Modeling Traffic Flow with Physics-Informed Neural Networks (PINNs) Using the LWR Equation

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MOTIVATION

- **Traffic congestion is a growing urban problem**
- **Simulating traffic helps in planning and optimization**
- **Traditional models exist, but they have limitations**

What is the LWR Model?

- Lighthill-Whitham-Richards model
- First-order PDE modeling traffic density
- Based on conservation of vehicles
- Flux defined as $q(p) = p(1 - p)$

What is a PINN?

- Neural network trained to satisfy PDEs
- Loss function includes physical laws
- Learns solution without ground truth labels

Our Scenario

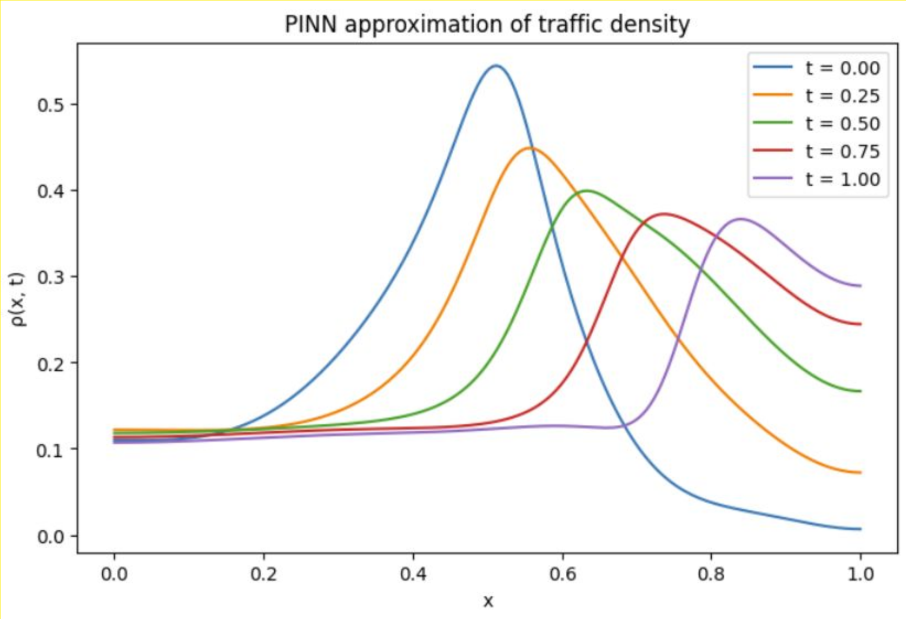
- Simulating a Gaussian traffic pulse
- 1D road segment from $x = 0$ to $x = 1$
- Neumann (zero-gradient boundary conditions)

Newark Architecture

- 3 hidden layers, 50 neurons each
- Tanh activation functions
- Inputs (x, t); Output: traffic density p(x, t)

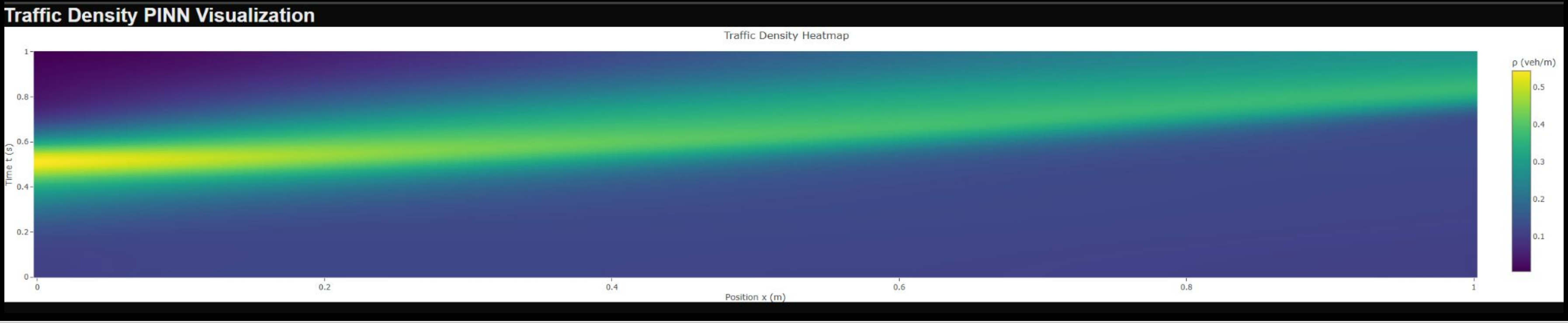
Training Process

- Loss = PDE residual + initial + boundary
- 20,000 collection points for PDE
- Trained with Adam optimizer for 5,000 epochs



Results - Heatmap

- PINN predicts full spatiotemporal density
- Captures shockwave moving left
- Rarefaction wave spreads right

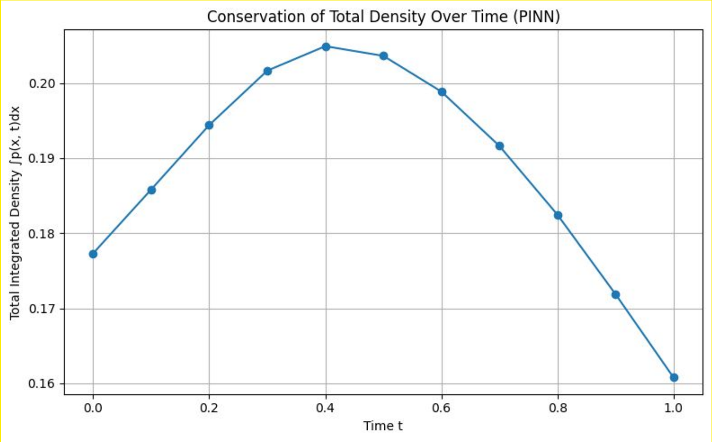


Results - Validation

- Matches LWR wave behavior
- Shows characteristic speeds: $c(p) = 1 - 2p$
- Boundary behavior is physically consistent

Results - Conservation Check

- Integral of p(x, t) shows fluctuation
- Max Deviation: ~15.7%
- Indicates mild loss of conservation



Limitations

- No direct comparison to numerical solvers
- PINNs smooth out sharp rocks
- Only tested on a single, simple case

Future Work

- Compare with traditional solvers (e.g. Godunov)
- Try more realistic scenarios
- Improve shock resolution with better architectures
- Explore hyperparameter tuning and adaptive sampling

Conclusion

- PINNs qualitatively captured LWR dynamics (shock/rarefaction)
- Captures key behaviors (shock, rarefaction)
- Promising tool with room for refinement