NERS 551 **Winter 2022**

Homework II Submission

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1 Question 1

This part of derivation is from the textbook. The time-depended diffusion equation without an independent source in operator notation is

$$\frac{1}{v}\frac{\partial \Phi}{\partial t} = (\boldsymbol{F_p - M})\boldsymbol{\Phi} + S_d \tag{1}$$

the reactor is assumed to be critical for $t \leq 0$.

$$0 = (\boldsymbol{F_{p0} - M_0})\boldsymbol{\Phi_0} + S_{d0} \tag{2}$$

The stationary prompt and delayed neutron sources can be combined to obtain the familiar form:

$$0 = (\boldsymbol{F_{p0} - M_0})\boldsymbol{\Phi_0} \tag{3}$$

with $\boldsymbol{F_0\Phi_0}$ explicitly containing the delayed neutrons. For a description of the degree of off-criticality from the concept of the reactivity, the operator F_d is added and subtracted

$$\frac{1}{v}\frac{\partial \Phi}{\partial t} = (\boldsymbol{F - M - F_d})\boldsymbol{\Phi} + S_d \tag{4}$$

and $F\Phi$ is given by Eq.(5.15) in the textbook but with the subscript 0 deleted. The quantity $F_d\Phi$ is the quasi-stationary delayed neutron source. Multiplying Eq.(4) with the initial adjoint flux and integrating with respect to energy and space yields

$$\frac{\partial}{\partial t}(\Phi_0^*, \frac{1}{v}\Phi) = (\Phi_0^*, \boldsymbol{(F - M)\Phi}) - (\Phi_0^*, \boldsymbol{(F_d)\Phi}) + (\Phi_0^*, \boldsymbol{S_d}) \tag{5}$$

where teh initial adjoint flux is the solution of the adjoint problem to Eq.(3). Then consider,

$$\Delta F = F - F_0 \tag{6}$$

and

$$\Delta M = M - M_0 \tag{7}$$

this yield,

$$\frac{\partial}{\partial t}(\Phi_0^*, \frac{1}{v}\Phi) = (\Phi_0^*, \boldsymbol{(\Delta F - \Delta M)\Phi}) - (\Phi_0^*, \boldsymbol{(F_d)\Phi}) + (\Phi_0^*, \boldsymbol{S_d}) \tag{8}$$

$$F(t) = (\Phi_0^*, F\Phi) \tag{9}$$

$$\Lambda(t)\dot{p}(t) = [\rho(t) - \beta(t)]p(t) + s_d(t) \tag{10}$$

with

$$s_d(t) = \frac{(\Phi_0^*), \sum_k \chi_{dk} \lambda_k C_k)}{F(t)} = \frac{F_0}{F_t} \sum_k \lambda_k \zeta_k(t) \tag{11}$$

The definition of quantities in the previous two equstions as they evolve from the derivation of the exact kinetics equations are the following:

$$\Lambda(t) = \frac{(\Phi_0^*, \frac{1}{v}\Phi)}{(\Phi_0^*, F\Phi)} = \frac{K_0}{F(t)} \tag{12}$$

and

$$\rho(t) = \frac{1}{F(t)} (\Phi_0^*, [\boldsymbol{\Delta F} - \boldsymbol{\Delta M}]\boldsymbol{\Phi}) \tag{13}$$

and

$$\beta(t) = \frac{1}{F(t)} (\Phi_0^*, \boldsymbol{F_d}\boldsymbol{\Phi}) = \sum_k \beta_k(t) \tag{14}$$

with

$$\beta_k(t) = \frac{1}{F(t)} (\Phi_0^*, \boldsymbol{F_{dk}}\boldsymbol{\Phi}) \tag{15}$$

and

$$\zeta_k(t) = \frac{1}{F_0} (\Phi_0^*, \boldsymbol{\chi_{dk}}\boldsymbol{C_k}) \tag{16}$$

