**RATIONAL EXPRESSIONS**

**3**

3.1 Undefined Rational Expressions

3.2 Multiplication and Division of Rational Expressions

3.3 Addition and Subtraction of Rational Expression

3.4 Complex Fractions

3.5 Additional Equations

3.6 Word Problems Involving Rational Expressions

**INTRODUCTION**

The Egyptian hieroglyphic numerical system is based on powers of 10. The symbol for

‘1 is I , a vertical staff, and for

10 is̯, a heel bone.

The Egyptian hieroglyphic numeral system inscriptions show the reciprocals of integers by using an elongated oval sign. The fractions and were written as and

respectively. I this chapter, we generalize the concept of rational numbers to that of a rational expression which is the quotient of two polynomials.

Find values that make rational expression undefined;

**3.1 UNDEFINED OF RATIONAL EXPRESSIONS**

**B**

**A**

**C**

Find values that make rational expression undefined;

Build fractions: and

Reduce rational expressions to lowest terms.

1. **Undefined Rational Expressions**

The following are rational expressions:

, , and

Since the denominators of these expressions must not be zero, we must place some restrictions. Their restrictions are as indicated:

For , cannot be 0 ( = 0) because we would have .This is not defined.

For , x = - 3. If x is -3, then = =

This is undefined.

For , m = - 2. If m is -2, then = . This is undefined.

For = , x = 2. If x = 3, then

= = . Again, this is undefined.

The following rule can be used to avoid stating repeatedly that the denominators of rational expressions must not be zero.

**Rule** *Avoiding Zero Denominators*

The variables in rational expression must not be replaced by numbers that will make the denominator zero.

The value that is not permissible replacement for the variables is sometimes called a ***restricted value***.

**Procedure**

*Determining Restricted Values of Rational Expressions*

Use the following steps to determine the restricted value(s). if any, of

a rational expression.

1. Set the denominator equal to 0.
2. Solve the resulting equation.
3. Any solution of the equation is a restricted value.

**EXAMPLE 1**

Find the value for which the rational expression is undefined.

1. b. c.

**Solution**

1. Set the denominator equal to zero and solve.

3x + 2 = 0

3x = -2 Subtract 2.

X = - Divided by 3.

Thus,

is undefined for x = .

1. Set the denominator equal to zero and solve.

(x + 5) (x - 1) = 0 Factor.

X + 5 = 0 or x- 1 = 0 Solve each equation.

X= -5 or x = 1

Thus

is undefined at x = -5 and x = 1.

1. Set the denominator equal to zero and solve.

` But the square of any real number is not negative, so there are no real numbers p for which the denominator is zero. Note that is greater than or equal to zero, so + 3 is always positive. There are no values for which is undefined.

**EXAMPLE 2**

Determine the values of the variables that must be excluded in the expression

Solution

Let

- = 0

(x + y)(x - y) = 0

X + y = 0 or x – y = 0

X = -y or x = y

Thus, x = -y, or x = y. This may be written as x ==y.

**B. Building Up Fractions**

In fractions, we learned how to recognize which ones are equal. This idea can be used to reduce or build fractions. The following are equal fractions:

= = = = = =

Below is another way of writing the fractions above:

= = Note: = 1

= =

= =

= =

= =

= =

a rational expression that is equivalent to another rational expression can be obtained b multiplying the numerator and denominator of the given rational expression be the same nonzero number or expression, C. in symbols, we have:

**Rule** *Fundamental Rule of Rational Expressions*

= (B = 0, C = 0)

Remember that

= because = 1

and multiply by 1 does not change the value of the expression. We have used the fundamental rule of rational expressions in building fractions equivalent to. That is, if we want to write with a denominator of 35 we write the problem as

= Note: 35 = 7.5

If we obtained 35 by multiplying 7 by 5, we need also to multiply the numerator by 5. We obtain

= Note: =

Similarly, we can write

with a denominator of. We first write the new equivalent expression

With the old denominator 5b factored out:

= = Write as ,

Since the multiplier is , we have

= = Multiply the numerator and denominator by

Thus,

=

**EXAMPLE 1**

Write with a denominator of 32.

**SOLUTION**

=

Write 32 as 8(4).

= ●

Since we multiplied the denominator by 4, multiply also the numerator by 4.

= =

Thus,

=

**EXAMPLE 2**

Write with a denominator of .

**Solution**

=

Since we multiplied the denominator by 5n, multiply also the numerator by 5n.

= =

Thus,

= .

**EXAMPLE 3**

Write