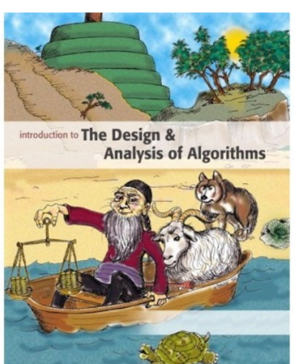
Introduction to

Algorithm Design and Analysis

[18] String Matching



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In the last class…

- **Dynamic programming**
  - Optimal Binary Search Tree
  - Separating Sequence of Word
  - Changing coins
- **Elements of Dynamic Programming**
  - Overlapping subproblems
  - Optimal substructure

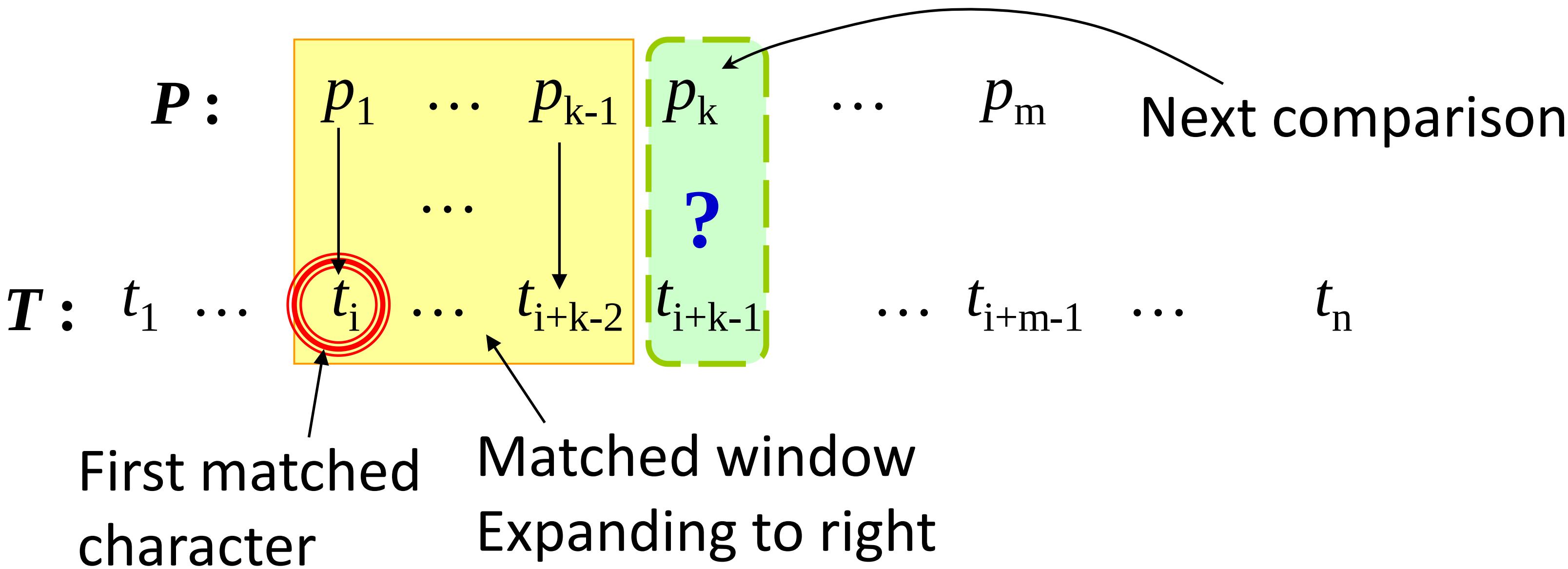
String Matching

- **Simple String Matching**
  - Brute force
- **KMP**
  - KMP Flowchart Construction
  - Jump at Fail
  - KMP Scan
- **Boyer-Moore**
  - Basic idea

Problem Description

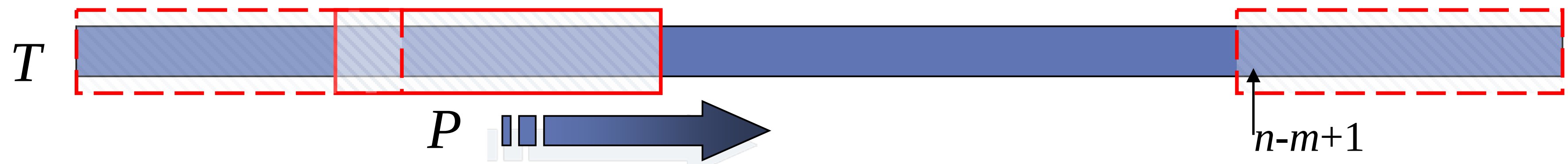
- Search the **text T**, a string of characters of length n
- For the **pattern P**, a string of characters of length m **(usually, m<<n)**
- The result
  - o If T contains P as a substring, returning the index starting the substring in T
  - o Otherwise: fail

BF Solution



Note: If it fails to match p_k to t_{i+k-1}, then backtracking occurs, a cycle of new matching of characters starts from t_{i+1}.In the worst case, nearly *n* backtracking occurs and there are nearly *m*-1 comparisons in one cycle, so **$\Theta(mn)$**

Brute-Force Matching Works



sliding window **worst-case: *m*(*n*-*m*+1)**

Average-case: (characters of *P* and *T* randomly chosen from Σ(|Σ|=d≥2)

