NERS551

Winter, 2022

Homework #2

Due TBA

Chapter 5

1. Starting with time-dependent neutron diffusion equation, derive the exact point kinetics equations for an initially critical reactor, Eqns 5.34.

2. Review Questions, p. 81 Ott

   a. What is dynamic reactivity and why is it different from the static reactivity? (Ques. 9)

   b. What is the major difference between beta-effective (5.62) and the corresponding one-group beta (3.50)? Explain these differences qualitatively for thermal and fast reactors. (Ques. 10)

   c. What are the basic assumptions in the point reactor model? (Ques. 12)

Chapter 6

3. Describe the two methods for combining the six precursor groups that result in different definitions for the one-group λ. Explain the differences in the two methods and the conditions for which each is most appropriate.

4. Write the 7 different point kinetics equations in order of decreasing sophistication of the description of the delayed neutron source. (Briefly explain the difference in the treatment of the delayed neutron source).

5. Inhour Equation (Ott Hmk Problem 1, p. 134)

   Use the Inhour Equation (Eq. 6.65) to perform the following analysis:

   a. Find the stable and prompt period branches for U-235 as fuel and Λ= 1E-4, 2E-5, 4E-7 seconds (Use kinetics data given in Table 2-III of the text).

   b. Find ρ in the one-delay-group approximation (Eq. 6.68) with the one-group λ.

   Plot the results in ($) for the three reactivities outside of the range of singularities for each value of Λ. Compare the reactivities and discuss the effects of Λ.

6. Solution of Point Kinetics Equations with One-Group Delayed Neutrons

Beginning with the one delay group kinetics equations (Eqns. 6.97a/b), show the detailed derivation of the solution of the point kinetics equations (Eqn. 6.109).

7. Transients with Constant Reactivity, Ott Hmk Problem 5, p. 136, with some modifications.

Develop a small program (in MATLAB or FORTRAN) to solve the point kinetics equations for six delayed neutron groups (see Appendix). Find the solution for the reactivity insertions below (assume an initially critical reactor).

   a. $\rho 1$ = 0.25 \$
   b. $\rho 1$ = 0.50 \$
   c. $\rho 1$ = -1.0 \$

For each reactivity, plot results for three cases: Λ= 1E-4, 2E-5, 4E-7 seconds.

Be sure to discuss the transient results, in particular the short-time behavior, the asymptotic behavior, and the Λ dependence. Please submit a copy of your source, program input and output through CANVAS.

Appendix

Solution of PKE w/ Six precursor Groups:

$$\dot{\zeta}_k(t) = -\lambda_k \zeta_k(t) + \beta_k p(t) \qquad k = 1..6 \tag{1}$$

$$\dot{p}(t) = \frac{\rho - \beta}{\Lambda} p(t) + \frac{1}{\Lambda} \sum_k \lambda_k \zeta_k(t) \tag{2}$$

Assume existence of α_n $(n = 1..7)$ satisfying:

$$\dot{\zeta}_k(t) = \alpha_n \zeta_k(t) \tag{3}$$

$$\dot{p}_k(t) = \alpha_n p(t) \tag{4}$$

Then

$$\alpha_n \zeta_k(t) = -\lambda_k \zeta_k(t) + \beta_k p(t) \qquad k = 1..6 \tag{5}$$

$$\alpha_n p(t) = \frac{\rho - \beta}{\Lambda} p(t) + \frac{1}{\Lambda} \sum_k \lambda_k \zeta_k(t) \tag{6}$$

In matrix form

$$\alpha_n \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \\ \zeta_4 \\ \zeta_5 \\ \zeta_6 \\ p \end{bmatrix} = \begin{bmatrix} -\lambda_1 & & & & & & \beta_1 \\ & -\lambda_2 & & & & & \beta_2 \\ & & -\lambda_3 & & & & \beta_3 \\ & & & -\lambda_4 & & & \beta_4 \\ & & & & -\lambda_5 & & \beta_5 \\ & & & & & -\lambda_6 & \beta_6 \\ \frac{\lambda_1}{\Lambda} & \frac{\lambda_2}{\Lambda} & \frac{\lambda_3}{\Lambda} & \frac{\lambda_4}{\Lambda} & \frac{\lambda_5}{\Lambda} & \frac{\lambda_6}{\Lambda} & \frac{\rho - \beta}{\Lambda} \end{bmatrix} \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \\ \zeta_4 \\ \zeta_5 \\ \zeta_6 \\ p \end{bmatrix} \tag{7}$$