Introducing the flux factorization and integrating the resultant equation with respect to r and E yields

$$\frac{\partial}{\partial t} (\Phi_0^*, \boldsymbol{\chi_{dk}}\boldsymbol{C_k}) = -\lambda_k (\Phi_0^*, \boldsymbol{\chi_{dk}}\boldsymbol{C_k}) + (\Phi_0^*, \boldsymbol{F_{dk}}\boldsymbol{\phi}) p(t) \tag{17}$$

which is divided by the time-independent quantity

$$F_0 = (\Phi_0^*, \boldsymbol{F_0}\boldsymbol{\phi_0}) \tag{18}$$

This yields the exact balance equation for the reduced precursors

$$\dot{\zeta_k}(t) = -\lambda_k \zeta_k(t) + \frac{F(t)}{F_0} \beta_k(t) p(t) \tag{19}$$

To obtain the more familiar form of the precursor balance equations , Eq.(17) is divided by the time-independent quantity K_0 instead of F_0, with K_0 expressed as

$$K_0 = F(t) \cdot \Lambda(t) \tag{20}$$

This yields, instead of Eq.19,

$$\dot{c_k}(t) = -\lambda_k c_k(t) + \frac{1}{\Lambda(t)} \beta_k(t) p(t) \tag{21}$$

with

$$c_k(t) = \frac{(\Phi_0^*, \boldsymbol{\chi_{dk}}\boldsymbol{C_k})}{K_0} = \frac{(\Phi_0^*, \boldsymbol{\chi_{dk}}\boldsymbol{C_k})}{F(t)\chi(t)} \tag{22}$$

The exact kinetics equations are used then in the form

$$\dot{p(t)} = \frac{p(t) - \beta(t)}{\Lambda(t)} p(t) + \frac{1}{\Lambda_0} \sum_k \lambda_k \chi_k(t) \tag{23}$$

2 Question 2

(a) static reactivity assumes the reactivity doesn't change when the control rod is inserted. In contrast, the dynamic reactivity assumes the reactivity constantly changes during the insertion of the control rod.

(b) the β_{eff} in (3.50) used the adjoint flux as a weighting function. This method considered the importance of the neutrons and therefore, should be considered as a more accurate representation of the β_{eff}. In thermal reactors, the delayed neutron is more important, use the adjoint flux as a weighting function models the reactor better. In a fast reactor, although the fast neutrons are the more important ones, such modelling still important (but not as important as that in a thermal reactor).

(c) The shape of the neutron flux and the neutron density distribution is ignored. The reactor is reduced to a point model.

3 Question 3

The two methods to combine the six precursors are

(a) direct weighting with *beta*

$$\bar{\lambda} = \frac{\sum_{i=1}^{6} \beta_i \lambda_i}{\sum_{i=1}^{6} \beta_i}$$

(b) inverse weighting with *beta*

$$\frac{1}{\bar{\lambda}} = \frac{\sum_{i=1}^{6} \beta_i \frac{1}{\lambda_i}}{\sum_{i=1}^{6} \beta_i}$$

Since the decay rate sometimes can vary by orders of magnitude, the first method bias towards the precursors with short decay time (larger λ), while the second method bias towards those with longer decay time (smaller λ). Therefore, the first method is better at modelling the reactors in a short time interval while the second method is better at modelling the reactors in a long time time interval.

4 Question 4

(a) kinetics without delayed neutrons:

$$\dot{p} = \frac{\rho}{\Lambda} p$$

This model only consider the prompt neutrons.

(b) prompt kinetic approximation; neglect of explicit delayed neutron source:

$$\dot{p} = \frac{\rho - \beta}{\Lambda} p$$

This model only consider the prompt neutrons.

(c) Constant delayed neutron source (CDS) approximation

$$\dot{p} = \frac{\rho - \beta}{\Lambda} p + \frac{1}{\Lambda} \beta p_0$$

This model assumes the delayed neutron source is a constant during the transient, which is a reasonable assumption in short-time analysis.

(d) Precursor Accumulation (PA) approximation

$$\dot{p} = \frac{\rho - \beta}{\Lambda} p + \frac{1}{\Lambda} [\beta p_0 + \bar{\lambda} \beta I(t)]$$

This model assumes the delayed neutrons come from the accumulation of precursors during a time period.

(e) One-delay-group kinetic

$$\dot{p} = \frac{\rho - \beta}{\Lambda} p + \frac{1}{\Lambda} \bar{\lambda} \zeta(t)$$

This model combines different groups of delayed neutrons into a single group.

