**SYSTEMS OF LINEAR EQUATIONS**

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* 1. Graph of a linear Equation
  2. Solving Systems of Equations Graphically
  3. Solving Systems of Equations Using Substitution
  4. Solving Systems of Equations Using Elimination
  5. More Real-World Problems (Application)
  6. Solving Systems of Linear Inequalities

**INTRODUCTION**

The first proof of a systematic method of solving systems of linear equations ids provided in the *Nine Chapters of the Mathematical Arts,* the oldest textbook of arithmetic in existence. Unfortunately, the original copies of the *Nine Chapters* were destroyed in 213 B.C. However, the Chinese mathematician Liu Hui wrote a commentary on the *Nine Chapters* in 213 A.D. and the information concerning the original work comes to us through the said commentary.

In Section 1.1, we review the graph of linear equation in two variables. In Section 1.2, we graph the equations and find their solution (the point of intersection of two graphs, if any). If the graphs are parallel lines, the system has no solution and is *Inconsistent.* If the graphs coincide (or are the same), the equations are *dependent* and has infinitely many solutions, Since the graphical method is not very useful when we need precise answers., we introduce two other ways of solving systems of equations: *substitutions,* where one equation is solved for a variable and the result is substituted in the other equation (Section 1.3) and *elimination,* a method that uses the addition property of equality to eliminate an unknown (Section 1.4). These methods are used in Section 1.5 in solving real-world problems. In section 1.6, we extend the ideas learned in solving systems of linear equations in solving systems of linear inequalities.

**1.1 GRAPHOF LINEAR EQUATION (REVIEW)**

**Objectives:**

**Graph linear equations in two variables using :**

a set of ordered pairs;

the intercepts; and

the slope- intercept method

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The *Cartesian* or *rectangular coordinate system* is used in graphing linear equations in two variables. The standard form for a linear equation in two variables is

*ax + by = c*

where a,b , and c are real numbers and a and b are not both equal to zero.

The graph of linear equation is a line. A linear equation in two variables can be graphed using s set of ordered pairs, the intercept method, and the slope-intercept method.

1. **Graphing Using Ordered Pairs**

To determine the solution of a linear equation in two variables, we still use table of values. To do this, we must choose convenient values for x and then solve the resulting equation to find the corresponding values for y. the process of graphing a line requires that we find ordered pairs that make the equation true.

**Procedure**

*To graph a linear equation:*

1. Make a table of ordered pairs that satisfy the equation.
2. Plot these ordered pairs on the Cartesian plane.
3. Draw a line through the plotted points.

**EXAMPLE 1**

Graph the equation 2x + y = 3.

**Solution**

Set up a table of values. Choose any permissible value for x, the independent variable, and obtain the corresponding value for y, the dependent variable. In solving the equation, we will use -2, -1, 0, 1, and 2 for x.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| x | -2 | -1 | 0 | 1 | 2 |
| y=2x+3 | -2(-2)+3 | -2(-1)+3 | -2(0)+3 | -2,(1)+3 | -2(2)+3 |
| y | 7 | 5 | 3 | 1 | -1 |

Therefore, (-2, 7), (-1, 5), (0, 3), (1,1) and (2,-1) are five possible solutions of the linear equation 2x + y = 3. Plot these ordered pairs. Draw a line through the plotted points.

The point on the line is the solutions of the given equation.

**EXAMPLE 2**

Graph the equation 4x-2y=3.

**Solution**

Solve for y:

4x-2y=3

-2y=-4x+3

Transform the equation into y-standard

Formby multiplying the equation by

or

Set up a table of values. Choose any value for x.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| x | -2 | -1 | 0 | 1 | 2 |
|  |  |  |  |  |  |
| y |  |  |  |  |  |
| (x, y) |  |  |  |  |  |

Therefore, , -1,- , , , andare solutions of the linear equation 4x-2y = 3.

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To graph a linear equation in two variables using ordered pairs:

* Plot the ordered pair

solutions found in

the table of values.

* Connect the points

with a line.

