Calcium channel models

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This note derives stochastic single-channel models of an R-type and T-type calcium channel from the data presented in Magee and Johnston, J Physiol (1995).

1 Data description

The data are single-channel recordings from patches of rat CA1 pyramidal neuron dendrite. They report channel open times, voltage dependencies for activation and inactivation.

Table 1. Dendritic single-channel properties

			Activation			Inactivation					
	γ (pS)	Open time (ms)	Range (mV)	$V_{½}$ (mV)	k	Range (mV)	$V_{½}$ (mV)	k	τ (ms)	Distinctive pharmacology	Likely correlate
Na^+	$15 \pm 0{\cdot}3$	n.d.	−50 to 0	−30	6	−90 to −30	−67	7	1 *‡	TTX	TTX-sen Na^+
LVA Ca^{2+}	$10 \pm 0{\cdot}4$	~1 *	−55 to −5	−32	7	−100 to −40	−70	6·5	~50 *	Low $[Ni^{2+}]$	T-type Ca^{2+}
HVA_m Ca^{2+}	17 ± 1	< 0·5	−20 to +30	+3	8	−80 to −15	−39	9·2	~100	Low $[Ni^{2+}]$	R-type Ca^{2+}
HVA_m Ca^{2+}	17 ± 1	~1	−10 to +30	n.d.	n.d.	−80 to −30	n.d.	n.d.	~100	Conotoxin	N-type Ca^{2+}
HVA_l Ca^{2+}	27 ± 3	< 0·5 and > 1†	−10 to +30	+9	6	n.d.	n.d.	n.d.	none	Bay K 8644	L-type Ca^{2+}

All channel properties determined at +10 mV unless otherwise specified. Ca^{2+} channel γ was determined in 110 mM $BaCl_2$ and all other properties in 20mM $BaCl_2$. γ, single-channel conductance; n.d., not determined; * determined at −20mV; † majority showed two open states; ‡ only fast τ shown; sen, sensitive.

Figure 1.1: Estimates of single channel properties from Magee and Johnston.

2 Channel models

Magee and Johnston fit the following steady-state activation and inactivation functions of voltage V respectively:

$$m_\infty(V) = \frac{1}{1 + \exp(\frac{V_{1/2} - V}{k})}$$

and

$$h_\infty(V) = \frac{1}{1 + \exp(\frac{V - V_{1/2}}{k})}$$

where m and h are the gating variables, $V_{1/2}$ is the half-maximum voltage and k is the slope factor.

We want to convert this model into a four-state single channel model (m_0h_0, m_1h_0, m_0h_1 and m_1h_1, i.e. the open state.) The eight transition rates (α for forward, β for backward) between these four states are derived from the activation functions above

$$x_\infty(V) = \frac{\alpha(V)}{\alpha(V) + \beta(V)}$$

and the time constants

$$\tau(V) = \frac{1}{\alpha(V) + \beta(V)}$$

For inactivation, we can take the τ_h from the table above, and assume it is independent of voltage. Although this is not usually true, time constants typically remain within a certain order of magnitude across the physiological voltage range, so it will not be a bad approximation. Then

$$\alpha_h(V) = \frac{h_\infty(V)}{\tau_h} \tag{2.1}$$

and

$$\beta_h(V) = \frac{1 - h_\infty(V)}{\tau_h} \tag{2.2}$$

For activation (m gates), we don't know τ_m but we do know the closing rate when open at a particular voltage V', from the 'open time' in the above table. We can assume this corresponds to $\beta_m(V') = 1/\Delta t_{open}$. We can then infer $\alpha_m(V')$ by rearranging the above:

$$\alpha_m(V') = \beta_m(V')\frac{m_\infty(V')}{1 - m_\infty(V')}$$

This will let us find α_m and β_m at a particular voltage V'. We can then make the assumption that τ_m is independent of voltage, and compute it as above, $\tau_m = \tau_m(V') = 1/(\alpha_m(V') + \beta(V'))$. With this we can then compute α_m and β_m for any voltage from the two earlier equations 2.1 and 2.2.

References

Magee, J C, and D Johnston. 1995. "Characterization of Single Voltage-Gated Na+ and Ca2+ Channels in Apical Dendrites of Rat CA1 Pyramidal Neurons." The Journal of Physiology 487 (Pt 1) (August): 67–90.