(f) two-delay-group kinetic

$$\dot{p} = \frac{\rho - \beta}{\Lambda} p + \frac{1}{\Lambda} [\lambda^1 \zeta^1(t) + \lambda^2 \zeta^2(t)]$$

This model combine different groups of delayed neutrons into two groups.

(g) six-delay-group kinetic

$$\dot{p} = \frac{\rho - \beta}{\Lambda} p + \frac{1}{\Lambda} [\sum_{i=1}^{6} \lambda^i \zeta^i(t)]$$

This model doesn't do simplification in terms of delayed neutron and is the most sophisticated modelling of the reactor

5 Question 5

The inhour equation is

$$\rho = \alpha \Lambda + \sum_K \frac{\beta_k \alpha}{\alpha + \lambda_k}$$

and its one-delay-group approximation is

$$\rho = \alpha \Lambda + \frac{\beta \alpha}{\alpha + \lambda}$$

The relationship between the reactivity ρ and the inverse period α is shown in Fig.1

In the figure, the solid curves correspond to six-group precursor models while the + symbols mark the curves for one-group approximation. As shown, the two methods give very close results. The value of Λ affects the behavior of the reactor when the inverse period is high. The larger the Λ, the faster the curve rises. Mathematically, this is because the linear term starts to dominate when *alpha* is large.

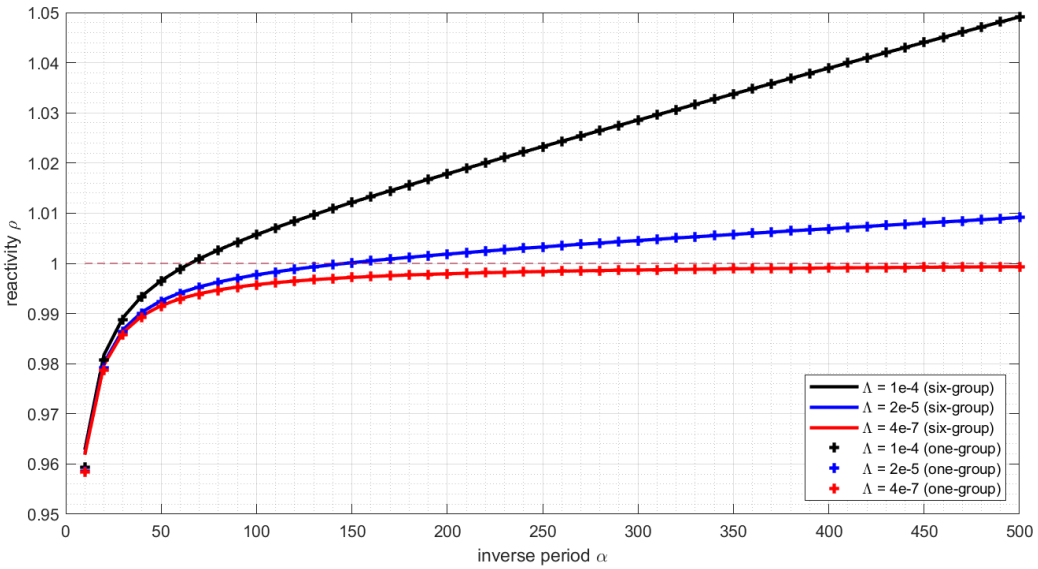

Figure 1: The relationship between the reactivity ρ and the inverse period α

6 Question 6

Begin with the one-delay group kinetics equations, derive teh solution to the point kinetic equations. The one-delay-group kinetics equation is

$$\dot{p} = \frac{\rho_1 - \beta}{\Lambda} p + \frac{1}{\Lambda} \lambda \zeta \tag{24}$$

and

$$\dot{\zeta} = -\lambda \zeta + \beta p \tag{25}$$

with the precursor accumation approximation (PA)

$$\dot{p} = \frac{\rho_1 - \beta}{\Lambda} p + \frac{\beta}{\Lambda} [p_0 + \lambda I(t)] \tag{26}$$

to case these to equations into the same mathematical form, the equations are differentialted with respect to time; eliminating of ζ yields second-order differential equations

$$\ddot{p} + (\lambda - \alpha_p)\dot{p} - \frac{\lambda}{\Lambda} \rho_1 p \tag{27}$$