This is a eigenvalue-eigenvector problem which can be solved using the MATLAB function "eigens", to obtain 7 eigenvalues α_n

$$\begin{bmatrix} \zeta_1 & \zeta_2 & \zeta_3 & \zeta_4 & \zeta_5 & \zeta_6 & p \end{bmatrix}_n^T = \begin{bmatrix} \zeta_{1,n} & \zeta_{2,n} & \zeta_{3,n} & \zeta_{4,n} & \zeta_{5,n} & \zeta_{6,n} & p_n \end{bmatrix}^T \text{ n=1..7}$$

α_n are exact the roots of in-hour equation.

Then from equation (3)(4), we know the final form of the solution:

$$\begin{bmatrix} \zeta_1(t) \\ \zeta_2(t) \\ \zeta_3(t) \\ \zeta_4(t) \\ \zeta_5(t) \\ \zeta_6(t) \\ p(t) \end{bmatrix} = \sum_{n=1}^{7} A_n \begin{bmatrix} \zeta_{1,n} \\ \zeta_{2,n} \\ \zeta_{3,n} \\ \zeta_{4,n} \\ \zeta_{5,n} \\ \zeta_{6,n} \\ p_n \end{bmatrix} \exp(\alpha_n t) \tag{8}$$

Apply the initial conditions to obtain 7 coefficients A_n n=1..7.

The initial conditions are

When $t = 0^-$

$$\dot{\zeta}_k(t) = 0 \tag{9}$$

and $p(0) = 1$ (10)

Then from equation (1)

$$\zeta_k(0) = \frac{\beta_k}{\lambda_k} p(0) = \frac{\beta_k}{\lambda_k} \tag{11}$$

Substitute (10) and (11) to (8), take t=0

$$\begin{bmatrix} \beta_1/\lambda_1 \\ \beta_2/\lambda_2 \\ \beta_3/\lambda_3 \\ \beta_4/\lambda_4 \\ \beta_5/\lambda_5 \\ \beta_6/\lambda_6 \\ 1 \end{bmatrix} = \sum_{n=1}^{7} A_n \begin{bmatrix} \zeta_{1,n} \\ \zeta_{2,n} \\ \zeta_{3,n} \\ \zeta_{4,n} \\ \zeta_{5,n} \\ \zeta_{6,n} \\ p_n \end{bmatrix} = \begin{bmatrix} \zeta_{1,1} & \zeta_{1,2} & \zeta_{1,3} & \zeta_{1,4} & \zeta_{1,5} & \zeta_{1,6} & \zeta_{1,7} \\ \zeta_{2,1} & \zeta_{2,2} & \zeta_{2,3} & \zeta_{2,4} & \zeta_{2,5} & \zeta_{2,6} & \zeta_{2,7} \\ \zeta_{3,1} & \zeta_{3,2} & \zeta_{3,3} & \zeta_{3,4} & \zeta_{3,5} & \zeta_{3,6} & \zeta_{3,7} \\ \zeta_{4,1} & \zeta_{4,2} & \zeta_{4,3} & \zeta_{4,4} & \zeta_{4,5} & \zeta_{4,6} & \zeta_{4,7} \\ \zeta_{5,1} & \zeta_{5,2} & \zeta_{5,3} & \zeta_{5,4} & \zeta_{5,5} & \zeta_{5,6} & \zeta_{5,7} \\ \zeta_{6,1} & \zeta_{6,2} & \zeta_{6,3} & \zeta_{6,4} & \zeta_{6,5} & \zeta_{6,6} & \zeta_{6,7} \\ p_1 & p_2 & p_3 & p_4 & p_5 & p_6 & p_7 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \end{bmatrix} = \begin{bmatrix} \beta_1/\lambda_1 \\ \beta_2/\lambda_2 \\ \beta_3/\lambda_3 \\ \beta_4/\lambda_4 \\ \beta_5/\lambda_5 \\ \beta_6/\lambda_6 \\ 1 \end{bmatrix} \tag{12}$$

A_n n=1..7. can be solved from this linear system. Then insert them to equation (8) to obtain the final solution.