Optimal Sampling for Uncertainty-of-Information Minimization in a Remote Monitoring System

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***Abstract*—In this paper, we study a remote monitoring system where a receiver observes a remote binary Markov source and decides whether to sample and transmit the state through a randomly delayed channel. We adopt uncertainty of information (UoI), defined as the entropy conditional on past observations at the receiver, as a metric of value of information. To address the limitations of prior UoI research that assumes one-time-slot delays, we extend our analysis to scenarios with random delays. We model the problem as a partially observable Markov decision process (POMDP) problem and simplify it to a semi-Markov decision process (SMDP) by introducing the belief state. We propose two algorithms: A globally optimal *bisection relative value iteration* (bisec-RVI) algorithm and a computationally efficient sub-optimal index-based threshold algorithm to solve the long-term average UoI minimization problem. Numerical simulations demonstrate that our sampling policies surpass traditional zero wait and AoI-optimal policies, particularly under conditions of large delay, with the sub-optimal policy nearly matching the performance of the optimal one.**

***Index Terms*—Remote monitoring, uncertainty of information, age of information, Markov decision process**

I. Introduction

To evaluate the information freshness, age of information (AoI) has been proposed in [1], [2], and has attracted extensive research attention in remote monitoring, industrial automation, and internet-of-things (IoT) applications [3]–[13]. Traditionally, AoI is known to be *state-agnostic* [14]–[16], *i.e.*, focusing solely on the timeliness of information without accounting for the dynamics and the semantics of the source. Today, some variants of AoI have been proposed in [17]–[26] to address the *state-agnostic* limitation. The researchers successfully derive their metrics of interest, such as the mutual information and the mean square error between the source and the latest receives message, as non-linear penalty functions of AoI. In this way, the problem of minimizing mutual information, or mean square error, can be transformed into a problem of minimizing non-linearly penalized AoI.

Uncertainty of information (UoI) is a new metric that addresses the *state-agnostic* limitation of AoI [27]. Defined as the entropy conditional on past observations at the receiver, UoI quantifies the receiver's uncertainty about the latest state of the source based on previously received, potentially

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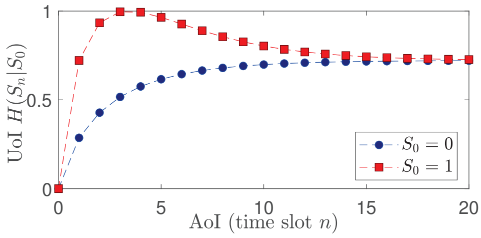

Fig. 1. UoI vs. AoI and the latest observed state S_0.

outdated observations. Unlike AoI, UoI integrates both the age and the historical observations, making it a *state-aware* metric. For example, consider a binary discrete-time Markov process with state transition probabilities $P[0|1] = 0.2$ and $P[1|0] = 0.05$. As illustrated in Fig. 1, UoI's dependence on AoI varies with the latest received state S_0, exhibiting a non-monotonic relationship. This phenomenon introduces new challenges in designing UoI-optimal sampling policies.

In [27] and [28], the UoI-optimal sampling and scheduling policy has been investigated in the one-time-slot system. In [27], a Whittle index-based multi-source scheduling policy for binary Markov process is derived. Then an index policy for general finite-state Markov processes under unreliable channels are further extended in [28]. Both of the studies idealize the transmission delay as one time slot. However, random and large delays are common in communication networks due to network loading, routing, and retransmission [29]. Under random delay, UoI-optimal sampling process is no longer a typical Markov decision process (MDP), which brings about new challange. This prompts us to formulate a new problem for UoI-minimization under random delay.

Up to this point, considerable research efforts have been devoted to optimizing metrics related to AoI in the presence of delays [30]–[35]. Authors proposes a new "selection-from-buffer" model for sending the features aimed at minimizing the general functions of AoI (monotonic or non-monotonic) with random transmission delay in [32]. As an extension to [32], authors minimizes the general functions of AoI through a channel with highly variable two-way random delay in [34]. And in [35], an optimal sampling policy is designed to minimize the average AoI when the statistics of delay are unknown. However, all of these AoI-related functions to be minimized are state-agnostic, distinct to minimization of UoI, which is a state-aware metric.

