Ex 4 Pen and Paper

→ Gradient of Negative Conditional Log-Likelihood

a).

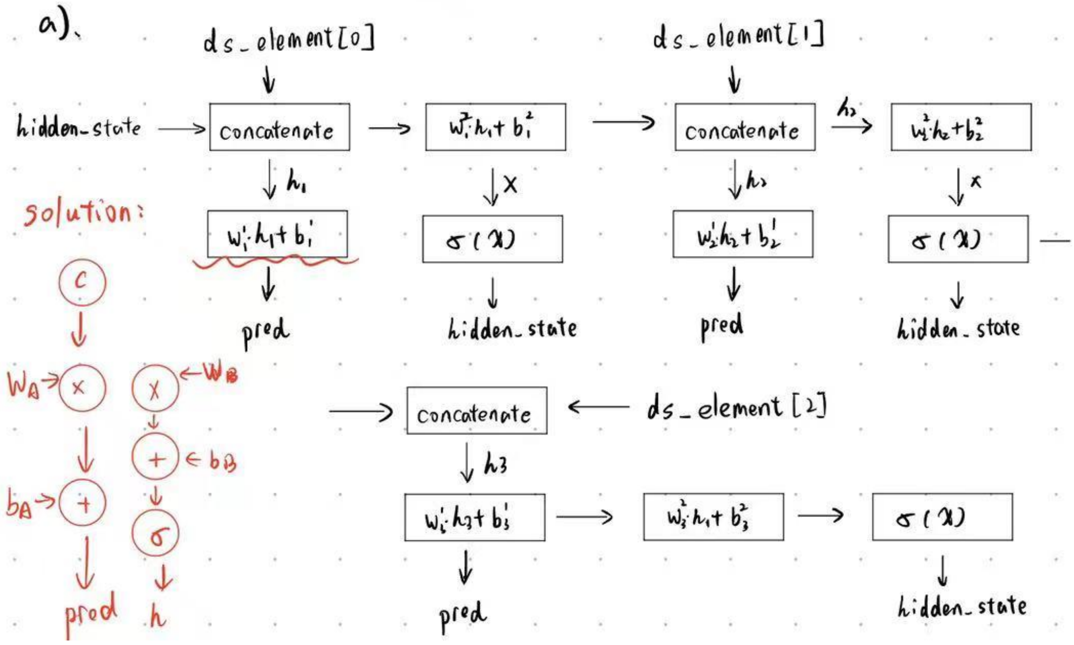
$$\nabla_w L(w) = -\sum_{n=1}^{N}\left(\frac{\partial \psi(x^n, y^n, w)}{\partial w} - \frac{\sum_y \exp\{\psi(x^n, y^n, w)\} \cdot \frac{\partial \psi(x^n, y^n, w)}{\partial w}}{\sum_y \exp\{\psi(x^n, y^n, w)\}} \right)$$

$$= -\sum_{n=1}^{N}\left(\frac{\partial \psi(x^n, y^n, w)}{\partial w} - \sum_y \frac{\exp\{\psi(x^n, y^n, w)\} \cdot \frac{\partial \psi(x^n, y^n, w)}{\partial w}}{\sum_{y'} \exp\{\psi(x^n, y'^n, w)\}} \right)$$

$$= -\sum_{n=1}^{N}\left(\frac{\partial \psi(x^n, y^n, w)}{\partial w} - \sum_y p(y^n \mid x^n, w) \cdot \frac{\partial \psi(x^n, y^n, w)}{\partial w} \right)$$

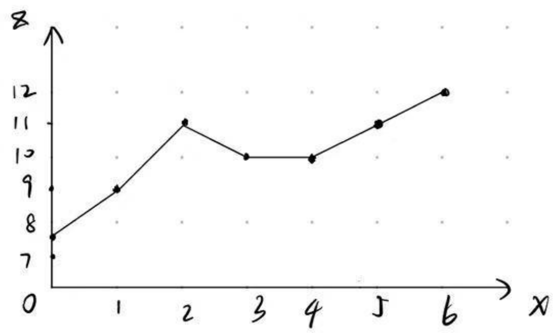
→ Inference Unrolling

a).



