Optimal scheduling

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Project Proposal

The employee scheduling, originally called the nurse scheduling problem (NSP) is the operations research problem of finding an optimal way to assign employees to shifts. It necessitates the assignment of suitably qualified personnel to particular shifts in order to meet an organization's service demands while observing workplace rules and trying to accommodate individual work preferences. Finding a schedule that satisfies all constraints manually can be much more difficult compared to computer calculations. This problem has been studied since before 1969, and is known to have NP-hard complexity[1].

1 Idea

Staff scheduling is applicable in a wide range of situations, including scheduling of medical personnel in hospitals, or scheduling of airline and hotel personnel. The main goals of staff scheduling are to make the most effective use of resources, to create a schedule with a balanced workload, and to accommodate individual preferences to the greatest extent possible.

Currently, the monthly schedules are generated manually using excel sheets; however, there are instances where the generated schedule violates certain constraints. Lack of breaks between shifts and double shifts are examples of poorly organized schedules. This results in poor production as a result of low employee morale and overwork, as well as an increased risk of accidents or mistakes due to tiredness of the employees.

Since this problem belongs to the wide category of timetabling problems, it is non-deterministic polynomial time (NP)-complete, as far as its computational complexity is concerned, meaning that the difficulty to find a solution rises exponentially to its size and a deterministic algorithm, giving an acceptable solution in polynomial time, cannot be found. Some calculations made using regular methods took up to 11 hours in execution time[1]. Therefore, alternative optimization methods, namely metaheuristics, have been developed in order to reach a (near) optimal solution for the staff scheduling problem. A lot of heuristic methods have been developed for dealing with this problem: genetic algorithms, tabu search, simulated annealing, variable neighborhood search, scatter search, iterated local search, particle swarm optimization, memetic algorithms, ant colony optimization, etc. The method which will be covered in this study will be simulated annealing.

1.1 Problem

Hard Constraint [Feasibility]

- Type 1 - Employee Coverage: for each shift, number of employees assigned within a range, number of Top Level Employees > 2, etc.
- Type 2 - Prohibited Working Patterns: Morning shift after a Night Shift, 3 consecutive Night Shifts, etc.

Soft Constraint [Optimality]

- Type 3 - Satisfaction/Preferences: total number of days-off $= 4$, total number of night-shifts $= 3$, etc.

Formulation:

- Let there be I employees scheduled, $i = 1, \dots, I$.
- Let the scheduling period be for K days, $k = 1, \dots, K$.

- Let each day have four shifts denoted by $S = s_1, s_2, s_3, s_4$.
- Define s_1, s_2, s_3 to be working shifts (morning, afternoon, night) and let s_4 represent a day off.
- Let $D_{i,k,s} \in 0, 1$ for each employee i, day k, shift s - that is, employee i is either working the shift or not.
- Let $H_{k,s}$ represent the organization's minimum coverage constraint for day k, shift s.

Hard constraints:

- For all days k and shifts s where $k = 1, \ldots, K$ and $s = s_1, s_2, s_3, s_4$:

$$\sum_{i=1}^{I} D_{i,k,s} \geq H_{k,s} \tag{1}$$

that is, each schedule must satisfy the organization's minimum coverage constraint.

- For all i, k where $k = 1, \ldots, K-1, i = 1, \ldots, I$:

$$D_{i,k,s_3} + D_{i,k+1,s_1} \leq 1 \tag{2}$$

that is, no employee may be scheduled to work a night shift (s_3) followed immediately by a morning shift (s_1).

- For all employees i and days k:

$$\sum_{s \in S} D_{i,k,s} = 1 \tag{3}$$

that is, each employee must be scheduled for exactly one shift each day.

Soft constraints:

- For each employee $i = 1, \ldots, I$ and for $k = 1, \ldots, K-2$:

$$D_{i,k,s_4} + D_{i,k+1,s_4} + D_{i,k+2,s_4} \leq 2 \tag{4}$$

that is, each employee may have no more than three days off in a row.

- For each employee $i = 1, \ldots, I$ and for $k = 1, \ldots, K-4$:

$$\sum_{j=k}^{k+6} D_{i,j,s} \leq 5 \tag{5}$$

that is, each employee may work no more than five days in a row.

Objective: To create a feasible employee roster while maximizing its quality, i.e., to create a roster that satisfies all hard constraints and, at the same time, satisfies the maximum possible number of soft constraints.

$$Minimize \quad \lambda * F(\sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{s=1}^{S} D_{i,k,s}) + (1-\lambda) * G(\sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{s=1}^{S} D_{i,k,s}) \tag{6}$$

where F is a linear function that takes in a schedule matrix, G is an exponential function for the organization's cost due to over/under-staffing and $\lambda \in [0, 1]$ is weight in the interest of employees. To form the objective we may choose λ to favor either the interest of the employees ($\lambda > 0.5$), the organization ($\lambda < 0.5$), or both interests equally ($\lambda = 0.5$).

Algorithm 1 Simulated Annealing Algorithm [2]

A total of N variables, N = number of employees * number of days
Assign initial values randomly, calculate C, set as optimum
Find a neighbor, compare objective value C
if dC < 0 **then**
    Accept the neighbor as new optimum
else
    if $U < e^{-dC/t}$ **then**
        Accept
    end if
    Temperature t reduces in two different rates (depend on Changes/Trials)
end if

2 Outcomes

A ready-made solution for employers on how to distribute employees by shifts, in the form of a code that will display the shift schedule.

3 Literature Review

The source code in [3], [4] provides a solution to a modified version of this problem using Google Operation Research Tools. The following constraints are taken into account: each job requires specific skills and needs to be done in specific locations; each employee has their own skills and their home can be near or very far from job's location; each of them is available on different time slots during the week.

The article [5] provides a basic cost optimization approach towards the problem using Gurobi Python Interface.

In [6], [7], a simulated annealing algorithm for use in a continually operating scheduling environment is presented. Their study aims to minimize the number of employees required to meet expected demand. According to the findings, the simulated annealing-based method outperforms other heuristics and converges quickly to a low-cost solution.

Branch and price methods are used for solving integer linear programming and mixed integer linear programming problems which include many variables. The branch and price methods have been applied to scheduling of train drivers in [8]. Their approach focuses on reducing the number of duties and optimizing the schedule for outside interruptions. A duty in this case consists of a sequence of trips carried out on a given day by a train driver.

4 Quality metrics

The solution should not violate any constraints, minimize the overtimes and maximize the satisfaction of employees in terms of desired days-off.

5 Rough plan

- Draft 24.03
- Simulated annealing approach 07.04
- Proper processing and visualization of results 14.04
- Extra: General Assignment problem 21.04
- Extra: Genetic algorithm approach

References

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[2] Tianhao Lu Sijia Zhu Fanying Chen, Shuyao Lu. Nurse scheduling.

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[5] Eric Stoltz. Scheduling with ease: Cost optimization tutorial for python. 2018.

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[7] Dr. René Bekker Marloes Boxelaar. Shift scheduling in a nursing home using simulated annealing. 2017.

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