To sum up, our motivation is twofold: (i) The optimization

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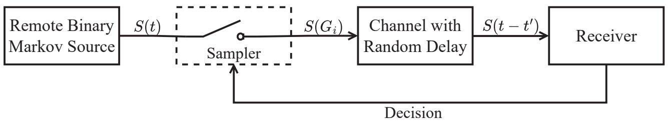

Fig. 2. System model of the considered remote monitoring system.

of UoI under random delay still remains largely unexplored. (ii) The UoI-minimization problem is distinct from other AoI-related optimization under random delay. In this paper, we study a remote monitoring system where a receiver observes a remote binary Markov source and decides whether to sample the source's state over a randomly delayed channel. We model the problem as a partially observed Markov decision process (POMDP), and simplify it into a semi-Markov decision process (SMDP) by introducing *belief state*. Specifically, the contributions of this paper are as follows:

- We formulate a new problem to minimize the average UoI under random delay.
- We propose an optimal policy, by applying a two layered *bisection relative value iteration (bisec-RVI)* algorithm.
- We develop a sub-optimal policy with computation efficiency, based on the special properties of belief state.

Numerical simulations illustrate that our proposed sampling policies outperform traditional zero wait and AoI-optimal policies. And the performance of the sub-optimal policy nearly match that of the optimal policy, especially under large delay.

II. System Model and Problem Formulation

A. System Model

We consider a discrete-time remote monitoring system in Fig. 2, where the states of a remote binary Markov source are delivered through a channel to a receiver. Based on the history observation of the source's state, the receiver decides whether to sample the current state or not, and transmits the decision to the sampler. The remote source is a discrete-time binary Markov process with a state at time t denoted by $S(t) \in \{0,1\}$. We assume that the one-step transition matrix of the Markov process $\mathbf{P}$ is known to the receiver, given by

$$\begin{array}{cc} & \begin{array}{cc} \text{State } 0 & \text{State } 1 \end{array} \\ \begin{array}{c} \text{State } 0 \\ \text{State } 1 \end{array} & \begin{bmatrix} 1-p & p \\ q & 1-q \end{bmatrix} \end{array} \tag{1}$$

where $0 < p \leq q < 1$, $p+q \neq 1$ [1]. Consequently, the n-step transition matrix of this Markov process is [36, Appendix A]

$$\mathbf{P}^n = \begin{bmatrix} 1-p^{(n)} & p^{(n)} \\ q^{(n)} & 1-q^{(n)} \end{bmatrix}, \tag{2}$$

where $p^{(n)} \triangleq \frac{p-p(1-p-q)^n}{p+q}$, $q^{(n)} \triangleq \frac{q-q(1-p-q)^n}{p+q}$, $n \in \mathbb{N}^+$.

The i-th state information sent over the channel is sampled and transmitted in time slot G_i, and is delivered at the receiver at time slot $R_i = G_i + Y_i$, where $Y_i \geq 1$ is the *independent and identically distributed* (i.i.d) random transmission delay of the i-th state information, satisfying $1 \leq E[Y_i] < \infty$. We assume that the sampler receives delay-free decision commands from the receiver [2], such that the receiver sends the decision and the sampler receives the decision at time slot G_i. Let $Z_i = G_i - R_{i-1} \geq 0$ represent the waiting time for sending the i-th packet after the $(i-1)$-th packet is received by the receiver.

B. Problem Formmulation

The UoI is a metric proposed in [27] to measure the uncertainty of the source at the receiver side given the history observations, given by

$$U(t) = H(S(t)|\mathbf{W}(t)), \tag{3}$$

where $\mathbf{W}(t) = (S(G_0), S(G_1), \cdots, S(t'))$ are the history observations at the receiver side up to time slot t, and $t' \triangleq \max_i\{G_i : R_i \leq t\}$ is the time stamp of the most recently received update. Leveraging the Markov property of $S(t)$, we have

$$\begin{aligned} U(t) &\triangleq -\sum_{i\in\{0,1\}} \mathrm{P}[S(t)=i|\mathbf{W}(t)] \log_2 \mathrm{P}[S(t)=i|\mathbf{W}(t)] \\ &= -\sum_{i\in\{0,1\}} \mathrm{P}[S(t)=i|S(t')] \log_2 \mathrm{P}[S(t)=i|S(t')], \end{aligned} \tag{4}$$

By the definition of t', we rewrite $U(t)$ as a piecewise function

$$U(t) = -\sum_{i\in\{0,1\}} P[S(t)=i|S(G_i)] \log_2 P[S(t)=i|S(G_i)], \quad \text{if } R_i \leq t < R_{i+1}, \forall i \in \mathbb{N}. \tag{5}$$