and

$$\ddot{p} - \alpha_p \dot{p} - \frac{\lambda}{\Lambda} \beta p = -\frac{\lambda}{\Lambda} \beta p_0 \tag{28}$$

where ρ_1 denotes the step reactivity inserted and α_p the corresponding absolute inverse prompt period.The inital condition for both cases are

$$p(0) = p_0$$

and

$$\dot{(}p)(0+) = \frac{\rho_1}{\Lambda} p_0$$

The solution to the linear differential equations with constant coefficients is composed of exponential functions with exponents found as roots of the characteristic equations

$$\alpha^2 + (\lambda - \alpha_p)\alpha - \frac{\lambda}{\Lambda}\rho_1 = 0 \tag{29}$$

with

$$\alpha_{1,2} = -\frac{\lambda - \alpha_p}{2} \pm [(\frac{\lambda - \alpha_p}{2})^2 + \frac{\lambda}{\Lambda}\rho_1]^{1/2} \tag{30}$$

It can be verified that the term in the parentheses is much larger than the term that follows. Consequently, the square root term can be approximated by the first two terms of a Taylor expansion.

$$\frac{\lambda - \alpha_p}{2}[1 + \frac{2}{(\lambda - \alpha_p)^2 \frac{\lambda}{\Lambda}\rho_1}] = \frac{\lambda - \alpha_p}{2} + \frac{\lambda\rho_1}{\Lambda(\lambda - \alpha_p)}. \tag{31}$$

This gives for the two roots,

$$\alpha_1 = \frac{\lambda\rho_1}{\Lambda(\lambda - \alpha_p)} = -\frac{\lambda\rho_1}{\Lambda\alpha_p} = \frac{\lambda\rho_1}{\beta - \rho_1}$$

$$\alpha_2 = \alpha_p - \lambda\frac{\beta}{\beta - \rho_1} = \alpha_p$$

So the solution to the equation is given by,

$$p(t) = A_1 exp(\alpha_1 t) + A_2 exp(\alpha_2 t) \tag{32}$$

with

$$A_1 = \frac{\beta}{\beta - \rho_1} p_0$$

and

$$A_2 = \frac{-\rho_1}{\beta - \rho_1} p_0$$

Therefore, the solution to the one-delay-group kinetics equation for a step-reactivity-induced transient is given by

$$p(t) = p_0[\frac{\beta}{\beta - \rho_1} exp(\alpha_1 t) + \frac{-\rho_1}{\beta - \rho_1} exp(\alpha_2 t)] \tag{33}$$

7 Question 7

The mathematics behind solving the point kinetic equations follows the appendix of the HW2. The source codes are posted at `git@github.com:chzhuo158/NERS551_WN22.git` inside the $./HW2/code$ folder. Solving the eigenvalues for different system, we can have the results in Table.1

As shown in the table, when ρ is 0.25$ and 0.50$, there's one positive eigen-value of the system. This indicates that in the long time, the p(t) will be dominated by this and exponentially rise. When ρ is $-1$$, all eigen-values are negative, therefore, p(t) will decay away in the long run. This behavior is indeed observed by showing $p(t)$ in Fig.2

Another finding is that the value of Λ doesn't affect the results too much. Since the bottom-right element in the matrix is dominantly large, Λ affects the largest eigen-value. Since the term associated with this eigen-value decay to zero in a very short time period, the following transient behavior basically are the same.

Table 1: The convergence behavior of different method and different relaxation parameters

ρ	0.25$			0.5$			0.5$		
Λ	1e-4	2e-5	4e-7	1e-4	2e-5	4e-7	1e-4	2e-5	4e-7
α_1	-12375.58	-61875.58	-3093750.58	-8250.87	-8250.87	-8250.87	-8.25E6	-8.25E6	-8.25E6
α_2	-3.06	-3.06	-3.06	-2.99	-2.99	-2.99	-3.15	-3.15	-3.15
α_3	-1.11	-1.11	-1.11	-1.05	-1.05	-1.05	-1.20	-1.20	-1.20
α_4	-0.21	-0.2	-0.2	0.19	0.19	0.19	-0.27	-0.27	-0.27
α_5	0.04	0.04	0.04	-0.19	-0.19	-0.19	-0.01	-0.01	-0.01
α_6	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.03	-0.03	-0.03
α_7	-0.06	-0.06	-0.06	-0.05	-0.05	-0.05	-0.11	-0.11	-0.11

