NUCL551
Winter, 2022
Homework #1

Due Date TBA

1. Derivation of Adjoint Equations

a. Write the expressions for the adjoint neutronics operators, M^* and F^*

b. For the 2 energy group diffusion equations, confirm the relation:

$$\sum_{i=1}^{2} \phi_g^* [F\phi]_g = \sum_{i=1}^{2} \phi_g [F^* \phi^*]_g$$

c. Demonstrate the equivalence of the eigenvalues of the real and adjoint equations (ie. show that $\lambda^* = \lambda$):

$$M\Phi = \lambda F\Phi$$

$$M^* \Phi^* = \lambda^* F^* \Phi^*$$

d. (Extra) Show that $\Sigma_s^*(E \to E') = \Sigma_s(E' \to E)$

2. Derive the following four formulas for exact perturbation reactivity

(see p. 47 of text Ott, Dynamics, Ch 4 and HW problem Homework 3, p. 53)

a) $\Delta\rho = \dfrac{\left(\Phi_0^*, [\lambda_0 \Delta F - \Delta M]\Phi\right)}{\left(\Phi_0^*, F\Phi\right)}$ **b)** $\Delta\rho = \dfrac{\left(\Phi_0^*, [\lambda \Delta F - \Delta M]\Phi\right)}{\left(\Phi_0^*, F_0\Phi\right)}$

c) $\Delta\rho = \dfrac{\left(\Phi^*, [\lambda_0 \Delta F - \Delta M]\Phi_0\right)}{\left(\Phi^*, F\Phi_0\right)}$ **d)** $\Delta\rho = \dfrac{\left(\Phi^*, [\lambda \Delta F - \Delta M]\Phi_0\right)}{\left(\Phi^*, F_0\Phi_0\right)}$

where: $\Delta\rho \equiv \lambda_0 - \lambda$, $\Delta F \equiv F - F_0$, $\Delta M \equiv M - M_0$

and:

Perturbed balance equation	$M\Phi = \lambda F\Phi$	(1)
Unperturbed adjoint equation	$M_0^* \Phi_0^* = \lambda_0 F_0^* \Phi_0^*$	(2)

3. Perturbation Theory Application I: Energy

A point (infinite medium) reactor has 1 fuel nuclide (N^1) and 1 poison nuclide (N^2).

The neutron balance equation:

$$M\Phi = \lambda F\Phi \tag{1}$$

With the appropriate assumptions for a LWR (e.g. $\chi_1 = 1$) this can be written as:

$$\begin{bmatrix} N^1(\sigma_{c1}^1 + \sigma_{f1}^1 + \sigma_{12}^1) & 0 \\ -N^1\sigma_{12}^1 & N^1(\sigma_{c2}^1 + \sigma_{f2}^1) + N^2\sigma_{c2}^2 \end{bmatrix} \begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix} = \lambda \begin{bmatrix} N^1\nu\sigma_{f1}^1 & N^1\nu\sigma_{f2}^1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix} \tag{2}$$

or:

$$\begin{bmatrix} \sigma_{c1}^1 + \sigma_{f1}^1 + \sigma_{12}^1 & 0 \\ -\sigma_{12}^1 & \sigma_{c2}^1 + \sigma_{f2}^1 + \alpha\sigma_{c2}^2 \end{bmatrix} \begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix} = \lambda \begin{bmatrix} \nu\sigma_{f1}^1 & \nu\sigma_{f2}^1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix}$$

where: $\alpha = N^2 / N^1$

Using the two group data given below:

$$\begin{array}{lllll} \sigma_{r1}^1 = 9b & \sigma_{c1}^1 = 3b & \sigma_{12}^1 = 6b & \sigma_{f1}^1 = 0b & N^1 = 1.0E24 \\ \sigma_{c2}^1 = 1b & \sigma_{f2}^1 = 1b & \nu = 3 & \sigma_{c2}^2 = 10b & N^2 = 0 \end{array}$$

Evaluate first order and exact perturbation theory results for the following cases:

a) $\delta N^2 = 0.01E24$
b) $\delta N^2 = 0.10E24$
c) $\delta\sigma_{f2}^1 = 0.01b$
d) $\delta\sigma_{f2}^1 = 0.50b$
e) $\delta\sigma_{12}^1 = 1.00b$

Summarize results in a Table and discuss the results:

Case	Direct Substraction	FOPT[1]	EPT[2]	Relative Error of FOPT
a) $\delta N^2 = 0.01E24$				
b) $\delta N^2 = 0.10E24$				
c) $\delta\sigma^1_{f2} = 0.01b$				
d) $\delta\sigma^1_{f2} = 0.50b$				
e) $\delta\sigma^1_{12} = 1.00b$				

[1]First order perturbation theory
[2]Exact perturbation theory

4. Perturbation Theory Application II: Space

(HW problems 4 and 5 a/b of Chapter 4 in Ott Dynamics, P53)

Consider a perturbation of $+\Delta\Sigma a$ for r < r_a in a critical (**homogenous**) spherical reactor. Assume r_a << R, with R being the critical radius. Find the corresponding change in reactivity, using the unperturbed flux a) from reaction rate and b) from first order perturbation formulation for a one-group approximation.

Apply the formulas of previous problem to the following data: R=50cm;

$$\Sigma_t = 1.625/cm,\ \Sigma_s = 1.5/cm,\ \Sigma_a = 0.125/cm,\ \Sigma_c = 0.073/cm,$$
$$\Sigma_f = 0.052, \nu\Sigma_f = 0.126, D = 0.25cm$$

Neglect the extrapolation length.

a. find *keff*
b. Introduce $\Delta\Sigma_c$, as a perturbation for r < r_a =5cm as a fraction c% of Σ_c, find $\Delta\rho(c)$ between 0 and 100%.