**B. Graphing Using the Intercepts**

The x-intercept and y-intercept of a graph are special solutions of a linear equation in two variables. The **x-intercept**is the point where the graph intersects or crosses the x-axis. The y-coordinate of this point is 0. The **y-intercept** is the point where the graph intersects or crosses the y-axis. The x-coordinate of this point is 0.

The x-intercept can be obtained by letting y= 0 and solve for x. Also, the y-intercept can be obtained by letting x = 0 solve for y.

**Procedure**

*To graph a linear equation in two variables using the intercepts:*

1. Determine the x-intercept.
2. Determined the y-intercept.
3. Plot the intercepts and label the coordinates.
4. Connect the intercepts with a line.
5. Check the graph by locating a third point on the graph and determine whether its coordinates satisfy the equation.

**EXAMPLE 1**

Graph the equation 2x + y = 4 by using the intercepts.

**Solution**

To determine the x-intercept, substitute 0 for y in the equationand solve for x. thus,

2x + y = 4

2x + (0) = 4 Solve for the value x.

X = 2.

The x-intercept is (2, 0).

To determine the y-intercept, substitute 0 for x in the equation and solve for y. thus,

2x + y = 4

2(0) + y = 4 Solve for y.

Y = 4.

The y-intercept is (0,4).

An alternative way to determine the y-intercept is to solve the equation for y. thus,

2x + y = 4 or

Y= -2x + 4.

The y-coordinate of the y-intercept is 4. The y-intercept is (0,4).

Plot the two intercepts on the Cartesian plane.

To check the graph, determine a point on the line and check to see if it is a solution of the equation. Let us use point (1,2).

2x + y = 4

2(1) + 2 = 4

2 +2 = 4

4 = 4

Since the equation is true for the point (1,2),the graph is correct.

**EXAMPLE 2**

Graph the equation x -3y= 6 using the intercept method.

**Solution**

To determine the x-coordinate of the x-intercept, substitute 0 for y in the equation and solve for x. thus,

X – 3y = 6;

X – 3(0) = 6, or

X = 6.

The x-intercept is (6, 0).

To determine the y-coordinate of the y-intercept, substitute 0 for x in the equation and solve for y. thus,

X – 3y – 6;

(0)-3y= 6, or

Y = -2.

The y-intercept is (0,-2).

Plot the two ordered pairs obtained on the Cartesian plane.

To check the graph, determine a point on the line and check to see if it is a solution of the equation. Let us use the point (3,-1), the graph is correct.

**C. GRAPHING USING THE SLOPE-INTERCEPT METHOD**

**The Slope-intercept Form of a Linear Equation**

The slope-intercept form of a line is given by y = mx + b, where *m* is the slope and *b* is the y-intercept

**Definition**

The **slope** of a line is the ratio of the amount of rise to the amount of run, or

The **y-intercept** is the point on the line that lies on the y-axis. It is the constant term (b) in the equation, y= mx + b.

We can easily graph the line if we know its slope and y-intercept. When we know one point, (x1, y1), and the slope *m* of the line, we can graph the line. Here’s how:

**Procedure***To graph a line when we know a point and its slope:*

1. Plot the known point , (x1, y1) and label its coordinates.
2. Count the rise and run from the located point. For a positive rise, count upward; for a negative rise, count downward. For a positive run, count to the right; for a negative run, count to the left.
3. Plot a point at this location.
4. Graph the line through these two points.

**Properties of the Slope of a Line**

1. If the line slants upward to the right, the slope is positive.
2. If the line slants downward to the right, the slope is negative.
3. If the line is parallel to the x-axis (horizontal line), the slope is zero.
4. If the line is parallel to the y-axis (vertical line), the slope is undefined.

**EXAMPLE 1**

Graph the equation 2x – y = 3 using the slope-intercept method.

**Solution**

First, solve for y.

2x – y = 3

-y = -2 + 3 Transpose 2x to the other side.

Y = 2x – 3 multiply both sides by -1.

Therefore, the y-intercept is (0,-3) and the slope is or 2.

Locate the y-intercept and use the

Slope to locate another point. That

is, from (0,-3) we to move 2

units up and then 1 unit to the

right.

Label the coordinates of both

points.

Draw a line through the two

points.

**EXAMPLE 2**

Graph the equation 5x + 2y = 1 using the slope-intercept method.

