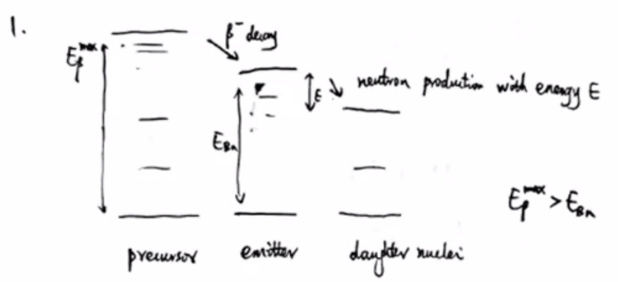
NERS551 Miterm

Status	To Do
URL	

1. (5) Explain the physics of delayed neutron emission (e.g. show the typical energy level scheme of precursor-emitter-daughter nuclei that leads to the production of delayed neutrons).



Delayed neutrons originate from the radioactive decay of nuclei produced in fission, and hence they are different for each fissile material. They are emitted by excited neutron-rich fission fragments (so-called **the delayed neutron precursors**) some appreciable time after the fission. How long afterward depends **on the precursor's half-life** since the neutron emission itself occurs in a very short time. The precursors usually undergo beta decay without any neutron emission, but a small fraction of them (highly excited nuclei) can undergo **the neutron emission** instead of the gamma emission.

E_max > E

Energy of the precirsor larger than emitter energy

2. (5) Provide a physical interpretation of the space, energy dependent fundamental mode adjoint flux $\phi*(r,E)$ and explain why it is used in the first order perturbation theory expression (Eqn. A.1 in Appendix of this exam).

2. +5

$\phi^*(\vec{r}, E)$ represents the importance of a neutron born at $\vec{r}$ with energy E.

In perturbation theory $$\delta\lambda = \frac{\langle \phi^w, (\lambda_0 \delta F - \Delta M)\phi_0 \rangle}{\langle \phi^w, F_0 \phi_0 \rangle} + \frac{\langle \phi^w, (\lambda_0 F_0 - M_0)\delta\phi \rangle}{\langle \phi^w, F_0 \phi_0 \rangle} + O(\delta^2)$$

if we want to eliminate the second first order term with $\delta\phi$ we need to select a proper weighting function ϕ^w.

Since $\langle \phi^*, (\lambda_0 F_0 - M_0)\delta\phi \rangle = \langle (\lambda_0 F_0^* - M_0^*)\phi^*, \delta\phi \rangle$

$+ O(\delta^2)$

if we want to eliminate the second first order term with $\delta\phi$ we need to select a proper weighting function ϕ^w.

Since $\langle \phi^*, (\lambda_0 F_0 - M_0)\delta\phi \rangle = \langle (\lambda_0 F_0^* - M_0^*)\phi^*, \delta\phi \rangle$

holds for all $\delta\phi$ and at the begining $\lambda_0 F_0^* \phi_0^* = M_0^* \phi_0^*$.

therefore if we choose ϕ_0^* as the weighting function. we can cancel out

$$\frac{\langle \phi_0^*, (\lambda_0 F_0 - M_0)\delta\phi \rangle}{\langle \phi_0^*, F_0 \phi_0 \rangle} = 0$$

- The adjoint flux represents the importance of a neutron born at r with energy E.
- In pertubation theory, if we want to eliminate the second first order term, with deltaPhi, we need to select a proper weighing factor phi_star.
- Since xxxx, if we choose ohi_star as the weighting function, we can eliminate that term.

3. (5) In standard operator notation the fundamental mode real and adjoint equations can be written as:

Fundamental Mode Real	$M\Phi = \lambda F\Phi$
Fundamental Mode Adjoint	$M^*\Phi^* = \lambda^* F^*\Phi^*$