For a specific window, the expected number of comparison is:

matched: $m\left(\frac{1}{d}\right)^m$

ummatched: for the case that the first unmatched character

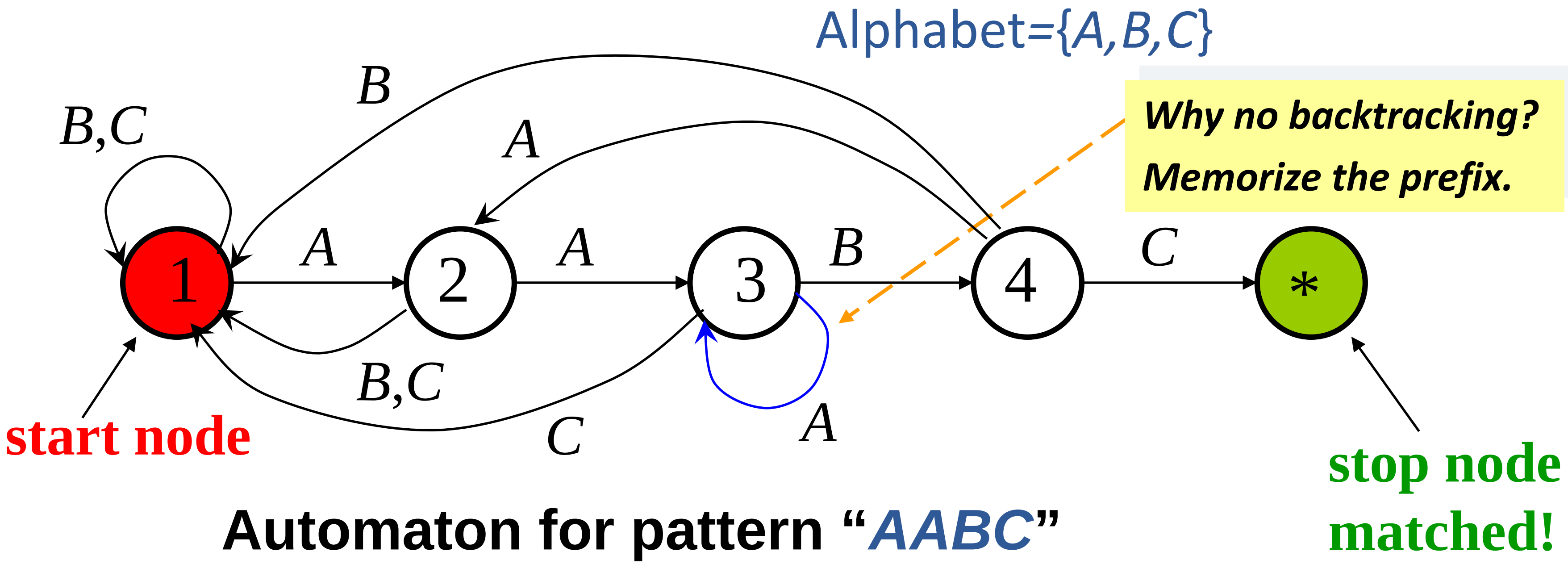
is the ith in the window, then, $i\left(\frac{1}{d}\right)^{i-1}\left(1-\frac{1}{d}\right)$

So, $\sum_{i=1}^{m}\left[i\left(\frac{1}{d}\right)^{i-1}\left(1-\frac{1}{d}\right)\right]+m\left(\frac{1}{d}\right)^m=1+\sum_{i=1}^{m}\left[(i+1)\left(\frac{1}{d}\right)^i-i\left(\frac{1}{d}\right)^i\right]=\frac{1-d^{-m}}{1-d^{-1}}\le 2$

Disadvantages of Backtracking

- More comparisons are needed
- Up to m-1 most recently matched characters have to be readily available for re-examination. (Considering those text which are too long to be loaded in entirety)

Automaton for Matching



Automaton for pattern "*AABC*"

Advantage: each character in the text is checked only once

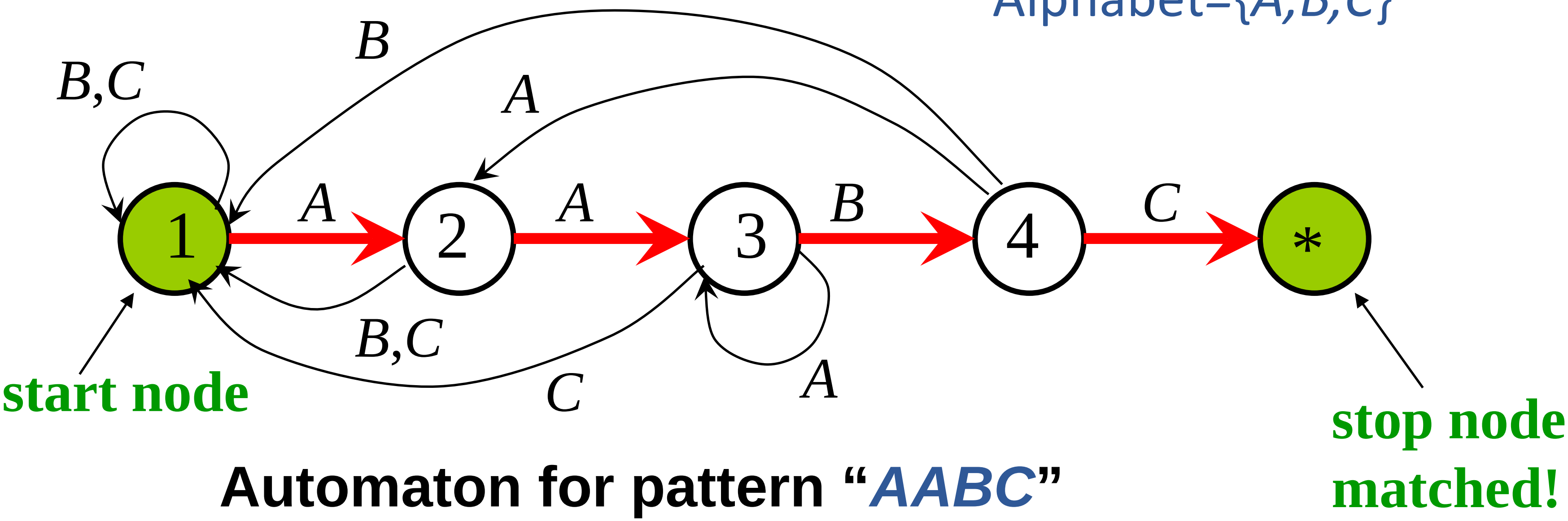
Difficulty: Construction of the automaton – too many edges(for a large alphabet) to defined and stored

