In equation 1, substitute the point (2, 1).

3x – y = 3

3(2) – (1) = 3

6 -1 =3

5 = 3

In equation 2, substitute the point (2, 1)

X + 7y = 1

(2) + 7(1) = 1

9 = 1

Since the point (2, 1) does not make each equation true, then (2, 1) is not a solution of the given system of linear equations.

To solve a system of equations means to determine all ordered pairs of real numbers that simultaneously satisfy all equations of the system. We can find the solution to any system of equations in two variables by:

1. Graphing each equation and finding the coordinates of the point of intersection of the graphs
2. Using the substitution method; and
3. Using the elimination method.

B. **Solving Systems of Linear Equations Graphically**

We know that the solution of a linear equation in two variables may be represented graphically. Therefore, if we graph both linear equations in a system, we should be able to determine the solution of the linear system. To solve a system of linear equations graphically, graph the equations on the same coordinate plane. The coordinates of the point of intersection are the numbers in the ordered pair solution.

**Steps in Solving a System of Linear Equations by Graphing**

1. Sketch the graph of each linear equation on the same coordinate plane.
2. Find the solution of the given system.
3. If the two lines intersect at one point, the solution is the ordered pair that corresponds to that point.
4. If the two lines are parallel, the system has no solution.
5. If the two equations have the same graph, the system has infinitely many solutions.
6. Check the solution in both equation.

**EXAMPLE 1**

P

Solving the given system graphically:

**Solution**

Find the x- and y-intercepts of each equation, and then sketch each graph.

3x + y = 1 Equation 1

x – y = 3 Equation 2

Solve for the x-intercept in equation 1. Let y = 0.

3x = y = 1

3x + (0) = 1

The x-intercept is

Next, solve for the y-intercept. Let x = 0.

3x + y = 1

3(0) + y =1

y = 1

The y-intercept is (0, 1).

Solve for x-intercept in equation 2. Let y = 0.

x – y = 3

x – (0) = 3

x = 3

The x-intercept is (3, 0).

Next, solve for the y- intercept. Let x = 0

x – y = 3

(0)– y = 3

y = -3

The y-intercept is (0, -3)..

Graph equations 1 and 2 on the Cartesian

plane. The lines intersect at point (1, 2)

This point will make each equation in

the system true.

**To check:**

Substitute the value of x and y in each equation.

In equation 1 :

3x + y = 1

3(1) + (-2) = 1

1 =1 True

In equation 2 :

x – y = 3

1. – (-2) = 3

1 + 2 = 3

3 = 3 True

Since (1, 2) makes each equation true, {(1, --2)} is the solution set of the given system.

**EXAMPLE 2**

Solve the given system graphically:

**Solution**

Express each equation in slope-intercept form. We can use the slope and the y—intercept to graph each equation.

x + 2y = 6 Equation 1

2x + 4y = 12 Equation 2

We find the slope and y-intercept in equation 1.

X + 2y = 6

2y = -x + 6 Transpose x to the other side.

Y =- Multiply the equation by

Therefore, *m = .*

In equation 2, we find its slope and its y-intercept.

2x = 4y = 12

4y = -2x + 12 Transpose 2x to the other side.

Y =- Multiply the equation by

Therefore, *m = .*

The graphs of the lines are shown at the

right. Notice that the two equations have

the same graph. This means that there are

infinitely many solutions to this system.

Every point on the graph of x + 2y = 6 lies

also on the graph of 2x + 4y = 12. This is

called a *consistent system* and the equations

are *dependent.­*

**Check**

Substitute some points in the system and

verify if they will make each equation true.

Using point (-4, 5); we get

In equation 1 : In equation 2 :

x + 2y = 6 2x + 4y =12

(-4) + 2 (5) = 6 2 (-4) + 4(5) = 12

-4 + 10 = 6 -8 + 20 = 12

6 = 6 True 12 = 12 True

Using point (0, 3); we get

In equation 1 : In equation 2 :

x + 2y = 6 2x + 4y =12

1. + 2(3) = 6 2(0) + 4(3) = 12

6 =6 True 12 = 12 True

Using point (4, 1); we get

In equation 1 : In equation 2 :

x + 2y = 6 2x + 4y = 12

(4) + 2(1) = 6 2(4) + 4(1) = 12

4 + 2 = 6 8 + 4 = 12

6 =6 True 12 = 12 True

Since some of the points on the lines make each equation true, all the points on the lines are included in the solution set.

