SOLUTIONS FOR THE FIRST ASSIGNMENT

Solutions for the problems from the text can be found starting on page 299. We won't include solutions to those problems here.

Question A.

1. If $ab = 0$, then $a = 0$ and $b = 0$. This statement is false. Here is a counterexample. Take $a = 1$ and $b = 0$. The $ab = 0$ is true. But it is not true that $a = 0$ and $b = 0$.

2. If $ab = 0$, then $a = 0$ or $b = 0$. This statement is true. It is a property of the real numbers which was stated in class (without proof).

3. If $a + b = 0$, then $a = 0$ and $b = 0$. This is false. Here is a counterexample. Take $a = 5$ and $b = -5$. Then $a + b = 0$ is true, but it is not true that $a = 0$ and $b = 0$.

4. If $ab = 0$ and $a + b = 0$, then $a = 0$ and $b = 0$. This statement is true. Here is a proof. Assume that $ab = 0$ and $a + b = 0$.

Since $ab = 0$, it follows that $a = 0$ or $b = 0$.

If $a = 0$, then $a + b = b$. Since $a + b = 0$, it then follows that $b = 0$. Hence it then follows that $a = 0$ and $b = 0$.

If $b = 0$, then $a + b = a$. Since $a + b = 0$, it then follows that $a = 0$. Hence it then follows that $a = 0$ and $b = 0$.

Therefore, we have proved that $a = 0$ and $b = 0$.

Question B. The statement in question is:

(S): If a divides b and a divides c, then a divides $b + c$.

This statement is true. Here is a proof. Assume that a divides b and a divides c. Since a divides b, we can write $b = am$ where $m \in \mathbf{Z}$. Since a divides c, we can write $c = an$ where $n \in \mathbf{Z}$. We then have

$$b + c = am + an = a(m + n)$$

Since $m, n \in \mathbf{Z}$, it follows that $m + n \in \mathbf{Z}$. Therefore, the equation $b + c = a(m + n)$ implies that a divides $b + c$, which is what we wanted to prove.

Question C. The converse of (S) is the following statement:

If a divides $b + c$, then a divides b and a divides c.

This statement is false. Here is a counterexample. Let $a = 5, b = 3, c = 7$. Then $b + c = 10$. Obviously, for that choice of a, b, and c, it is true that a divides $b + c$. But it is not true that a divides b and a divides c.

Question D. As defined in class, a sequence $\{a_n\}$ of real numbers is said to be *bounded* if there exists a real number M such that $|a_n| < M$ for all $n \in \mathbf{Z}^+$. A sequence $\{a_n\}$ is said to be *unbounded* if it is not bounded. Here is a definition without words of negation.

A sequence $\{a_n\}$ of real numbers is said to be *unbounded* if for every real number M, there exists at least one $n \in \mathbf{Z}^+$ such that $|a_n| \geq M$.

Question E. We will simply list the subsets of $\{1, 2, 3, 5, 7\}$ in a systematic way:

Subsets with zero elements or five elements:

$$\phi, \qquad \{1, 2, 3, 5, 7\}$$

where ϕ denotes the empty set.

Subsets with one or four elements:

$$\begin{array}{ll} \{1\}, & \{2, 3, 5, 7\} \\ \{2\}, & \{1, 3, 5, 7\} \\ \{3\}, & \{1, 2, 5, 7\} \\ \{5\}, & \{1, 2, 3, 7\} \\ \{7\}, & \{1, 2, 3, 5\} \end{array}$$

Subsets with two or three elements:

$$\begin{array}{ll} \{1, 2\}, & \{3, 5, 7\} \\ \{1, 3\}, & \{2, 5, 7\} \\ \{1, 5\}, & \{2, 3, 7\} \\ \{1, 7\}, & \{2, 3, 5\} \\ \{2, 3\}, & \{1, 5, 7\} \\ \{2, 5\}, & \{1, 3, 7\} \end{array}$$

$$\begin{array}{ll} \{2,7\}, & \{1,3,5\} \\ \{3,5\}, & \{1,2,7\} \\ \{3,7\}, & \{1,2,5\} \\ \{5,7\}, & \{1,2,3\} \end{array}$$

The above lists all subsets of $\{1, 2, 3, 5, 7\}$. The number of subsets is $1+1+5+5+10+10 = 32$.

Question F. The statement in question is false. Here is a counterexample. Take

$$A = \{1,\ 3\}, \quad B = \{1,\ 2,\ 3\}, \quad \textit{and} \quad C = \{1,\ 2\}$$

Thus, A, B and C are subsets of $\mathbf{Z}^+$. By definition, we have

$$A + C = \{1+1,\ 1+2,\ 3+1,\ 3+2\} = \{2,\ 3,\ 4,\ 5\}$$

and

$$B + C = \{1+1,\ 1+2,\ 2+1,\ 2+2,\ 3+1,\ 3+2\} = \{2,\ 3,\ 3,\ 4,\ 4,\ 5\} = \{2,\ 3,\ 4,\ 5\}$$

Hence $A + C = B + C$, but $A \neq B$.