An Automata-theoretic View on String Matching

- **Matching a specific pattern P**
  - Construct an automaton A for P (important)
  - "Inject" the text T into A (so easy)
- **String matching: matching any pattern P**
  - Design an "automaton factory" algorithm
    - Which can construct an automaton A for any pattern $\mathcal{P}$
  - Match the given pattern P
    - Using the automaton just constructed
- **What is KMP?**
  - A specific "automaton factory"

Re-look at the Automaton

Alphabet={*A*,*B*,*C*}

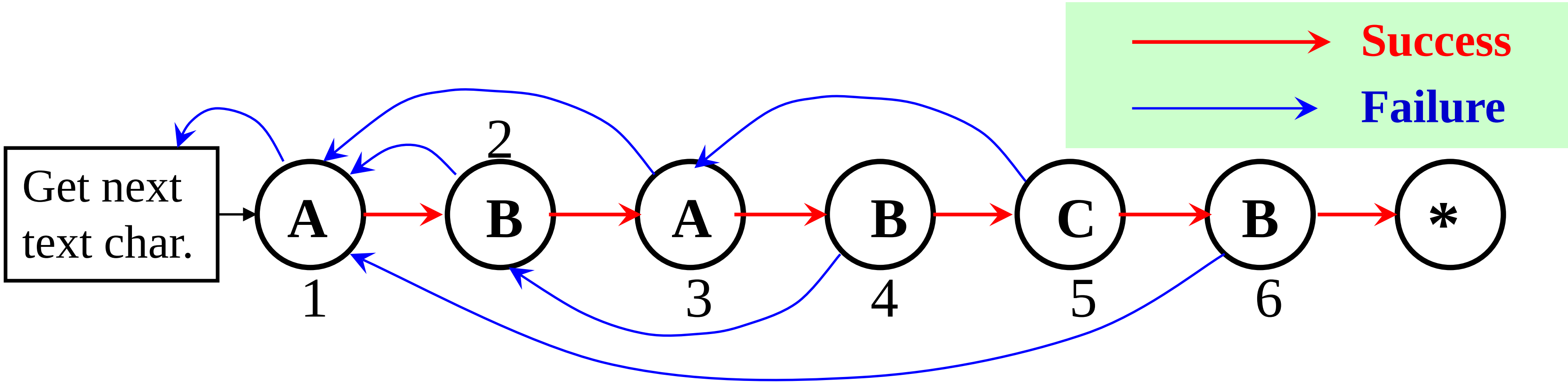


Automaton for pattern "*AABC*"

There is only one path to success,

However, many paths leading to Fail.

The KMP Flow Chart



1 2 3 4 5 6 7 8 9 10 11

An example: *T*="A C A B A A B A B A", *P*="ABABCB"

KMP cell number	1	2	1	0	1	2	3	4	2	1	2	3	4	5	3	4
Text being scanned	1	2	2	2	3	4	5	6	6	6	7	8	9	10	10	11
	A	C	C	C	A	B	A	A	A	A	B	A	B	A	A	-
Success or Failure	s	f	f	C	s	s	s	f	f	s	s	s	s	f	s	F

get next char.

Matched Frame

P: ABABABCB

T: … ABABAB *x* …

to be compared next

matched frame

If *x* is not C

P: ABAB ABCB

T: … ABABAB *x* …

The matched frame move to right for 2 chars, which is equal to moving the pointers backward.