For short-hand notations, we introduce $H(p) \triangleq -p\log_2(p) - (1-p)\log_2(1-p)$ as the entropy of a binary information. With (2) in hand, (5) can be rewritten as

$$U(t) = \begin{cases} H(p^{(t-G_i)}), \text{ if } S(G_i)=0, R_i \leq t < R_{i+1}, \forall i \in \mathbb{N} \\ H(q^{(t-G_i)}), \text{ if } S(G_i)=1, R_i \leq t < R_{i+1}, \forall i \in \mathbb{N}, \end{cases} \tag{6}$$

where $t - G_i$ for $R_i \leq t < R_{i+1}, \forall i \in \mathbb{N}$ is what exactly AoI indicates. As a result, UoI is an observation-based non-monotonic function of AoI.

A sampling policy is a sequence $\pi = (G_1, G_2, \cdots) \in \Pi$, which is time stamps of sampling times for each packet. Alternatively, the sampling policy can also be expressed as $\pi = (Z_1, Z_2, \cdots) \in \Pi$, which is the sequence of waiting times for each packet. (see [37, Appendix B] for a detailed explanation.) Our goal is to find both an optimal sampling policy that minimizes the time-average expected sum-UoI:

$$\inf_{\pi\in\Pi} \limsup_{T\to\infty} \frac{1}{T}\mathbb{E}\left[\sum_{t=0}^{T-1} U(t)\right]. \tag{7}$$

[1] Without loss of generality, we assume that $p \leq q$. And when $p+q=1$, UoI is constant that can not be changed by any sampling policy. For a similar reason, we also assume that $p+q \neq 0$ and $p+q \neq 2$.

[2] A remote monitoring system with two-way random delay can be extended based on this work.

This is a new problem different from previous studies focused on optimizing cost functions of AoI with random delay. The uniqueness resides in the fact that UoI is not only dependent on the age of the latest observation (*i.e.* $t-G_i$), but also customized by the latest observation $S(G_i)$, as shown in (6). Compared to UoI, the current functions of AoI used as metrics to design sampling policy under random delay, as we know, have nothing to do with the contents of the transmission information and remains invariant [32]–[34].

III. Optimal Sampling policy

In this section, we propose an optimal sampling policy by using *bisec-RVI* algorithm to minimize the long-term average expected sum-UoI.

A. Belief State

In our system model, the receiver is tasked with determining sampling actions based on delayed and imperfect observations $\mathbf{W}(t)$ to minimize the time-average UoI. This problem is commonly modeled as a POMDP. A fundamental approach in solving a POMDP involves transforming it into MDP by utilizing a concept known as *belief state*. In this subsection, we explore the *belief state* as the probability of $S(t)=1$ given the observations $\mathbf{W}(t)$, given by

$$\Omega(t)=\mathrm{P}[S(t)=1|\mathbf{W}(t)]. \tag{8}$$

Similar to the process to obtain (6), we can prove that for $R_i \le t < R_{i+1}$, $\forall i\in\mathbb{N}$, $\Omega(t)$ can be expressed by

$$\Omega(t)=\begin{cases} p^{(t-G_i)}, & \text{if } S(G_i)=0 \\ 1-q^{(t-G_i)}, & \text{if } S(G_i)=1. \end{cases} \tag{9}$$

The evolution of the *belief state* is given in the following lemma:

Lemma 1. *Given $\Omega(t)=\omega$, $\Omega(t+k)$ can be explicitly calculated by*

$$\Omega(t+k)=\frac{p-p(1-p-q)^k}{p+q}+\omega(1-p-q)^k, \tag{10}$$

where $\omega\in\{p^{(n)},1-q^{(n)}\}$, $n,k\in\mathbb{N}$. For short-hand notations, we leverage $\tau^k(\omega)$ to denote the right-hand side of (10).

Proof. Please refer to [37, Appendix A]. □

Corollary 1. *The equilibrium belief state of $\Omega(t)$ is*

$$\omega^* \triangleq \lim_{k\to\infty}\tau^k(\omega)=\frac{p}{p+q}. \tag{11}$$

Proof. Since $0<|1-p-q|<1$, we have that $\lim_{k\to\infty}(1-p+q)^k=0$, and thus we obtain the limit. □

Since $H(p)=H(1-p)$, by combing (6) and (9) we have

$$U(t)=H(\Omega(t)). \tag{12}$$

Then the problem (7) can be rewritten as:

$$\bar{p}_{\mathrm{opt}}=\inf_{\pi\in\Pi}\limsup_{T\to\infty}\frac{1}{T}\mathbb{E}\left[\sum_{t=0}^{T-1}H(\Omega(t))\right], \tag{13}$$

where $\bar{p}_{\mathrm{opt}}$ is the optimum value of (7).