Show that the fundamental mode real and adjoint eigenvalues are identical:

$$\lambda^* = \lambda$$

---

3 Multiply ϕ^* on both sides of the real equation.

+5 we get $\phi^* M\phi = \phi^* \lambda F\phi$

integrate over all energy and space:

$$\langle \phi^*, M\phi \rangle = \langle \phi^*. \lambda F\phi \rangle$$

apply the definition of adjoint:

$$\langle \phi^*. M\phi \rangle = \langle M^*\phi^*. \phi \rangle$$

$$\langle \phi^*. \lambda F\phi \rangle = \langle (\lambda F)^*\phi^*. \phi \rangle = \langle \lambda F^*\phi^*. \phi \rangle$$

i.e. $\langle M^*\phi^*. \phi \rangle = \langle \lambda F^*\phi^*. \phi \rangle$ (1)

multiply ϕ on both sides of the adjoint equation and integrate over all energy and space:

$$\langle \phi, M^*\phi^* \rangle = \langle \phi. \lambda^* F^*\phi^* \rangle$$

$\Rightarrow \langle M^*\phi^*, \phi \rangle = \langle \lambda^* F^*\phi^*. \phi \rangle$ (2)

$(1)-(2)$: $0 = \langle (\lambda-\lambda^*) F^*\phi^*, \phi \rangle$ holds for all ϕ and ϕ^*.

therefore we require $\lambda - \lambda^* = 0$ i.e. $\lambda^* = \lambda$

Chapter 5

4. (5) What is the "effective" delayed neutron fraction, β_{eff} and why is it used in reactor analysis? Explain how and why the β_{eff} differ in a fast and thermal reactor?

+5 4. β_{eff} is ~~defined~~ defined as: $\beta_{eff,k} = \bar{\beta}\, \gamma_{dk}$

where $\gamma_{dk} = \dfrac{\int_0^\infty \chi_{dk}(E)\,\phi^*(E)\,dE}{\int_0^\infty \chi(E)\,\phi^*(E)\,dE}$

which means β_{eff} is defined by the adjoint weighted delayed neutron spect

Eq 5.64

β_{eff} is used because the neutrons born in different positions and with different energies have different importance to the system. So we use $\phi^*(\vec{r}, E)$, which describes the importance of a neutron born at $\vec{r}$ with energy E, to weight the β_i.

In a fast reactor. $\gamma_{dk} \approx 0.82$. because the delayed neutrons have less energy. the probability to cause a fast fission is lower. therefore β_{eff} is smaller than β. And the difference between groups β are small.

In a thermal reactor, β_{eff} is larger than β. Since the delayed neutrons are born with less energy, the probability to escape the resonance becomes higher, so delayed neutrons are more important.

Chapter 6

5. (5) Describe the physical basis for the two methods used to derive the one group precursor decay constant from six delayed groups. Write the Equation for each method and explain the types of transients for which each is an appropriate approximation.

5. Method 1. from the precursor equation

$$\frac{d\xi_k}{dt} = -\lambda\xi_k + \beta_k P$$

we sum up both sides.

$$\sum_{k=1}^{6} \frac{d\xi_k}{dt} = \sum_{k=1}^{6} (-\lambda_k\xi_k + \beta_k P)$$

+3 Precursor Accumulate Approx compare with one-group equation $\frac{d\xi}{dt} = \lambda\xi + \beta P$

$$\Rightarrow \lambda^{\beta} = \frac{\sum_{k=1}^{6} \lambda_k \xi_k}{\sum_{k=1}^{6} \xi_k} = \frac{\sum_{k=1}^{6} \beta_k \xi_k}{\sum_{k=1}^{6} \beta_k} = \lambda^{\beta}$$

which is β weighted λ

Method 2: From the ~~[illegible]~~ static state case.

$$\frac{d\xi_k}{dt} = 0 \text{ . therefore } -\lambda_k\xi_k + \beta_k P = 0$$

$$\Rightarrow \xi_k = \frac{\beta_k P}{\lambda_k}$$

therefore: $$\lambda = \frac{\sum_{k=1}^{6} \lambda_k \xi_k}{\sum_{k=1}^{6} \xi_k} = \frac{\sum_{k=1}^{6} \lambda_k \frac{\beta_k P}{\lambda_k}}{\sum_{k=1}^{6} \frac{\beta_k P}{\lambda_k}} = \frac{\sum_{k=1}^{6} \beta_k}{\sum_{k=1}^{6} \frac{\beta_k}{\lambda_k}} = \lambda^{inv}$$

which is inverse β weighted λ

Short time transient: λ^{β}. since short ~~[illegible]~~ life precursors (λ_k larger) have more importance

long-time transient: λ^{inv}. since long life precursors (λ_k smaller) have more importance.