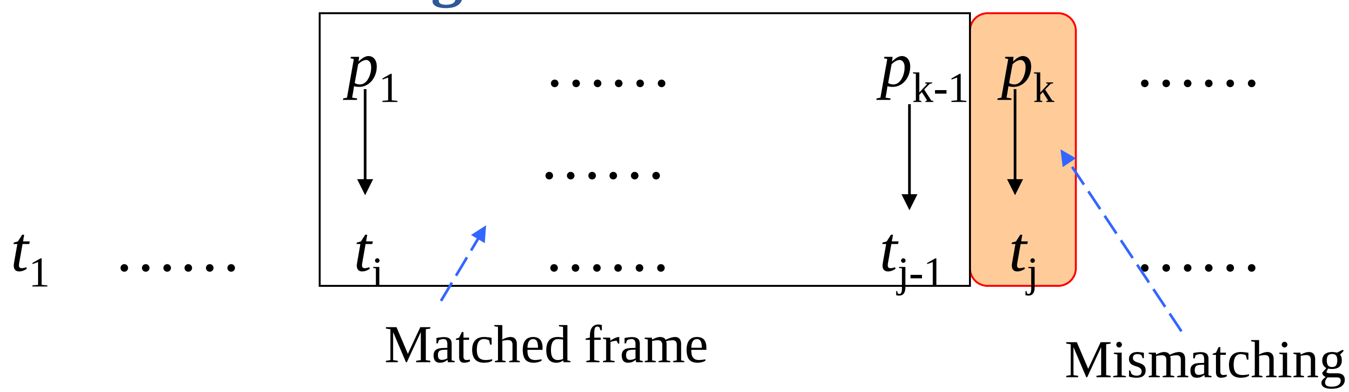
Moving for 4 chars may result in error.

P: ABABABCB

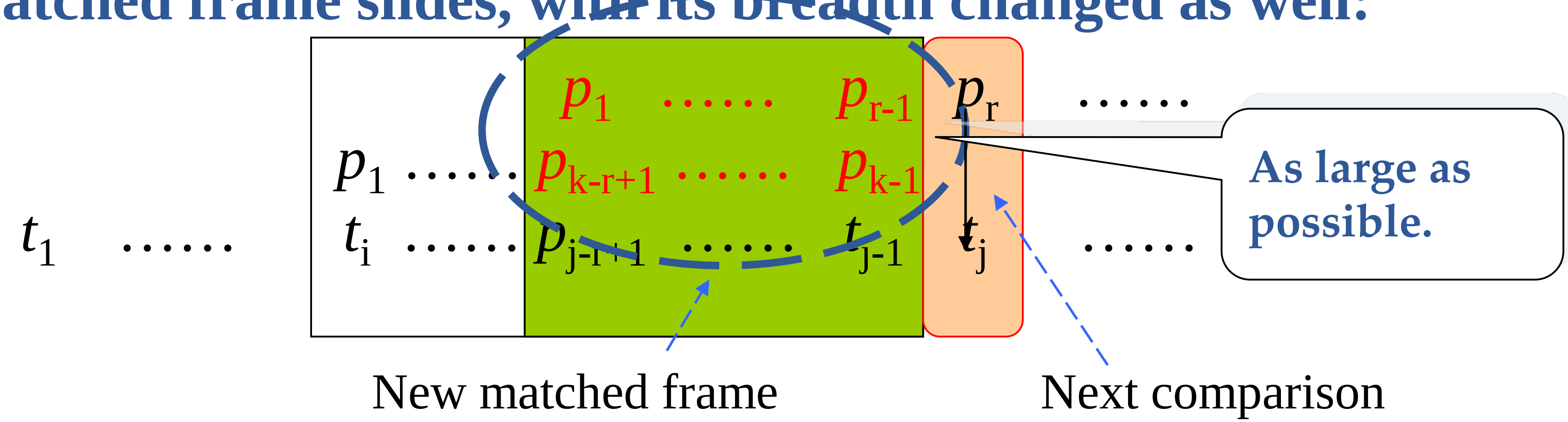
T: … ABABABABCB …

Sliding the Matched Frame

When mismatching occurs:



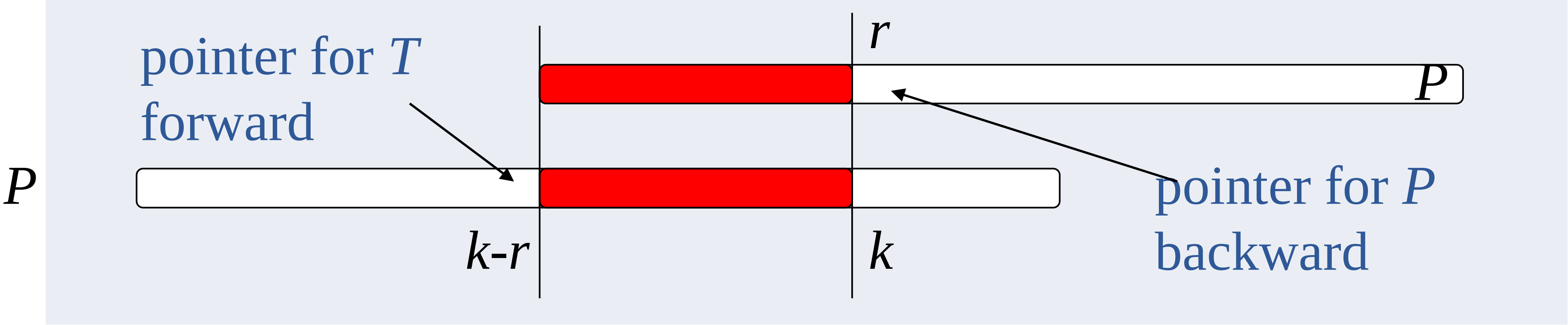
Matched frame slides, with its breadth changed as well:



Fail Link

Which means:
When fail at node k, next comparison is p_k vs. p_r

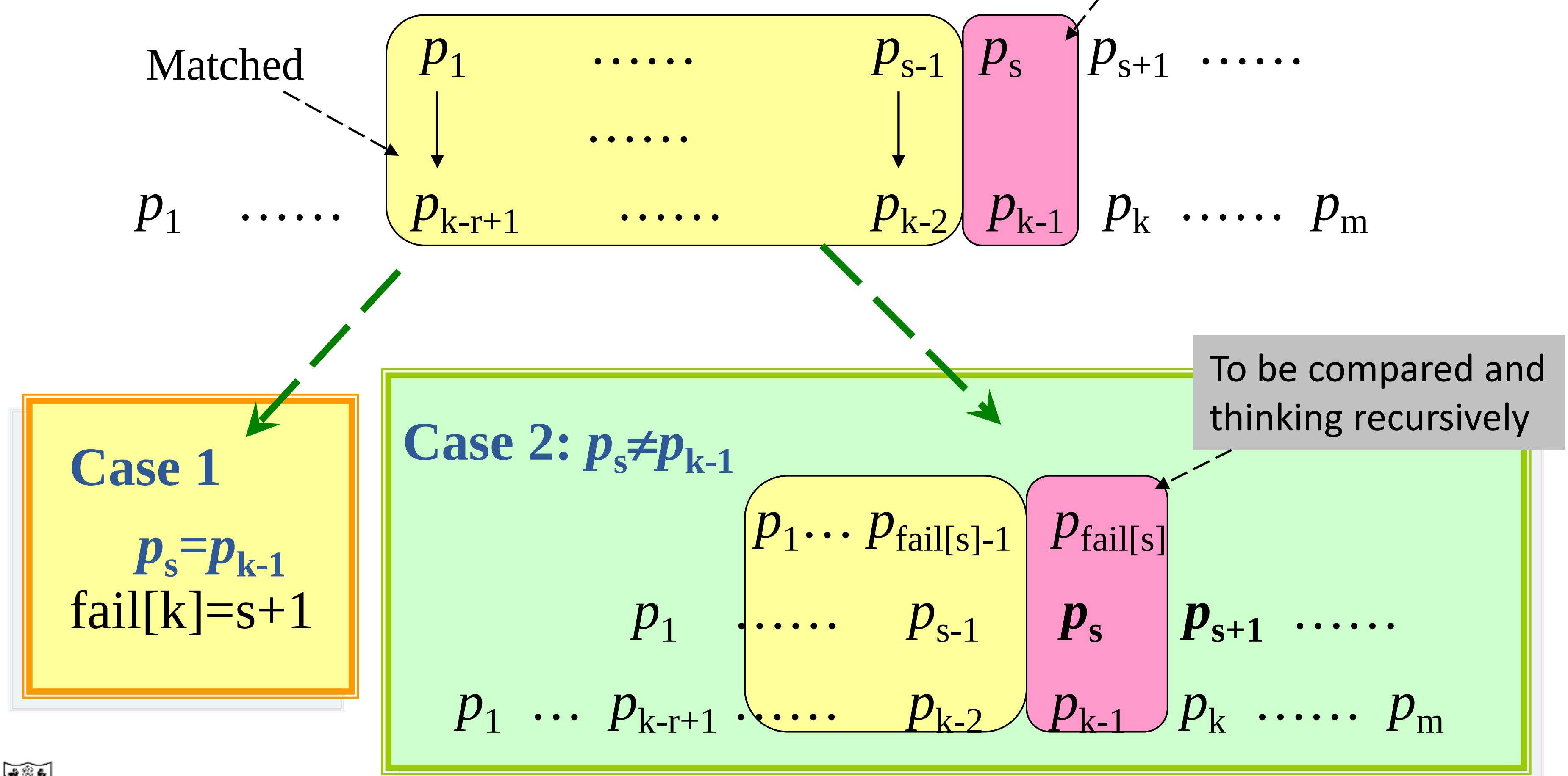
- Out of each node of KMP flowchart is a fail link, leading to node r, where r is the largest non-negative interger satisfying $r<k$ and $p_1,\ldots,p_{r-1}$ matches $p_{k-r+1},\ldots,p_{k-1}$. **(stored in fail[k])**



- Note: r is independent of T.

Computing the Fail Links

Thinking recursively, let fail[k-1]=s:



Recursion on fail[s]

Thinking recursively, at the beginning, s=fail[k-1]:

Case 2: $p_s \neq p_{k-1}$

$p_1 \cdots p_{\text{fail[s]-1}}$ $p_{\text{fail[s]}}$

p_1 ...... p_{s-1} $\boldsymbol{p_s}$ $\boldsymbol{p_{s+1}}$ ......

p_1 ... p_{k-r+1} ...... p_{k-2} p_{k-1} p_k ...... p_m

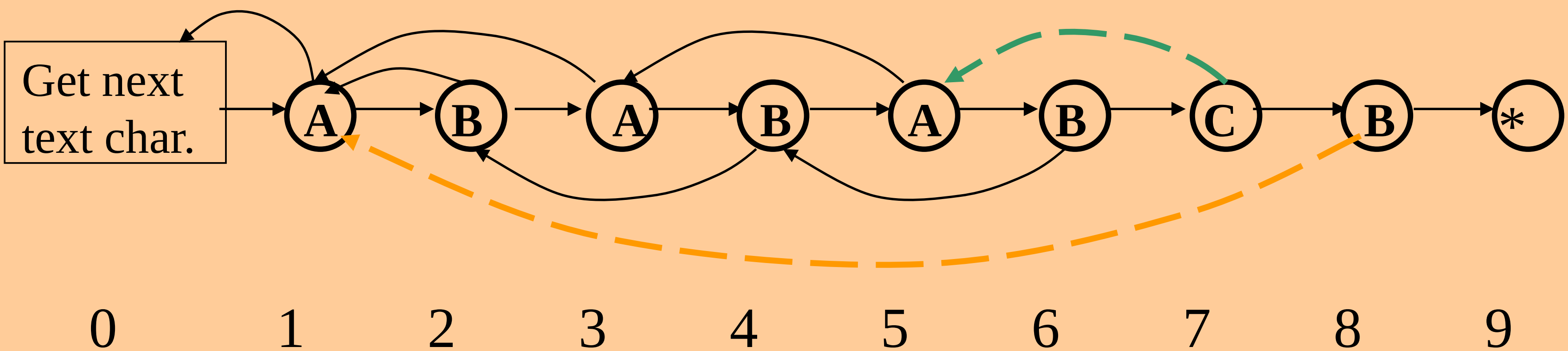
p_s is replaced by $p_{\text{fail[s]}}$, that is, new value assumed for s