B. An Optimal sampling Policy

We present an optimal policy for (13) as follows:

Theorem 1. *If Y_i's are i.i.d. with a finite mean $\mathbb{E}[Y_i]$, given $\Omega(R_i)=\omega$, then $\pi^*=(Z_1^*,Z_2^*,\cdots)$ is an optimal solution to* (13)*, which satisfies the following optimality equation:*

$$\begin{aligned} &g(\bar{p}_{\mathrm{opt}})+\widetilde{V}(\omega,\bar{p}_{\mathrm{opt}}) \\ &=\inf_{Z_{i+1}\in\mathbb{N}}\{c(\omega,Z_{i+1},\bar{p}_{\mathrm{opt}})+r(\omega,Z_{i+1},\bar{p}_{\mathrm{opt}})\}, \end{aligned} \tag{14}$$

$$g(\bar{p}_{\mathrm{opt}})=\inf_{Z_{i+1}\in\mathbb{N}}\{c(p,Z_{i+1},\bar{p}_{\mathrm{opt}})+r(p,Z_{i+1},\bar{p}_{\mathrm{opt}})\}, \tag{15}$$

where

$$c(\omega,Z_{i+1},\bar{p}_{\mathrm{opt}})=\mathbb{E}\left[\sum_{k=0}^{Z_{i+1}+Y_{i+1}-1}(H(\tau^k(\omega))-\bar{p}_{\mathrm{opt}})\right], \tag{16}$$

$$\begin{aligned} r(\omega,Z_{i+1},\bar{p}_{\mathrm{opt}})=&\mathbb{E}\Big[\tau^{Z_{i+1}+Y_{i+1}}(\omega)\widetilde{V}(1-q^{(Y_{i+1})},\bar{p}_{\mathrm{opt}}) \\ &+(1-\tau^{Z_{i+1}+Y_{i+1}}(\omega))\widetilde{V}(p^{(Y_{i+1})},\bar{p}_{\mathrm{opt}})\Big], \end{aligned} \tag{17}$$

for all $\omega\in\{p^{(n)},1-q^{(n)}\}$, $n\in\mathbb{N}$.

Proof sketch. The problem (13) can be cast as an infinite-horizon average cost SMDP [36, Chapter 11]. Recall that $Z_{i+1}=G_{i+1}-R_i$ as the waiting time for sending the $(i+1)$-th packet after the i-th packet is received by the receiver. Given $\Omega(R_i)=\omega$, the Bellman optimality equation of the average cost problem is

$$\begin{aligned} V^*(\omega)=\inf_{Z_{i+1}\in\mathbb{N}}\Bigg\{&\mathbb{E}\left[\sum_{k=0}^{Z_{i+1}+Y_{i+1}-1}(H(\tau^k(\omega))-\bar{p}_{\mathrm{opt}})\right] \\ &+\mathbb{E}\Big[\tau^{Z_{i+1}+Y_{i+1}}(\omega)V^*(1-q^{(Y_{i+1})}) \\ &+(1-\tau^{Z_{i+1}+Y_{i+1}}(\omega))V^*(p^{(Y_{i+1})})\Big]\Bigg\}, \end{aligned} \tag{18}$$

for all $\omega\in\{p^{(n)},1-q^{(n)}\}$, $n\in\mathbb{N}$, where $V^*(\omega)$ is the relative value function associated with the average cost problem (13).

We assume that Y_i's are random variables with limited values, thus the states of ω are finite and countable. The equation (18) can be converted to (14) and (15). This inversion can be interpreted as a transformation from the SMDP to an equivalent MDP [36, Chapter 11]. Please refer to [37, Appendix B] for details of the proof. □

Applying Dinkelbach's method for nonlinear fractional programming as shown in [38] and [39, lemma 2], we get two assertions: (i) $\bar{p}_{\mathrm{opt}}\lesseqgtr\beta$ if and only if $g(\beta)\lesseqgtr 0$. (ii) $\bar{p}_{\mathrm{opt}}$ is the unique root of $g(\beta)=0$. Consequently, we can find the value of $\bar{p}_{\mathrm{opt}}$ by finding the root of $g(\beta)=0$ as shown in Algorithm 1, which is a two-layered *Bisec-RVI* algorithm.