**EXAMPLE 3**

Solve the given system graphically.

**Solution**

We can use a set of ordered pairs in graphing the lines. We assume some values for x and then solve for y.

2x + y = 3 Equation 1

4x + 2y = 12 Equation 2

In equation 1 :

2x + y = 3

Y = - 2x + 3

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| x | -2 | -1 | 0 | 1 | 2 |
| Y = -2x + 3 | -2 (-2) + 3 | -2 (-1) + 3 | -2(0) + 3 | -2(1) + 3 | -2(2) + 3 |
| y | 7 | 5 | 3 | 1 | -1 |
| (x, y) | (-2, 7) | (-1, 5) | (0, 3) | (1,1) | (2, -1) |

In equation 2 :

4x +2y = 12

2y = -4x + 12

Y = -2x + 6

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| x | -2 | -1 | 0 | 1 | 2 |
| y= -2x = 6 | -2(-2) + 6 | -2(-2) + 6 | -2(-1) + 6 | -2(1) + 6 | -2(2) + 6 |
| Y | 10 | 8 | 6 | 4 | 2 |
| (x, y) | (-2, 10) | (-1, 8) | (0, 6) | (1,4) | (2,2) |

We plot the points and graph the lines. The graphs are shown below. Notice that the lines are parallel. Hence, there is no point of intersection. This means that the system has no solution or inconsistent and the linear equations are independent. We can show this analytically by nothing that the slopes of the lines are the same.

When solving a system of linear equations in two variables graphically, one of these possibilities will occur.

1. The graphs may intersects

* One solution exists, that is, the point of intersection.
* The system is consistent.
* The equations are independent.

1. The graphs maybe parallel

* No solutions exist.
* The system is inconsistent.
* The equations are dependent.

1. The graphs may coincide

* An infinite number of solutions exist. The solutions are all ordered pairs (x, y) that satisfy equation of the system.
* The system is consistent.
* The equations are dependent.

**C. Finding Inconsistent and Consistent Systems**

Algebraically, we can easily determine whether a system is consistent (has a solution) or inconsistent (has no solution) or the equation are dependent (infinitely many solutions) if the following properties are satisfied.

If are real numbers where and not both zero, and not both 0, then any linear system of the form,

+ y = and

+ y =

Has a unique solution when

= (1)

Has no solution when

== (2)

And has an indefinite number of solutions when

== (3)

The following table will help us further.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Type of Graphs | Slopes | y-intercept | Number of Solutions | Type of System |
| Intersecting Lines | Different | Same or different | One | Consistent |
| Parallel Lines | Same | Different | None | Inconsistent |
| Coincide line | Same | Same | Infinite | Consistent |

**EXAMPLE 1**

Determine whether the given systems of linear equations have one solution (consistent system), no solution (inconsistent system), or infinite number of solutions (dependent equations, consistent system).

1. b. c.

**Solution**

1. We first write each equation in standard form. Thus,

2x + 3y = 5

4x – 7y = 8

And then note that

=

Hence, the system has only one solution.

1. We first write each in standard form. Thus,

4x + 6y = 7

2x +3y = 2

And then note that

Hence, the system has no solution.

1. We first write each in standard form. Thus,

2x + 2y = 6

3x + 3y = 9

And then note that

Hence, the system has an infinite number of solutions.

**EXERCISE 1.2**

1. Determine whether each statement is true or false. Explain your answer.
2. The ordered pair (2, 1) is a solution of the equation 3x + y = 7
3. The ordered pair (1, 2) satisfies the equations 3x + y = 7 and 4x + 3y = 16.
4. The ordered pair (-1, 4) satisfies the equations 5x – y = -9 and 2x – 3y = -14.
5. No ordered pair satisfies y = 4x – 3 and y = 3x + 4.
6. The equations y = 2x + 4 and y = 3x – 6 are independent.
7. The equations y = 3x + 4 and y = 3x -7 are inconsistent.
8. The graphs of dependent equations are the same.
9. Answer each.
10. What does the solution of a system of linear equations represent?
11. Define: a. consistent system of equations

b. inconsistent system of equations

1. Suppose that you have a system of linear equations of the form

y = , and

y =

what can you say about the graph of the system when –

1. = and = ? How many solutions do you have? Explain.
2. = and = ? How many solutions do you have? Explain.
3. = ? How many solutions do you have? Explain.
4. When you are solving a system of linear equations using the graphical method, how can you tell if the system is –

a. consistent?

b. inconsistent?