6. (10) Beginning with the kinetics equations with one group of delayed neutrons (Eqn. A.2 in Appendix of the exam), derive the following inhour equation:

$$\rho = \alpha\Lambda + \frac{\beta\alpha}{\alpha + \lambda}$$

6. PKE : $\frac{dP}{dt} = \frac{\rho_0 - \beta}{\Lambda} P + \frac{1}{\Lambda}\lambda \xi$ (1)

precursor : $\frac{d\xi}{dt} = -\lambda \xi + \beta P$ (2)

Assume $P(t) = P_{as}\, e^{\alpha t}$

$\xi(t) = \xi_{as}\, e^{\alpha t}$

---

6. PKE : $\frac{dP}{dt} = \frac{\rho_0 - \beta}{\Lambda} P + \frac{1}{\Lambda}\lambda \xi$ (1)

precursor : $\frac{d\xi}{dt} = -\lambda \xi + \beta P$ (2)

Assume $P(t) = P_{as}\, e^{\alpha t}$

$\xi(t) = \xi_{as}\, e^{\alpha t}$

insert into (1) and (2)

$\alpha P_{as} \cancel{e^{\alpha t}} = \frac{\rho_0 - \beta}{\Lambda} P_{as} \cancel{e^{\alpha t}} + \frac{1}{\Lambda}\lambda \xi_{as} \cancel{e^{\alpha t}}$ (3)

$\alpha \xi_{as} \cancel{e^{\alpha t}} = -\lambda \xi_{as} \cancel{e^{\alpha t}} + \beta P_{as} \cancel{e^{\alpha t}}$ (4)

cancel out $e^{\alpha t}$ on both sides and we obtain the relation between P_{as} and ξ_{as}

$$P_{as} = \frac{\alpha+\lambda}{\beta}\xi_{as} \qquad (5)$$

insert (5) into (3) :

$$\alpha\frac{\alpha+\lambda}{\beta}\cancel{\xi_{as}} = \frac{\rho-\beta}{\Lambda}\frac{\alpha+\lambda}{\beta}\cancel{\xi_{as}} + \frac{\lambda}{\Lambda}\cancel{\xi_{as}}$$

cancel out ξ_{as} and ~~[illegible]~~ rearrange. we get

$$\rho = \alpha\Lambda + \beta - \frac{\lambda\beta}{\alpha+\lambda}$$

+10

$$= \alpha\Lambda + \frac{\alpha\beta+\lambda\beta-\lambda\beta}{\alpha+\lambda}$$

$$= \alpha\Lambda + \frac{\beta\alpha}{\alpha+\lambda}$$, which is the inhour equation

7. (10) A control rod bank is partially withdrawn from an initially critical reactor at low power and room temperature (i.e. assume feedback effects are not important). The signal of a neutron detector increases immediately to 125% of its value prior to withdrawal and then increases on a slow exponential (subprompt critical).

Assume the following kinetics data:

$$\beta = 0.0065 \quad \lambda = 0.08s^{-1} \quad \Lambda = 6\times10^{-5}\,s$$

a. Calculate the reactivity worth of the control rod bank withdrawal (in $)

b. Estimate the stable period of the reactor a.

$$\rho^{\circ} = \frac{\beta}{\beta-\rho_1}\rho_1 \Rightarrow \frac{\beta}{\beta-\rho_1} = 125\%$$

$$\Rightarrow \rho_1 = 0.2 \$$$

b. $\alpha = \frac{\lambda\rho_1}{\beta-\rho_1}$ (according to Appendix A3)

$$= \frac{0.08 \times 0.2}{1-0.2}$$

-10

$$= 0.02 \text{ s}$$

8. (20) In the molten salt reactor (MSR) the fuel moves with the coolant and the time-dependent precursor concentration equation for precursor group k that would account for this fuel movement can be written as follows:

$$\frac{dC_k(\vec{r})}{dt} = \beta_k \int \nu\Sigma_f \phi dE - \lambda_k C_k(\vec{r}) - \vec{\nabla} \bullet \left(\vec{v}C_k(\vec{r})\right)$$

