.

## E. Exponential transformation parameter

First evaluate new step power with estimation of  $\alpha^n$  as,

$$\alpha^{n} = \frac{1}{\Delta t_{n-1}} \ln \frac{p^{n-1}}{p^{n-2}},$$
 (23)

Then check if the transformed solution is closer to linear than original solution

$$\left| p^{n} - e^{\alpha^{n} \Delta t_{n}} p^{n-1} \right| \le \left| p^{n} - p^{n-1} - (p^{n-1} - p^{n-2}) / \gamma \right|,$$
 (24)

If (24) is true, then the transformation parameter is accepted, otherwise re-evaluate solution without transformation.

$$\alpha^n = 0.$$

#### F. Linear feedback and first order heat conduction

From now on,  $\rho^{i+1}$  is not given, but can be determined from (25)

$$\rho(t) = \rho_{im}(t) + \gamma_d \int_0^t (H(t') - P_0) \exp(\lambda_H(t' - t)) dt'$$
 (25)

 $\rho_{im}$  imposed reactivity

$$\gamma_d = \frac{H \bullet 1s}{mc_p} \frac{d\rho}{dT_f}$$
 (26)

 $\gamma_d \le 0$  is the feedback constant. If  $\gamma_d = 0$ , the solution of this section should be same as that from previous section.

 $\lambda_H \ge 0$  is time constant.

If  $\lambda_H > 0$ , the feedback reactivity is,

$$\rho_d(t) = \rho(t) - \rho_{im}(t) = \gamma_d \int_0^t (H(t') - P_0) \exp(\lambda_H(t' - t)) dt'$$
 (27)

$$\rho_d^n = \gamma_d \int_0^{t_n} (H(t') - P_0) \exp(\lambda_H(t' - t_n)) dt'$$

$$\rho_d^n = e^{-\hat{\lambda}_H^n} \rho_d^{n-1} - P_0 \gamma_d \Delta t_n \kappa_0(\hat{\lambda}_H^n) + \gamma_d e^{-\hat{\lambda}_H} \int_0^{\Delta t_n} H(\tilde{t}) \exp(\lambda_H \tilde{t}) d\tilde{t}$$
(28)
$$\hat{\lambda}_H^n = \lambda_H \Delta t_n$$

Take linear approximation of H(t')

$$Q(\tilde{t}) = f_{fp}(\tilde{t})p(\tilde{t})e^{-\alpha\tilde{t}} = Q^n w + Q^{n-1}(1-w)$$
(29)
$$\tilde{\lambda}_H^n = (\lambda_H + \alpha)\Delta t_n$$

$$\int_0^{\Delta t_n} Q(\tilde{t}) \exp(\lambda_H + \alpha) \tilde{t} d\tilde{t} = e^{\tilde{\lambda}_H^n} \Delta t_n \left\{ Q^n \kappa_1(\tilde{\lambda}_H^n) + Q^{n-1} \left[ \kappa_0(\tilde{\lambda}_H^n) - \kappa_1(\tilde{\lambda}_H^n) \right] \right\} (30)$$

Substitute (30) to (25) and rearranging

$$\begin{split} \rho_d^n &= e^{-\widehat{\lambda}_H^n} \rho_d^{n-1} - \eta P_0 \gamma_d \Delta t_n \kappa_0 \left( \widehat{\lambda}_H^n \right) \\ &\quad + e^{\alpha \Delta t_n} \gamma_d \Delta t_n \left\{ Q^n \kappa_1 (\widetilde{\lambda}_H^n) + Q^{n-1} \left[ \kappa_0 \left( \widetilde{\lambda}_H^n \right) - \kappa_1 (\widetilde{\lambda}_H^n) \right] \right\} \end{split}$$

$$\rho^{n} = a_{1}p^{n} + b_{1}$$
 (31)

where

$$a_1 = f_{fp}^n \gamma_d \Delta t_n \kappa_1 \left( \tilde{\lambda}_H^n \right) \ \ (32)$$

$$b_{1} = \rho_{im}^{n} + e^{-\widehat{\lambda}_{H}^{n}} \rho_{d}^{n-1} - P_{0} \gamma_{d} \Delta t_{n} \kappa_{0}(\widehat{\lambda}_{H}^{n}) + e^{\alpha \Delta t_{n}} \gamma_{d} \Delta t_{n} f_{fn}^{n-1} p^{n-1} \left[ \kappa_{0}\left(\widetilde{\lambda}_{H}^{n}\right) - \kappa_{1}\left(\widetilde{\lambda}_{H}^{n}\right) \right]$$
(33)

