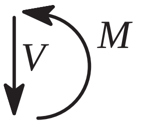
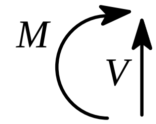


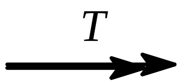
FORMULÆ FOR ME 211 INTRODUCTION TO SOLID MECHANICS

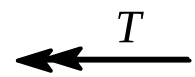


Vectors, forces and moments:

$$\boldsymbol{F} = \boldsymbol{i}F_x + \boldsymbol{j}F_y + \boldsymbol{k}F_z = \{F_x, F_y, F_z\} = \boldsymbol{e}|\boldsymbol{F}| \ ; \quad |\boldsymbol{F}| = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$\boldsymbol{e} = \left\{ \frac{F_x}{|\boldsymbol{F}|}, \frac{F_y}{|\boldsymbol{F}|}, \frac{F_z}{|\boldsymbol{F}|} \right\} = \{\cos\theta_x, \cos\theta_y, \cos\theta_z\}$$

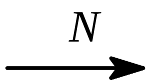


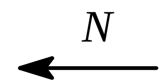


$$\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix} = (yF_z - zF_y)\boldsymbol{i} + (zF_x - xF_z)\boldsymbol{j} + (xF_y - yF_x)\boldsymbol{k} \ .$$

$$\boldsymbol{p} \cdot \boldsymbol{q} = p_x q_x + p_y q_y + p_z q_z = |\boldsymbol{p}||\boldsymbol{q}| \cos\theta \ ,$$

where θ is the angle between the vectors $\boldsymbol{p}$ and $\boldsymbol{q}$.





Properties of areas: Coordinates of the centroid are $\bar{x}, \bar{y}$ where

$$A = \iint dA \ ; \quad A\bar{x} = \iint x dA \ ; \quad A\bar{y} = \iint y dA \ ; \quad I_x = \iint y^2 dA \ ; \quad I_y = \iint x^2 dA \ .$$

In Cartesian coordinates, $dA = dxdy$. In polar coordinates, $dA = rdrd\theta$.

$$A = \sum A_i \ ; \quad A\bar{x} = \sum A_i x_i \ ; \quad A\bar{y} = \sum A_i y_i \ ; \quad I_x = \sum I_x^i + \sum A_i(\bar{y}_i - \bar{y})^2 \ ,$$

where I_x^i is the second moment of A_i about its own centroidal axis and I_x is the centroidal second moment of the composite area. For a circular section of radius a and diameter $d = 2a$,

$$I = \frac{\pi d^4}{64} = \frac{\pi a^4}{4} \ ; \qquad J = \frac{\pi d^4}{32} = \frac{\pi a^4}{2} \ .$$

For a rectangular section of height h and width b about a horizontal axis

$$I = \frac{bh^3}{12}$$

Bending: Relations between loading w, shear force V, bending moment M, slope θ and deflection v for a beam.

$$\frac{dV}{dx} = -w(x) \ ; \quad \frac{dM}{dx} = V(x) \ ; \quad EI\frac{d\theta}{dx} = M(x) \ ; \quad \frac{dv}{dx} = \theta(x) \ .$$

$$\frac{M}{I} = -\frac{\sigma_x}{y} = \frac{E}{R} = E\frac{d\theta}{dx} \ .$$

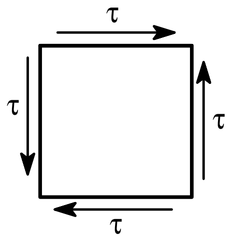
Definition of strain: Normal strain: if ϵ_x is uniform along the bar,

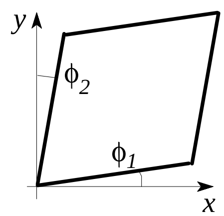
$$\epsilon_x = \frac{\Delta}{L} = \frac{u_B - u_A}{L_{AB}} \ ,$$

where Δ is the extension of the bar and L is its original length.
If ϵ_x varies along the bar, use

$$\epsilon_x = \frac{\partial u_x}{\partial x} \ .$$

Shear strain: If the initial rectangular element deforms as shown,





$$\gamma_{xy} = \phi_1 + \phi_2 = \frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y} \ .$$

Axial loading: Axial force N, cross-sectional area A.

$$\sigma_x = \frac{N}{A} \ ; \qquad u_B - u_A = \frac{N_{AB} L_{AB}}{E_{AB} A_{AB}} \ .$$

Torsion: (axis of the bar is along z), torque T, rotation ϕ about the axis.

$$\frac{T}{J} = \frac{\tau_{z\theta}}{r} = G\frac{d\phi}{dz} = \frac{G(\phi_B - \phi_A)}{L_{AB}} \ ; \qquad \phi_B - \phi_A = \frac{T_{AB} L_{AB}}{G_{AB} J_{AB}} \ .$$

Power transmitted is $P = T\Omega$ where Ω is the angular velocity of the shaft in radians per second.