$C =$ Precursor Concentration (1)

$\phi =$ Neutron Flux

$\vec{v} =$ Fluid velocity

This problem asks you to determine the steady-state axial (1D) distribution for the lumped precursor concentration ***C(z)*** in the reactor assuming a constant fluid velocity **v** and a constant, uniform axial flux shape with one group of delayed neutrons. For this case equation (1) can be reduced to:

$$v\frac{dC(z)}{dz} = -\lambda C(z) + \beta\overline{\psi} \tag{2}$$

$$where$$

$$\overline{\psi} = \int \nu\Sigma_f \phi dE$$

a. Use Eq.(2) to develop an expression for ***C(z)***

Hint: For the differential equation with the form:

$$\frac{dy}{dx} = -ay + b \tag{3}$$

$$y(x) = y(0)\exp(-ax) + \frac{b}{a}(1 - \exp(-ax)) \tag{4}$$

According to (3) and (4), the solution is

$$C(z) = C(0)\exp\left(-\frac{\lambda}{v}z\right) + \frac{\beta}{\lambda}\bar{\psi}\left(1 - \exp\left(-\frac{\lambda}{v}z\right)\right) \tag{8.1}$$

b. Assume the precursor concentration distribution is known at the entrance to the reactor and is **C(0).** Integrate your solution from part **a** over the axial height of the reactor, **H**, to determine an expression for the average precursor concentration in the reactor.

Integrate $C(z)$ from 0 to H

$$\bar{C} = \frac{1}{H}\int_0^H \left(C(0) - \frac{\beta}{\lambda}\bar{\psi}\right)\exp\left(-\frac{\lambda}{v}z\right)dz + \frac{1}{H}\int \frac{\beta}{\lambda}\bar{\psi}\,dz \tag{8.2}$$

$$\bar{C} = \frac{v}{\lambda H}\left(C(0) - \frac{\beta}{\lambda}\bar{\psi}\right)\left(1 - \exp\left(-\frac{\lambda}{v}z\right)\right) + \frac{\beta}{\lambda}\bar{\psi} \tag{8.3}$$

c. Show that for the case of stagnant precursors and fluid **v=0** the expression in part b reduces to:

$$\overline{C} = \frac{\beta\overline{\psi}}{\lambda} \tag{5}$$

When $\mathbf{v} = 0$, (8.3) turns to be

$$\bar{C} = 0 + \frac{\beta}{\lambda}\bar{\psi} = \frac{\beta}{\lambda}\bar{\psi} \tag{8.4}$$

d. Based on Eqn (3) and your analysis in parts a, b, and c discuss the relative magnitude of the delayed neutron fraction β for the reactor when the fluid is stationary and when it is moving.

The $\bar{\beta}$ is

$$\bar{\beta} = \frac{\lambda\bar{C}}{\bar{\psi}} = \frac{v}{H}\left(\frac{\lambda C(0)}{\bar{\psi}} - \beta\right)\left(1 - exp\left(-\frac{\lambda}{v}z\right)\right) + \beta \tag{8.5}$$

Then

$$\frac{\bar{\beta}}{\beta} = \frac{v}{H}\left(\frac{\lambda C(0)}{\bar{\psi}\beta} - 1\right)\left(1 - exp\left(-\frac{\lambda}{v}z\right)\right) + 1 \tag{8.6}$$

Typically

$$\lambda C(0) < \beta\bar{\psi} \tag{8.7}$$

Therefore

$$\frac{\bar{\beta}}{\beta} < 1$$

e. Quantify the relative magnitude by assuming the core height = 300 cm, the fluid velocity is 50cm/s, and C(0)=0, ie all the precursors in the fluid have decayed while the fuel is in the ex-core loop and before it reenters the core) Also, use the same kinetics data from problem 7 (where β from this problem is for the stagnant, non-flowing precursors):

$$\beta = 0.0065 \quad \lambda = 0.08 s^{-1} \quad \Lambda = 6\times 10^{-5} s$$

Plug the point kinetics data and parameter into the (8.6)

$$\frac{\bar{\beta}}{\beta} = \frac{50}{300}(-1)\left(1 - exp\left(-\frac{24}{50}\right)\right) + 1 = 0.936$$