Insert (31) to (20):

$$e^{-\alpha^{n}\Delta t_{n}} p^{n} - p^{n-1} = \theta \Delta t_{n} e^{-\alpha^{n}\Delta t_{n}} \left( \left( \frac{a_{1} p^{n} + b_{1} - \beta^{eff,n}}{\Lambda^{n}} - \alpha^{n} \right) p^{n} + \frac{\tau^{n}}{\Lambda_{0}} p^{n} + \frac{1}{\Lambda_{0}} \hat{S}_{d}^{n} \right) + \left( 1 - \theta \right) \Delta t_{n} \left( \left( \frac{\rho^{n-1} - \beta^{eff,n-1}}{\Lambda^{n-1}} - \alpha^{n} \right) p^{n-1} + \frac{S_{d}^{n-1}}{\Lambda_{0}} \right)$$
(34)

Rearrange eq. (34)

$$a(p^n)^2 + bp^n + c = 0 (35)$$

Where

$$a = \frac{\theta \Delta t_n a_1}{\Lambda^n}$$

$$b = \left[ \theta \Delta t_n \left( \left( \frac{b_1 - \beta^{eff,n}}{\Lambda^n} - \alpha^n \right) + \frac{\tau^n}{\Lambda_0} \right) - 1 \right]$$

$$c = \theta \Delta t_n \frac{1}{\Lambda_0} \hat{S}_d^n + e^{\alpha^n \Delta t_n} \left[ (1 - \theta) \Delta t_n \left( \left( \frac{\rho^{n-1} - \beta^{eff,n-1}}{\Lambda^{n-1}} - \alpha^n \right) p^{n-1} + \frac{S_d^{n-1}}{\Lambda_0} \right) + p^{n-1} \right]$$
(38)

If 
$$a < 0$$
, then  $\frac{-b + \sqrt{b^2 - 4ac}}{2a} < 0$ , so  $p^n = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$ , if  $a < 0$  (39)  
If  $a = 0$ , then  $p^n = \frac{c}{b}$ , (40)

Equation (40) is equivalent to (21).

#### G. P.K. Solution Scheme

Only formula with red numbers need to be coded, the sequence of these formula in your program should be as follows:

- 1. Determine transformation parameter with eq. (23)
- 2. Prepare  $\frac{\Omega_k^n}{n}$  and  $\frac{\hat{\zeta}_k^n}{n}$  with eq. (15),(16) and (17), the exponential integration functions (10)~(13) are called
- 3. Prepare  $\tau^n, \hat{S}_d^n, S_d^{n-1}$  with eq. (18),(19)
- 4. Prepare a1,b1 with eq. (32) and (33)
- 5. Prepare a,b,c with eq. (36), (37), and (38)
- 6. Evaluate the flux level with eq. (39) or (40)

Check if exponential transformation is acceptable with eq. (24), if not, redo step 3 and 4, 5 with  $\alpha = 0$ Evaluate power level, precursors and reactivity with eq. (22), (14) and (31)