Then, proceeding on new s, that is:
If case 1 applys ($\boldsymbol{p_s = p_{k-1}}$): fail[k]=s+1, or
If case 2 applys ($\boldsymbol{p_s \neq p_{k-1}}$): another new s

An Example

Constructing the KMP flowchart for *P* = "ABABABCB"

Assuming that fail[1] to fail[6] has been computed



fail[7]: ∵ fail[6]=4, and p_6=p_4, ∴ fail[7]=fail[6]+1=5 (case 1)

fail[8]: fail[7]=5, but $p_7 \neq p_5$, so, let s=fail[5]=3, but $p_7 \neq p_3$, keeping back, let s=fail[3]=1. Still $p_7 \neq p_1$. Further, let s=fail[1]=0, so, fail[8]=0+1=1.(case 2)

Getting the KMP Flow Chart

Input: ***P***, a string of characters; ***m***, the length of *P*
Output: **fail**, the array of failure links, filled

```
void kmpSetup (char [] P, int m, int [] fail)
  int k, s;

  fail[1]=0;
  for (k=2; k≤m; k++)
    s=fail[k-1];
    while (s≥1)
      if (p_s = = p_k-1)
        break;
      s=fail[s];
    fail[k]=s+1;
```

For loop executes m-1 times, and while loop executes at most m times since fail[s] is always less than s.
So, the complexity is roughly ***$O(m^2)$***

Number of Comparisons

$\leq 2m-3$

```
fail[1]=0;
  for (k=2; k≤m; k++)
    s=fail[k-1];
    while (s≥1)
      if (p_s = = p_k-1)
        break;
      s=fail[s];
    fail[k]=s+1;
```

These 2 lines combine to increase *s* by 1, done ***m*-2** times

Success comparison:
at most once for a specified *k*, totaling at most ***m*-1**

Unsuccessful comparison:
Always followed by decreasing of *s*. Since: *s* is initialed as 0,

s increases by one each time

s is never negative

So, the counting of decreasing can not be larger than that of increasing

KMP Scan

Input: *P* and *T*, the pattern and text; *m*, the length of *P*; *fail*: the array of failure links for *P*.

Output: index in *T* where a copy of *P* begins, or -1 if no match

```
int kmpScan(char[ ] P, char[ ] T, int m, int[ ] fail)
  int match, j,k; //j indexes T, and k indexes P
  match=-1; j=1; k=1;
  while (endText(T,j)=false)
    if (k>m) match=j-m; break;
    if (k= =0) j++; k=1;
    else if ( t_j= =p_k) j++; k++; //one character matched
    else k=fail[k]; //following the failure link
  return match
```