Bisection search method is applied to the outer layer to get a fixed β for every step and finally get the optimal value $\bar{p}_{\mathrm{opt}}$. In the inner layer, as the value of β has been fixed by the outer layer, we only need to use RVI to find convergent $g(\beta)$. The RVI algorithm here is the same as it in MDP. The details about RVI algorithm have been neglected since it has been a mature technique to solve an infinite-horizon MDP [36, Section 8.5.5]. Similar algorithms are proposed in [25], [40] and [41] to achieve age-optimal or mean square error (MSE)-optimal sampling.

Algorithm 1: Bisec-RVI algorithm

```
Input: l = 0, u = 1, tolerance ε > 0
while u − l ≥ ε do
    β := (l + u)/2;
    Run RVI to solve g(β) and Ṽ(ω, β);
    if g(β) > 0 then
        l := β;
    else
        u := β;
Output: p̄_opt = β
```

IV. Sub-optimal Index-based Policy

The optimal policy for (13) uses *bisec-RVI* algrithm, which requires repeatedly executing the RVI algorithm in the inner layer until the value to be found converges, resulting in high computing complexity. In this section, we explore a sub-optimal but computation-efficient index-based policy in the sequel. We assume that the transmission process spends a long time, *i.e.*, the value of $\mathbb{E}[Y_i], \forall i \in \mathbb{N}$ is large enough. The large transmission delay may be due to long distance, time-varying channel conditions, too many packets in the channel and so on. According to (11), we can conclude that the transition probability from receiving time R_i to R_{i+1} ($i \in \mathbb{N}$) is

$$\mathrm{P}[\Omega(R_{i+1}) = 1 - q^{(Y_{i+1})} | \Omega(R_i)] = \tau^{Z_{i+1}+Y_{i+1}}(\omega) = \omega^*. \tag{19}$$

As the state transition probabilities are constant, the second and third terms of the right-hand-side of (18) are irrelevant to the waiting time, making the Bellman optimality equation easier to solve. Along this line, we can finally get an index-based sampling policy.

We depict the details of the problem as follows. On the basis of the assumption, we consider a sampling policy $\psi = (Z_1, Z_2, \cdots) \in \Pi$ and try to optimize the following problem:

$$\bar{p}_{\mathrm{nopt}} = \inf_{\psi\in\Pi} \limsup_{T\to\infty} \frac{1}{T}\mathbb{E}\left[\sum_{t=0}^{T-1} H(\Omega(t))\right]. \tag{20}$$

After that, we present a sub-optimal index-based sampling policy for problem (13) using the solution to (20).

We first introduce an index function as

$$\eta(\omega) \triangleq \inf_{Z_i\in\mathbb{N}^+} \frac{1}{Z_i}\sum_{k=0}^{Z_i-1} \mathbb{E}\Big[H(\tau^{k+Y_i}(\omega))\Big], \tag{21}$$

where $\omega \in \{p^{(n)}, 1-q^{(n)}\}$, $n \in \mathbb{N}$. Then we have a theorem as follows:

Theorem 2. *If Y_i's are i.i.d. with a finite mean $\mathbb{E}[Y_i]$, then $\psi = (Z_1(\beta_\psi), Z_2(\beta_\psi), \cdots)$ is an optimal solution to (20), where*

$$Z_{i+1}(\beta_\psi) = \min\{k \in \mathbb{N} : \eta(\Omega(t+k)) \geq \beta_\psi, t = R_i(\beta_\psi)\}, \tag{22}$$

and β_ψ is the unique root of

$$\mathbb{E}\left[\sum_{t=R_i(\beta_\psi)}^{R_{i+1}(\beta_\psi)-1} H(\Omega(t))\right] - \beta_\psi \mathbb{E}[R_{i+1}(\beta_\psi) - R_i(\beta_\psi)] = 0, \tag{23}$$

where $R_i(\beta_\psi) = G_i(\beta_\psi) + Y_i$ is the receiving time of the i-th state information submitted to the channel, and $\Omega(t)$ is the belief state at time slot t. Moreover, β_ψ is exactly the optimum value of (15), i.e., $\beta_\psi = \bar{p}_{\mathrm{nopt}}$.