→ Surface Integration

x_i	0.0	1.0	2.0	3.0	4.0	5.0	6.0
$\frac{dz}{dx}(x_i)$	1.5	2.0	-1.0	0.0	1.0	1.0	0.0
$z(x_i)$	7.5	9.0	11.0	10.0	10.0	11.0	12.0



→ Volumetric Fusion Formation

a).

make $f(D) = \sum_i w_i \cdot (d_i - D)^2$

$$\frac{df(D)}{dD} = \sum_i w_i \cdot 2(d_i - D) \cdot (-1)$$

$$= \sum_i 2w_i D - 2w_i \cdot d_i$$

$$= 0$$

$\Rightarrow \quad D \cdot \sum_i w_i = \sum_i w_i \cdot d_i$

$\Rightarrow \quad D = \sum_i w_i \cdot d_i \Big/ \sum_i w_i$

$\because f(D)$ gets the minimum when $D = \frac{\sum_i w_i \cdot d_i}{\sum_i w_i}$

make $D = D(x)$, $w_i = w_i(x)$, $d_i = d_i(x)$, then

$D(x) = \frac{\sum_i w_i(x) d_i(x)}{\sum_i w_i}$ is the solution to the problem

$D^* = \arg\min \sum_i w_i (d_i - D)^2$.

b).

① $\because W(x) = \sum_i w_i(x)$

$\therefore W_{i+1}(x) - W_i(x) = \sum_{i+1} w_i(x) - \sum_i w_i(x)$

$= w_{i+1}(x)$

$\therefore W_{i+1}(x) = W_i(x) + w_{i+1}(x)$

② $\because D(x) = \frac{\sum_i w_i(x) \cdot d_i(x)}{\sum_i w_i(x)}$

$\therefore D_{i+1}(x) = \frac{\sum_{i+1} w_i(x) \cdot d_i(x)}{W_{i+1}(x)}$

$= \frac{W_i(x) \cdot D_i(x) + w_{i+1}(x) \cdot d_{i+1}(x)}{W_i(x) + w_{i+1}(x)}$

→ Drawing the p/q Reflectance Map for a Fixed R

a).

① $R = 0.5$:

$n^T \cdot s = R$

$$\Rightarrow \frac{1}{\sqrt{p^2+q^2+1}} = 0.5$$

$$\Rightarrow p^2+q^2 = 3$$

② $R = 0.9$:

$$\frac{1}{\sqrt{p^2+q^2+1}} = 0.9$$

$$\Rightarrow p^2+q^2+1 = \frac{100}{81}$$

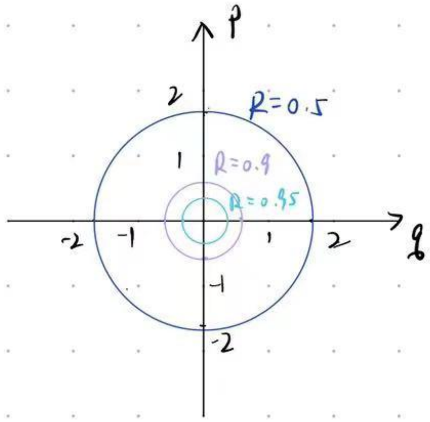
$$\Rightarrow p^2+q^2 = \frac{19}{81}$$

③ $R = 0.95$:

$$\frac{1}{\sqrt{p^2+q^2+1}} = 0.95$$

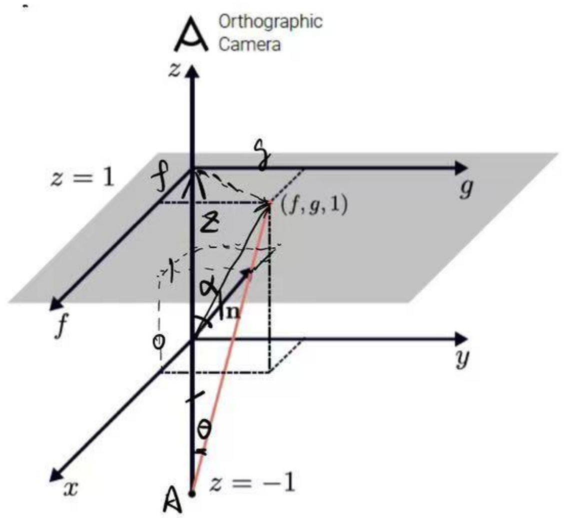
$$\Rightarrow p^2+q^2+1 = \frac{100^2}{95^2}$$

$$\Rightarrow p^2+q^2 = \frac{(5\sqrt{39})^2}{95^2}$$

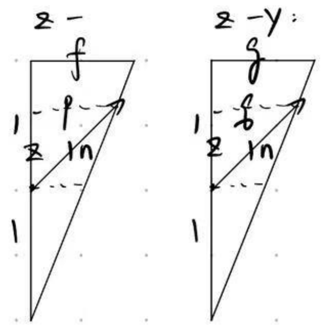


Reflection Map

b).



①



$$\begin{cases} \dfrac{f}{p/\sqrt{p^2+q^2+1}} = \dfrac{2}{1+|Z|} = \dfrac{2}{1+\frac{1}{\sqrt{p^2+q^2+1}}} \\[2ex] \dfrac{g}{q/\sqrt{p^2+q^2+1}} = \dfrac{2}{1+|Z|} = \dfrac{2}{1+\frac{1}{\sqrt{p^2+q^2+1}}} \end{cases}$$

$$\Rightarrow \begin{cases} f = \dfrac{2p}{1+\sqrt{p^2+q^2+1}} \\[2ex] g = \dfrac{2q}{1+\sqrt{p^2+q^2+1}} \end{cases}$$

c).

$$* \begin{cases} f = \dfrac{2p}{1+\sqrt{p^2+q^2+1}} \\ g = \dfrac{2q}{1+\sqrt{p^2+q^2+1}} \end{cases}$$

① $R = 0.5$:

$\because p^2+q^2+1 = 4$

$\therefore \frac{3}{2}f = p, \quad \frac{3}{2}g = q$

$\therefore \frac{9}{4}f^2 + \frac{9}{4}g^2 = 3 \Rightarrow f^2+g^2 = \frac{4}{3}$

② $R = 0.9$:

$\because p^2+q^2+1 = \frac{100}{81}$

$\frac{18\times 18}{19\times 81}$

$\therefore \frac{181}{162}f = p, \quad \frac{181}{162}g = q$

$\therefore (\frac{19}{18})^2 f^2 + (\frac{19}{18}g)^2 = \frac{19}{81}$

$\Rightarrow f^2+g^2 = 0.211$

③ $R = 0.95$:

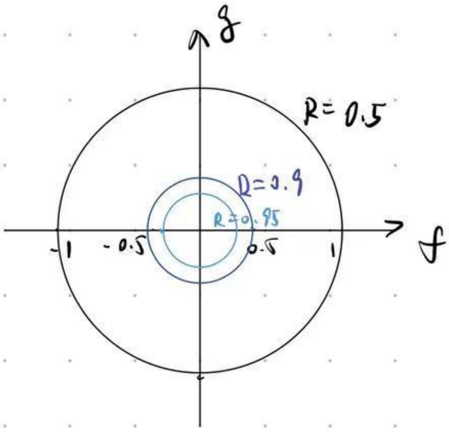
???

$\because p^2+q^2+1 = \frac{100^2}{95^2}$

$\therefore f = \dfrac{2p}{1+\frac{100}{95}} = \frac{190}{195}p, \quad g = \frac{190}{195}q$

$\therefore (\frac{39}{38})^2 \cdot f^2 + (\frac{39}{38})^2 g^2 = \frac{25\times 39}{95^2}$

$\Rightarrow f^2+g^2 = 0.103$



→ Marching cubes

a).