Each time a new cycle begins, $p_1, \ldots p_{k-1}$ matched

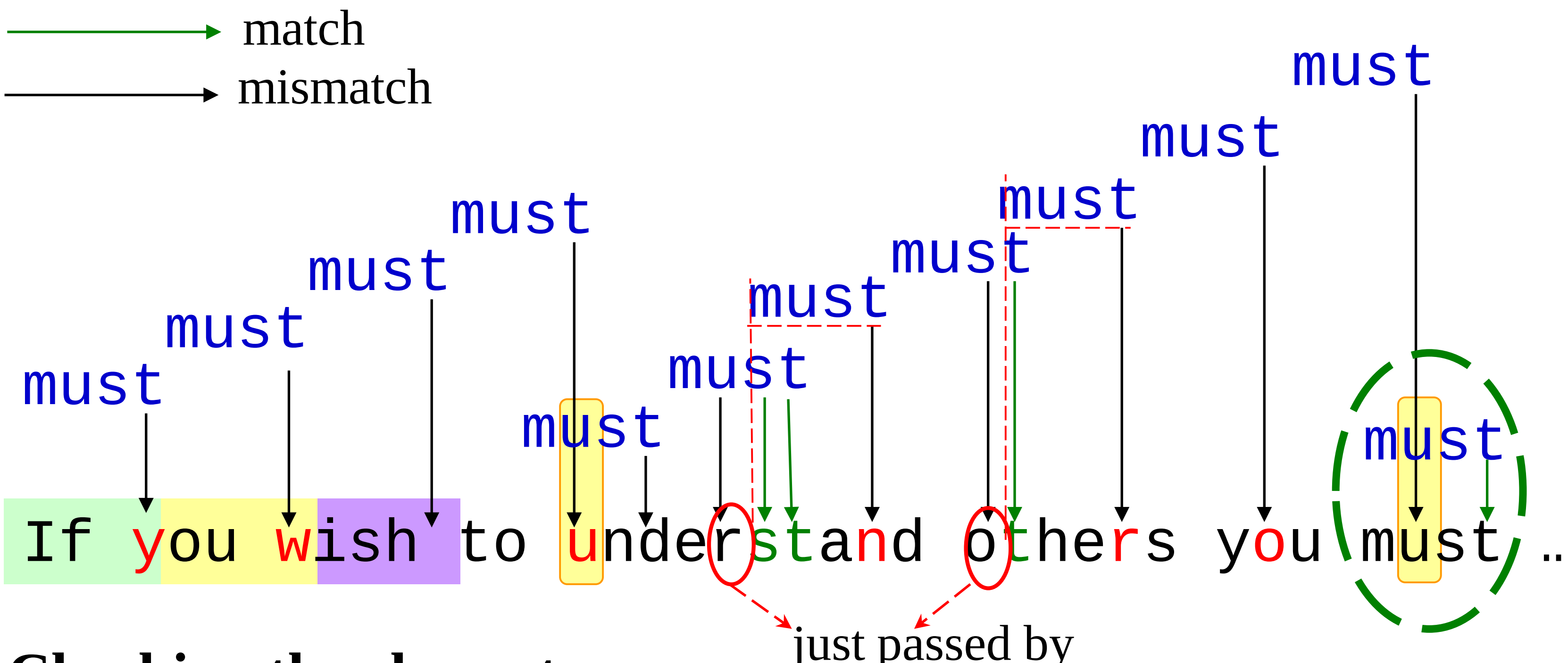
Matched entirely

Executed at most 2n times, why?

Skipping Characters

- **Longer pattern contains more information about impossible positions in the text.**
    - For example: if we know that the pattern doesn't contain a specific character.
- **It does not make the best use of the information by examining characters one by one forward in the text.**

An Example

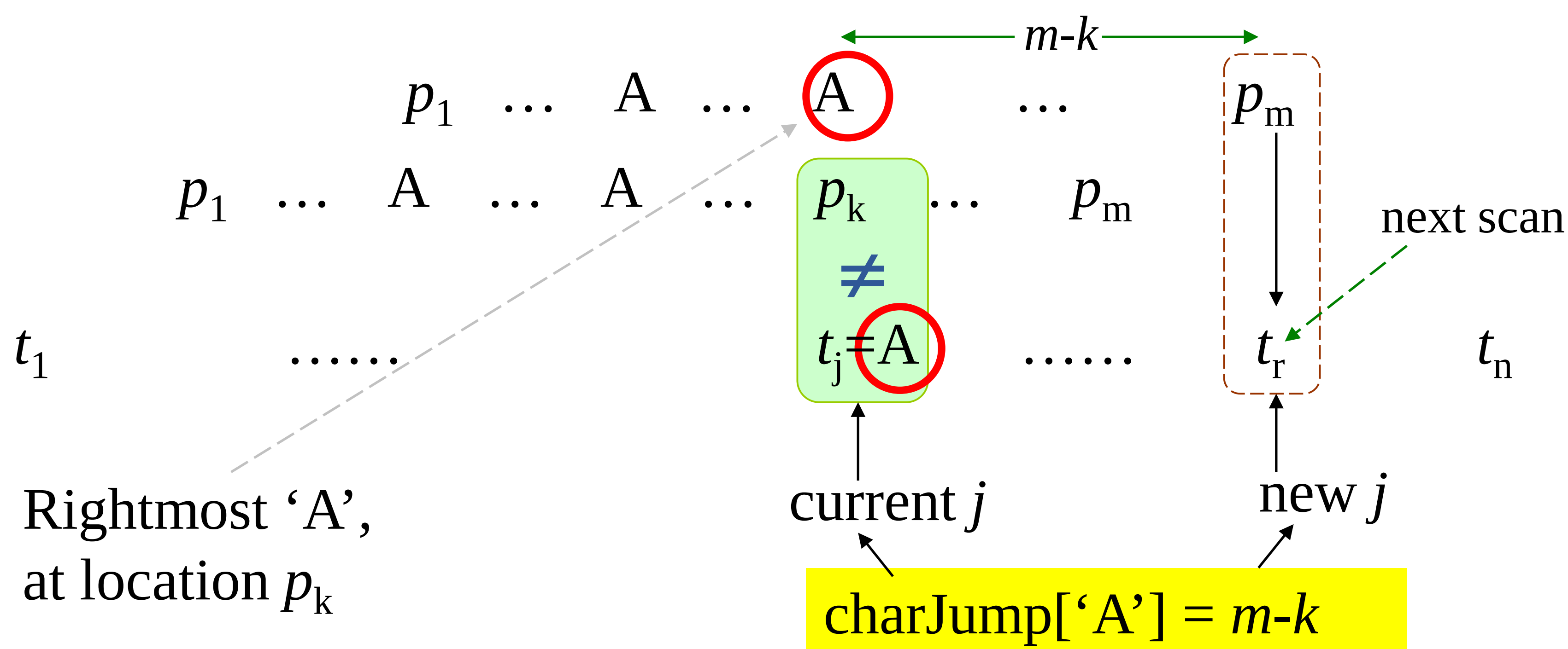


Checking the characters in *P*, in reverse order

The copy of the *P* begins at t_{38}. Matching is achieved in 18 comparisons

Distance of Jumping Forward

- **With the knowledge of *P*, the distance of jumping forward for the pointer of *T* is determined by the character itself, independent of the location in *T*.**