**Solution**

First, solve for y.

5x + 2y = 1

2y = -5x + 1 transpose 5x to the other side.

multiply the equation by .

Therefore, m = , the slope of the line and b =, the y-intercept.

The slope tell us that from

The point (0), we have

To move 5 units down and

Then 2 units to the right.

**EXERCISE 1.1**

1. Find three ordered pairs which are solution of the given equations. Then graph the

Equation on a coordinate or Cartesian plane.

1. X = y+ 1 11. X – 4y = 1
2. X = y + 1 12. X + 2y = 6
3. 3x + y =1 13. 2x = y + 4
4. 2y = x + 1 14. X + 10 = 2y
5. X + 1 = y +1 15. 4x = y + 5
6. X +2y = 7 16. 8x – 2y = -2
7. Y + 3x = 10 17. 3x – 2y +6 = 0
8. 2x = y + 2 18. 4x + 3y = 6
9. 3x – y = -2 19.
10. 2x = y – 3 20. 6y = 6 + 9x

1. Graph each equation using the intercept method.
2. X – 3y = 6 9. 5x – 4y = 20
3. 2x + 5y = 10 10. 4x + 5y = 20
4. 6y + = 6 11. 4x + y = 2
5. 4x – 3y = 12 12. 3x – 2y =1
6. 3x + 5y =15
7. X + 4y = 4 13.2x – y = 5
8. X – 2y = 6 14. X = 3y
9. 2x – 3y = 12 15.y = -2x
10. Sketch the graph of each equation using the slope-intercept method.
11. y = 7x + 5 11.
12. y= 2x- 2 12. 2(y – x) = 3
13. y=7 – x 13. 5(x + y) = 7
14. y= 3(x + 1)
15. y = 6 ( x + 5) 15. X – 5y = 3
16. Y =2 (x -7) 16. 16y -2x =5
17. Y =4 (2 –x) 17. 3(5-2y)=x-2
18. Y =5(x – 7) 18. X – 5y = 1
19. Y =3x + 4 19. Y = 7 (x - 2)
20. Y = 5x – 1 20. 2y – x = 4

**1.2 SOLVING SYSTEMS OF LINEAR EQUATIONS GRAPHICALLY**

**Objectives:**

Determine whether given ordered pair is a solution of a system of linear equation.

Solve systems of linear equation graphically; and

Determine whether a system of linear equations is consistent, inconsistent, or dependent.

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A system of linear equations is a set of more than one linear equation that is to be solved at the same time. The solution set of a system of linear equations is the intersection of the solution sets of each individual equation. We use brace to show that we are looking for simultaneous solution. The following are examples of systems of linear equations:

1. 3.

2. 4.

**Definition**

A pair of equations of the form

and

is called a **system of linear equations** in two variables.

1. **Determining the solution of a system of a linear equations**

An ordered pair is a **solution** of a system of linear equations if it is a solution to every linear equation in the system. To determine whether an ordered pair is a solution of a system of linear equation in two variables, substitute the coordinates of the ordered pair into each linear equation. If all the resulting equations are true, then the ordered pair is a solution of the system.

**EXAMPLE 1**

Determine whether (2, 1) is a solution of the given system of linear equations:

**Solution**

We substitute the value for x and y in each equations that is x = 2 and y = 1. If the point makes each equation true, then the given point is a solution of the system.

Let 3x – y =5 be equation 1 and

X + 4y = 6 be equation 2.

In equation 1, substitute (2, 1).

3x – y = 5

3 (2) – 1 = 5

6 -1 = 5

5 =5 true

In equation 2, substitute (2, 1).

X + 4y = 6

(2) + 4(1) = 6

6 = 6 true

Since the point (2, 1) makes each equations true, therefore, (2, 1) is a solution of the given system of linear equations.

**EXAMPLE 2**

Determine whether (2, 1) is a solution of the system of equations 3x – y = 3 and x + 7y = 1.

**Solution**

We substitute the value x = 2 and y = 1 in each equation. If the point makes each equation true, then the given point is a solution of the system.

Let 3x – y = 3 be equation 1 and

X + 7y = 1 be equation 2.