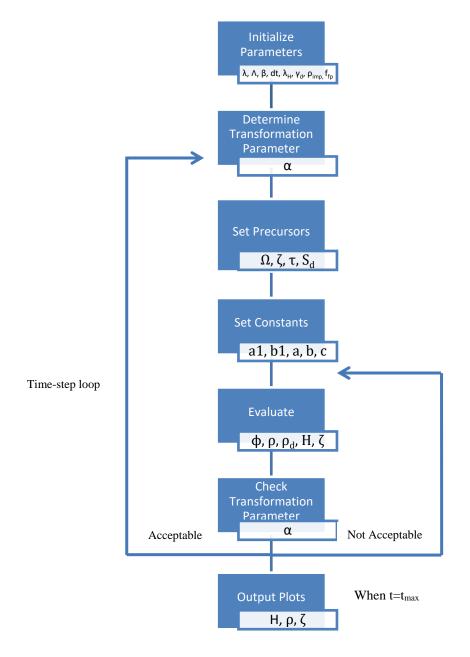
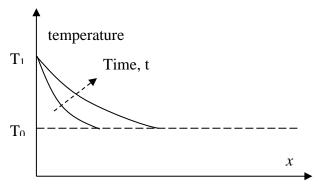


Figure A-1: Code mapping scheme for the numerical solution of the point kinetics equations.

#### APPENDIX B

# **Analytic Derivation of Heat Conduction Time Constant**

An uniform rod with constant density  $\rho$ , heat capacity  $C_p$  and conductivity k, has infinite length and been insulated in radial direction. There is no heat source in this rod. The rod has initially temperatures  $T_0$  everywhere, and the temperature at x=0 jump to  $T_1$  when t=0 and keep it forever. Find the temperature distribution in this rod with the time change  $T(x,t), x \in [0,\infty), t \in [0,\infty)$ .



Answer: One dimension heat conduction equation without heat source:

$$\rho C_p \frac{\partial T(x,t)}{\partial t} = k \frac{\partial^2 T(x,t)}{\partial x^2} \tag{1}$$

Defined:  $a = \frac{k}{\rho C_p}$  then equation (1) can be written as:

$$\frac{\partial T(x,t)}{\partial t} = \alpha \frac{\partial^2 T(x,t)}{\partial x^2} \tag{2}$$

Let 
$$\eta = \frac{x}{\sqrt{4at}}$$
 (3)

The equation (2) becomes ordinary differential equation,

$$\frac{d^2T(\eta)}{d\eta^2} = -2\eta \frac{dT(\eta)}{d\eta} \tag{4}$$

Let 
$$f(\eta) = \frac{dT(\eta)}{d\eta}$$
 (5)

Then equation (4) can be reduce to first order ODE,

$$\frac{df(\eta)}{d\eta} = -2\eta f(\eta) \tag{6}$$

The general solution of equation (6) is

$$f(\eta) = ce^{-\eta^2} \tag{7}$$

Where c is to be determined.

Substitute equation (5) into (7),

$$\frac{dT(\eta)}{d\eta} = ce^{-\eta^2} \tag{8}$$

The general solution of equation (8) is

$$T(\eta) = b + c \int_0^{\eta} e^{-w^2} dw$$
 (9)

Where b is to be determined.

Now, we can determine b and c by boundary conditions:

$$T(\eta = 0) = T_1 = b \tag{10}$$

$$T(\eta = \infty) = T_0 = T_1 + c \int_0^\infty e^{-w^2} dw$$
 (11)

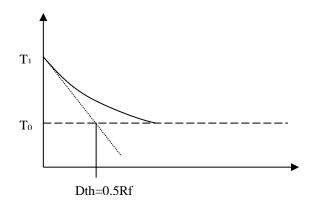
$$c = \frac{T_0 - T_1}{\int_0^\infty e^{-w^2} dw} = \frac{T_0 - T_1}{\sqrt{\pi/2}} = -\frac{2(T_1 - T)_0}{\sqrt{\pi}}$$
(12)

Substitute (10) and (12) into (9), we get solution of equation (4) as

$$T(\eta) = T_1 - \frac{2(T_1 - T_0)}{\sqrt{\pi}} \int_0^{\eta} e^{-w^2} dw$$
 (13)

