Ex 5 Pen and Paper

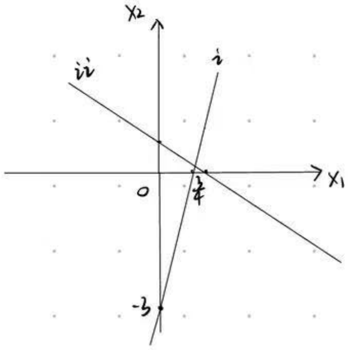
→ 2D Occupancy Network

a). $w_1x_1+w_2x_2+b=0$, $w=(w_1, w_2)$, $x=\begin{pmatrix}x_1\\x_2\end{pmatrix}$

b).

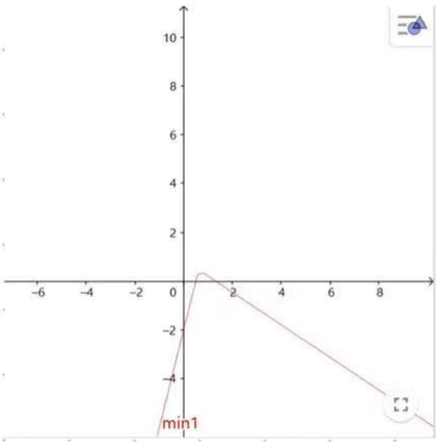
i). $-4x_1+x_2+3=0$

ii). $2x_1+3x_2-2=0$

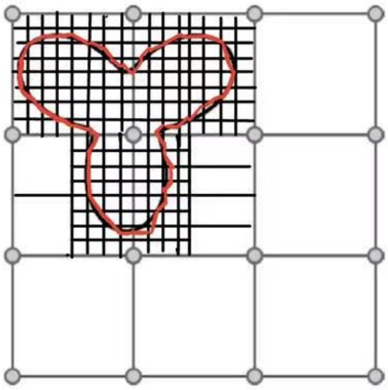


c).

l: $2\max(0, -4x_1+x_2+3)+3\max(0, 2x_1+3x_2-2)-2=0$



d).



→ Differential Volumetric Rendering

a). $x_0 = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$, $x_1 = \begin{pmatrix} \frac{2}{\sqrt{2}} \\ \frac{2}{\sqrt{2}} \end{pmatrix}$

$f(x_0) = \begin{pmatrix} \frac{-3}{\sqrt{2}} + 3 \\ \frac{-3}{\sqrt{2}} + 3 \end{pmatrix}$, $f(x_1) = \begin{pmatrix} -\frac{6}{\sqrt{2}} + 3 \\ -\frac{6}{\sqrt{2}} + 3 \end{pmatrix}$

$$\therefore \hat{p} = \begin{pmatrix} \frac{2}{\sqrt{2}} \\ \frac{2}{\sqrt{2}} \end{pmatrix} - \begin{pmatrix} 3 - \frac{6}{\sqrt{2}} \\ 3 - \frac{6}{\sqrt{2}} \end{pmatrix} \cdot \frac{\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}}{\begin{pmatrix} \frac{-3}{\sqrt{2}} \\ \frac{-3}{\sqrt{2}} \end{pmatrix}}$$

$$= \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

b). $\theta_t = W_t \cdot \hat{p} + b_t = 5 - 2 = 3$

$\therefore I(\hat{p}) = \sigma(\theta_t) = (1 + e^{-3})^{-1} = 0.953$

c). $L = \sqrt{(\hat{I} - I)^2}$ $L = (\hat{I} - I)^2$

① $\frac{\partial L}{\partial \hat{I}} = \frac{|\hat{I} - I|}{\hat{I} - I} = 1$ $2(\hat{I} - I) = 0.706$

② $\frac{\partial t_{\theta t}(\hat{p})}{\partial W_f} = \frac{\partial t_{\theta t} \cdot (\hat{p})}{\partial \hat{p}} \cdot \frac{\partial \hat{p}}{\partial W_f}$