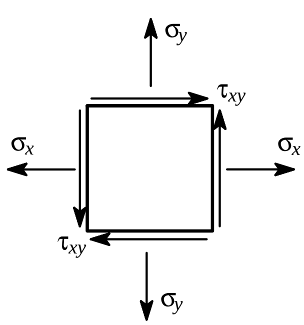
$$1 \text{ rpm } = \frac{\pi}{30} \text{ rad/s} \ ; \quad 1 \text{ W } = 1 \text{ Nm/s} \ ; \quad 1 \text{ hp } = 550 \text{ ft lb/s}$$

Multiaxial Hooke's law:

$$\begin{aligned} \epsilon_x &= \frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E} + \alpha\Delta T \\ \epsilon_y &= \frac{\sigma_y}{E} - \frac{\nu\sigma_x}{E} - \frac{\nu\sigma_z}{E} + \alpha\Delta T \\ \epsilon_z &= \frac{\sigma_z}{E} - \frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E} + \alpha\Delta T \ . \end{aligned}$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G} \ ; \quad \gamma_{zx} = \frac{\tau_{zx}}{G} \ ; \quad \gamma_{xy} = \frac{\tau_{xy}}{G} \ ; \qquad G = \frac{E}{2(1+\nu)} \ .$$

Stress transformation equations: Sign convention for stress components in two dimensions:

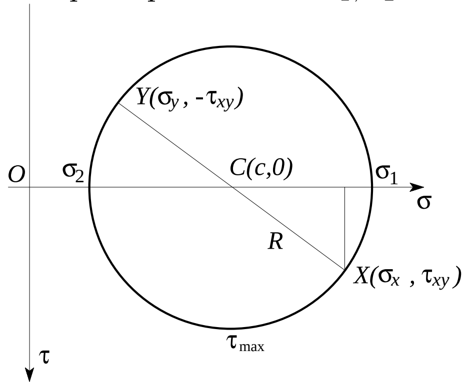


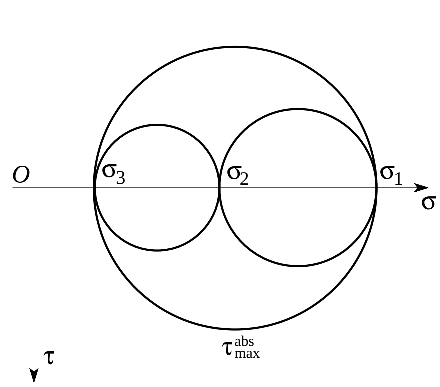
For stresses on a specified plane (rotated counterclockwise through θ from x) use

$$\sigma_{x'} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta\cos\theta$$

$$\tau_{x'y'} = \tau_{xy}(\cos^2\theta - \sin^2\theta) + (\sigma_y - \sigma_x)\sin\theta\cos\theta$$

For principal stresses σ_1, σ_2 or maximum shear stress use Mohr's circle:-





where

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \; ; \quad c = \frac{\sigma_x + \sigma_y}{2} \; ; \quad \sigma_1, \sigma_2 = c \pm R \; ; \quad \tau_{\text{max}} = R \; .$$

If the stress element is rotated through an angle θ, the points X, Y on the circle will move around the circle through an angle 2θ in the same direction. If the third principal stress σ_3 is known, sketch three Mohr's circles and the absolute maximum shear stress $\tau_{\text{max}}^{\text{abs}}$ is the radius of the largest circle.

Strain transformation equations: (for strain gages)

$$\epsilon_{x'} = \epsilon_x \cos^2\theta + \epsilon_y \sin^2\theta + \gamma_{xy} \sin\theta \cos\theta$$

$$\gamma_{x'y'} = \gamma_{xy}(\cos^2\theta - \sin^2\theta) + 2(\epsilon_y - \epsilon_x) \sin\theta \cos\theta$$

Discontinuity functions

$$\begin{aligned} < x - a >^n &= 0 \qquad\qquad ; \; x < a \\ &= (x-a)^n \; ; \; x > a \end{aligned}$$

$$\int < x - a >^n dx = \frac{< x - a >^{n+1}}{n+1} \; ; \quad \int \delta(x-a) dx = H(x-a) = < x - a >^0$$

Trigonometric identities and integrals:

$$\sin^2 x + \cos^2 x = 1 \; ; \quad \cos^2 x = \frac{1 + \cos(2x)}{2} \; ; \quad \sin^2 x = \frac{1 - \cos(2x)}{2} \; ;$$

$$\sin(2x) = 2 \sin x \cos x \; ; \quad \cos(2x) = \cos^2 x - \sin^2 x \; .$$

$$\int_0^{n\pi/2} \cos^2 x dx = \int_0^{n\pi/2} \sin^2 x dx = \frac{n\pi}{4} \; , \text{ where } n \text{ is any integer} \; .$$