Substitute (3) into (13), we get solution of equation (2) as

$$T(x,t) = T_1 - \frac{2(T_1 - T_0)}{\sqrt{\pi}} \int_{0}^{\frac{x}{\sqrt{4at}}} e^{-w^2} dw$$
 (14)



$$\frac{\partial T(x,t)}{\partial x}\bigg|_{x=0} = -\frac{T_1 - T_0}{\sqrt{\pi at}}$$

Penetration depth 
$$D_{th}(t) = \left(T_1 - T_0\right) / \left(-\frac{T_1 - T_0}{\sqrt{\pi a t}}\right) = \sqrt{\pi a t}$$

Time constant  $D_{th}(t) = \frac{1}{2}R_f$   $t = \frac{R_f^2}{4\pi a}, \ \lambda_H = \frac{1}{t} = \frac{4\pi a}{R_f^2} = \frac{4\pi k}{\rho C_p R_f^2}$ 

for UO2 
$$a = 0.01cm^2 / sec$$
  $t = \frac{R_f^2}{4\pi a} = \frac{0.42^2}{4*3.14*0.01} = 1.4s$ 

# Appendix C

# **PARCS Spatial Kinetics Solution**

Time	Reactivity	Generation Time	Normalization Factor	Relative Power
0.000000E-00 1.000000E-03 2.000000E-03 3.000000E-03 4.000000E-03 5.000000E-03 7.000000E-03 8.000000E-03 9.000000E-03 1.000000E-02	0.000000E-00 2.490108E-04 5.626118E-04 9.798401E-04 1.636867E-03 2.441198E-03 3.472841E-03 4.825114E-03 6.548127E-03 8.431524E-03 1.064335E-02	2.571145E-05 2.571145E-05 2.571149E-05 2.571157E-05 2.571163E-05 2.571171E-05 2.571182E-05 2.571195E-05 2.571208E-05 2.571233E-05	1.000000E+00 1.000034E+00 1.000088E+00 1.000169E+00 1.000266E+00 1.000387E+00 1.000542E+00 1.000744E+00 1.000944E+00 1.001112E+00 1.001301E+00	1.000000E-06 1.000078E-06 1.000221E-06 1.000460E-06 1.000810E-06 1.001305E-06 1.001974E-06 1.002862E-06 1.003980E-06 1.005293E-06 1.006865E-06
	Data Removed			
2.820000E+02 2.830000E+02 2.840000E+02 2.850000E+02 2.860000E+02 2.870000E+02 2.890000E+02	1.366502E-03 1.437802E-03 1.327996E-03 1.403029E-03 1.290550E-03 1.369222E-03 1.254154E-03 1.336344E-03	2.589218E-05 2.589207E-05 2.589219E-05 2.589219E-05 2.589208E-05 2.58920E-05 2.58920E-05	9.767156E-01 9.762291E-01 9.767213E-01 9.762258E-01 9.767269E-01 9.762224E-01 9.767327E-01 9.762189E-01	2.636842E-01 2.635812E-01 2.635939E-01 2.635884E-01 2.637033E-01 2.637126E-01 2.636022E-01 2.637216E-01
2.900000E+02 2.910000E+02 2.920000E+02 2.930000E+02 2.940000E+02 2.950000E+02 2.960000E+02 2.980000E+02 2.990000E+02 3.000000E+02	1.218766E-03 1.304389E-03 1.184351E-03 1.273311E-03 1.150855E-03 1.243108E-03 1.118289E-03 1.213740E-03 1.086586E-03 1.185198E-03 1.055737E-03	2.589220E-05 2.589208E-05 2.589221E-05 2.589221E-05 2.589221E-05 2.589209E-05 2.589222E-05 2.589223E-05 2.589209E-05 2.589223E-05 2.589223E-05	9.767384E-01 9.762153E-01 9.762153E-01 9.762116E-01 9.767501E-01 9.762078E-01 9.767560E-01 9.762039E-01 9.767620E-01 9.767680E-01	2.637216E-01 2.636087E-01 2.637304E-01 2.636150E-01 2.637390E-01 2.636211E-01 2.637474E-01 2.636270E-01 2.636327E-01 2.637637E-01