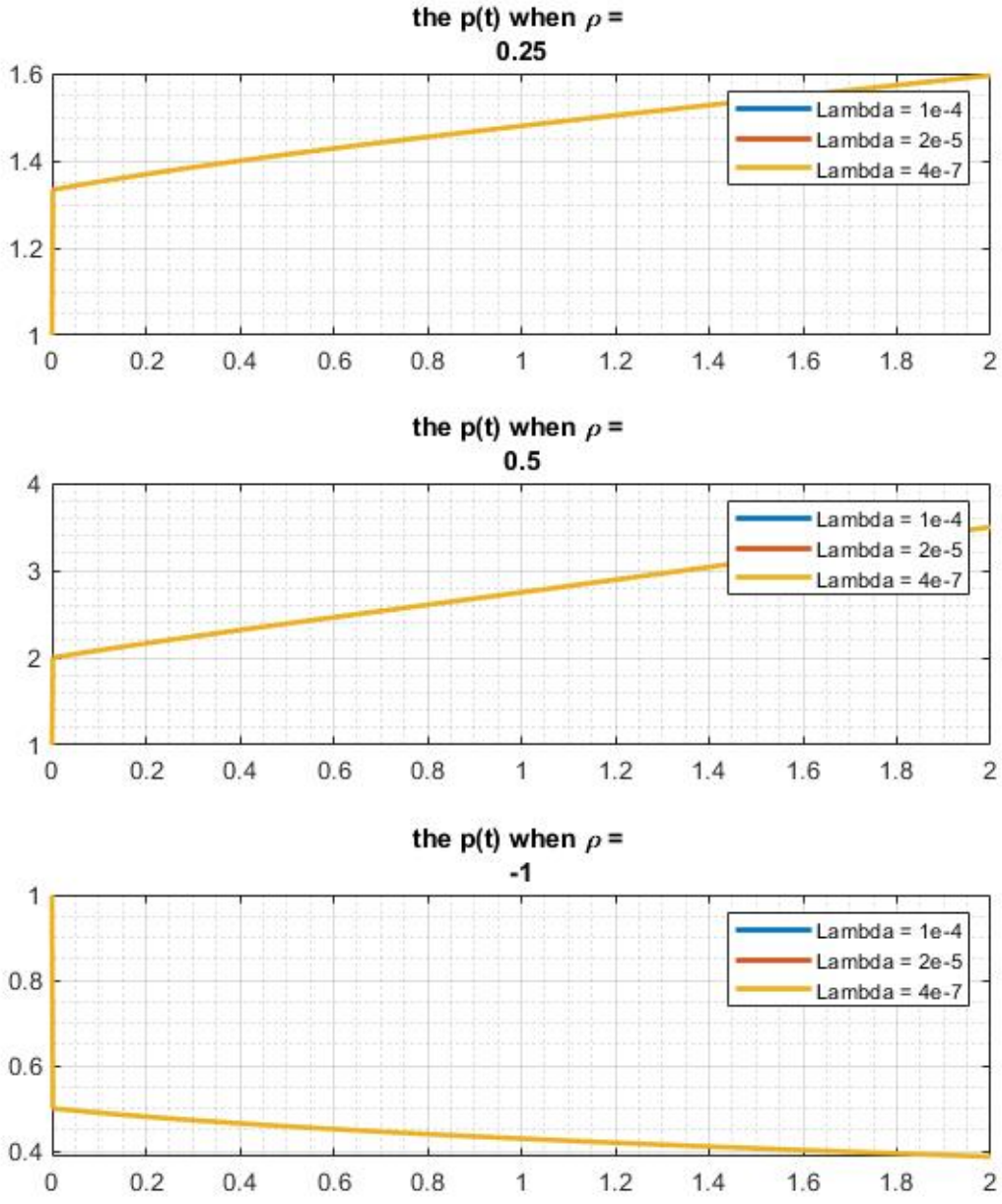

Figure 2: The relationship between the p(t) and the transient time t