$= \frac{\partial \sigma(W_t \cdot \hat{p} + b_t)}{\partial \hat{p}} \cdot \frac{\partial \hat{p}}{\partial W_f}$

$= 0.953 \times (1 - 0.953) \times W_t \cdot \frac{\partial \hat{p}}{\partial W_f} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

③ $\frac{\partial t_{\theta t}(\hat{p})}{\partial b_f} = \frac{\partial \sigma(W_t \cdot \hat{p} + b_t)}{\partial \hat{p}} \cdot \frac{\partial \hat{p}}{\partial b_t} = 0$

④ $\frac{\partial t_{\theta t}(\hat{p})}{\partial \hat{p}} = 0.953 \times (1 - 0.953) \times \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0.0896 \\ 0.1344 \end{pmatrix}^T$

d). $\frac{\partial \hat{p}}{\partial W_f} = -\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \cdot \left[(-4, 1) \times \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \right]^{-1} \cdot (1, 1)$

$= \begin{pmatrix} \frac{1}{3}, & \frac{1}{3} \\ \frac{1}{3}, & \frac{1}{3} \end{pmatrix}$

$$\frac{\partial \hat{p}}{\partial b_f} = -\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix} \cdot \left[(-4,1) \times \begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix}\right]^{-1} \cdot 1$$

$$= \begin{pmatrix}\frac{1}{3}\\ \frac{1}{3}\end{pmatrix}$$

$$\therefore \frac{\partial L}{\partial W_f} = 1 \cdot \left[\begin{pmatrix}0.0896\\ 0.1344\end{pmatrix}^T \times \begin{pmatrix}\frac{1}{3} & \frac{1}{3}\\ \frac{1}{3} & \frac{1}{3}\end{pmatrix}\right]$$

$$= [0.0747, 0.0747]$$

$$\frac{\partial L}{\partial b_f} = (0.0896, 0.1344) \times \begin{pmatrix}\frac{1}{3}\\ \frac{1}{3}\end{pmatrix}$$

$$= 0.0747$$

→ Neural Network Layers

a).

(1) ① $16^2 \times 256 + 256 = 65792$

② 16×16

(2) ① $3 \times 3 \times 32 \times 64 + 64 = 18496$

② 3×3

(3) ① $3 \times 3 \times 32 \times 32 \times 2 + 64 = 18496$

② 5×5, 9×9

(4) ① $32 \times 32 \times 3 \times 3 \times 2 + 64 = 18496$

② 3×3, 7×7

(5) ① 0

② 2×2

b).

	Predicted scores s	softmax(s)	CE loss
①	$(-1, +3, +0, +1)^T$		
②	$(+2, +3, -1, -1)^T$		
③	$(+0, -1, -1, +2)^T$		
④	$(+1, +0, +3, +3)^T$		

①: (0.0152, 0.8310, 0.0414, 0.1125)

CEloss: 4.19, Incorrectly

② (0.2619, 0.7120, 0.0130, 0.0130)

CEloss: 0.34, Correctly

③ (0.1096, 0.0403, 0.0403, 0.8098)

CEloss: 3.21, Incorrectly

④ (0.0619, 0.0228, 0.4576, 0.4576)

CEloss: 0.78, Correctly

→ Detection Metrics

a).

$IoU_A = \frac{6}{11} = 0.545 > 0.5$ ∴ correct

$IoU_B = \frac{2}{3} = 0.667 > 0.5$ ∴ correct

$IoU_D = \frac{4}{11} = 0.363 < 0.5$ ∴ incorrect

b). TP = 2, FP = 1 (2), FN = 1 (?)

c). TP = 1, FP = 1 3, FN = 2

precision = $\frac{1}{2} = 0.5$ $\frac{1}{4} = 0.25$

recall = $\frac{1}{3} = 0.333$