Computing the *charJump*[]

Input: Pattern string *P*; *m*, the length of *P*; alphabet size *alpha*=|Σ|

Output: Array *charJump*, indexed 0,…, *alpha*-1, storing the jumping offsets for each char in alphabet.

```
void computeJumps(char[ ] P, int m, int alpha, int[ ] charJump
    char ch;
    int k;
    for (ch=0; ch<alpha; ch++)
        charJump[ch]=m; //For all char no in P, jump by m
    for (k=1; k≤m; k++)
        charJump[p_k]=m-k;
```

$\Theta(|\Sigma|+m)$

The increasing order of k ensure that for duplicating symbols in *P*, the jump is computed according to the rightmost

Scan by charJump

Horspool's Algorithm

```
int horspoolScan(char[] P, char[] T, int m, int[] charjump)
    int j=m-1, k, match=-1;
    while (endText(T,j) = = false) //up to n loops
        k=0;
        while (k<m and P[m-k-1] = = T[j-k])//up to m loops
            k++;
        if (k= = m) match=j-m; break;
        else j=j+charjump[T[j]];
    return match;
```

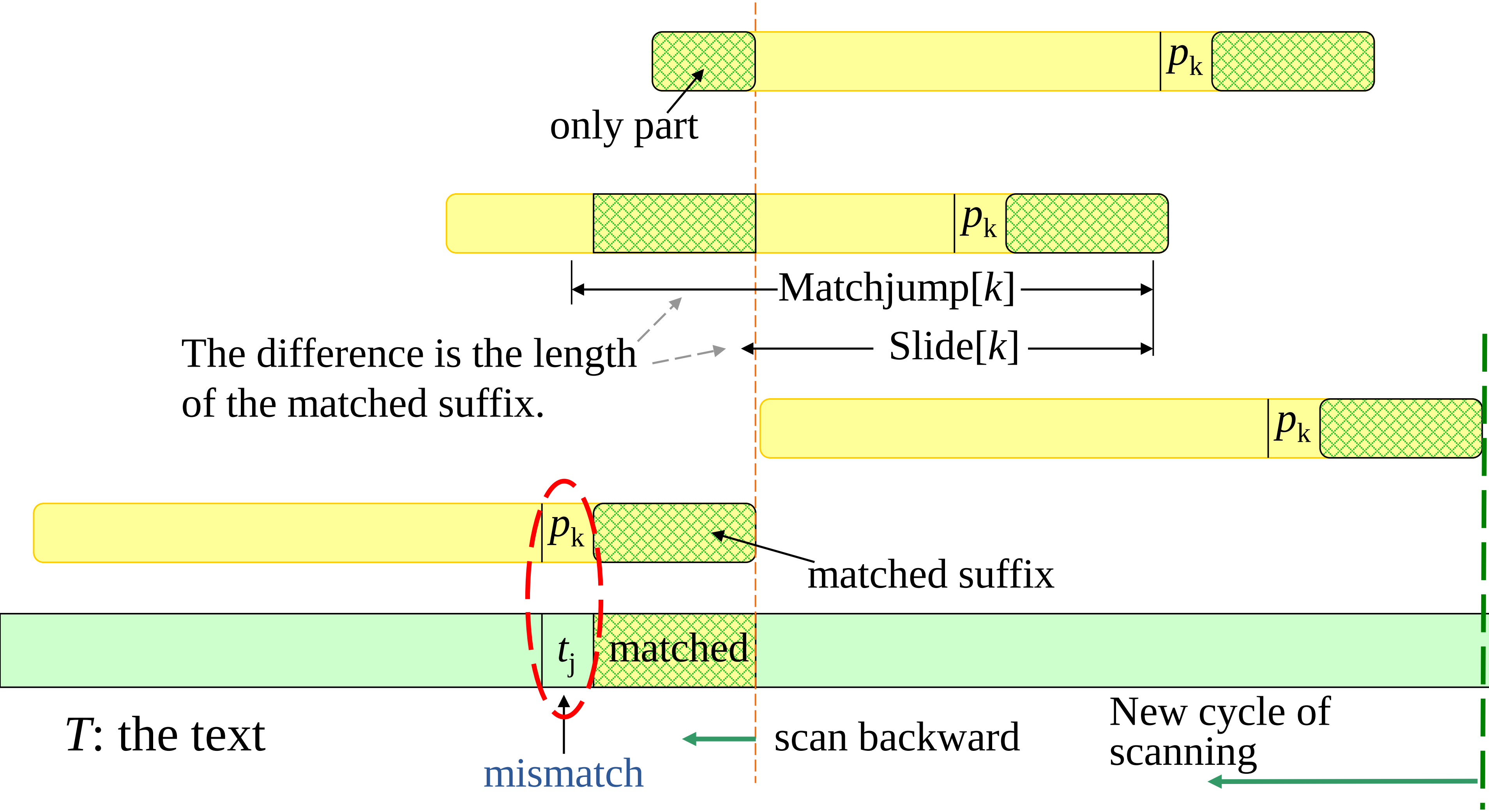
An example:

Search 'aaaa……aa' for 'baaaa'

Note: charjump['a']=1

So, in the worst case: $\Theta(mn)$

Boyer-Moore Algorithm



Performance

- **The performance depends on**
  - Size of the alphabet
  - Repetition within the strings

Thank you!

Q & A

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