Proof sketch. The problem (20) can be cast as an infinite-horizon average cost SMDP. Given $\Omega(R_i) = \omega$, the Bellman optimality equation of the average cost problem is

$$V_\psi(\omega) = \inf_{Z_{i+1}\in\mathbb{N}}\left\{\mathbb{E}\left[\sum_{k=0}^{Z_{i+1}+Y_{i+1}-1}(H(\tau^k(\omega)) - \bar{p}_{\mathrm{nopt}})\right]\right\} + \mathbb{E}\Big[\omega^* V_\psi(1-q^{(Y_{i+1})}) + (1-\omega^*)V_\psi(p^{(Y_{i+1})})\Big], \tag{24}$$

for all $\omega \in \{p^{(n)}, 1-q^{(n)}\}$, $n \in \mathbb{N}$, where $V_\psi(\omega)$ is the relative value function associated with the average cost problem (20). Theorem 2 is proven by directly solving (24). The details are provided in [37, Appendix C]. □

Theorem 2 signifies that the optimal solution to (20) is an index-based threshold policy, where the index function depends on the belief state. Specifically, the state of the source is submitted in time slot t if two conditions are satisfied: (i) The channel is idle in time slot t, (ii) the index $\eta(\Omega(t))$ exceeds a threshold β_ψ (*i.e.*, $\eta(\Omega(t)) \geq \beta_\psi$). where the threshold β_ψ is exactly the optimum value of (20).

For notational simplicity, we rewrite (23) as

$$f(\beta_\psi) = f_1(\beta_\psi) - \beta_\psi f_2(\beta_\psi) = 0, \tag{25}$$

where $f_1(\beta_\psi) = \mathbb{E}\Big[\sum_{R_i(\beta_\psi)}^{R_{i+1}(\beta_\psi)-1} H(\omega(t))\Big]$ and $f_2(\beta_\psi) = \mathbb{E}[R_{i+1}(\beta_\psi) - R_i(\beta_\psi)]$. Then algorithm 2 is a low-complexity algorithm to find the optimal objective value $\bar{p}_{\mathrm{nopt}}$.

Compared the relative value function (24) with (18), the core difference is that the state transition probabilities in (24) are irrelevant to the waiting time, which leads to an index policy for problem (20). According to the convergence of $\tau^k(\omega)$ to ω^* in (11), we induce that if $\mathbb{E}(Y_i)$ is large enough, the index policy is the optimal policy for (13), *i.e.*, $\pi^* = \psi$.

V. Numerical Results

This section presents numerical results that demonstrate the performance of the proposed policies. As shown in [27,

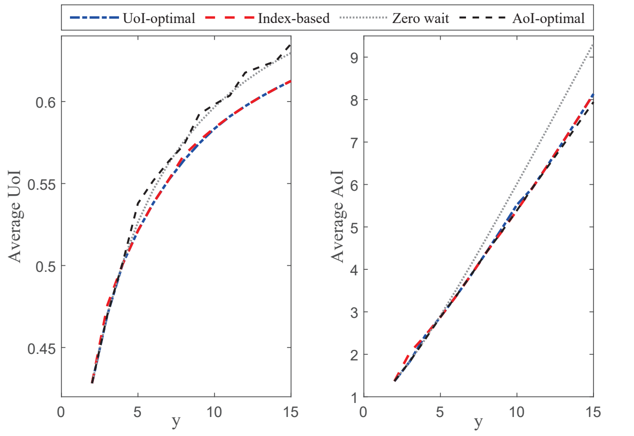

Fig. 3. Average UoI and Average AoI v.s. y with i.d.d random delay, where $P[Y_i = 1] = 0.8$ and $P[Y_i = y] = 0.2$, the dynamics of the Markov source depicted as $p = 0.05$ and $q = 0.2$.

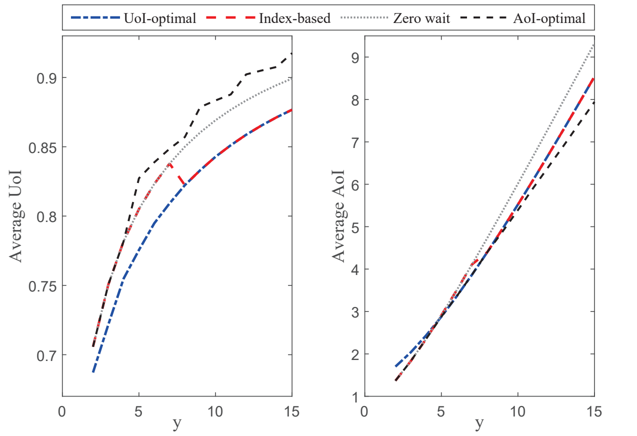

Fig. 4. Average UoI and Average AoI v.s. y with i.d.d random delay, where $P[Y_i = 1] = 0.8$ and $P[Y_i = y] = 0.2$, the dynamics of the Markov source depicted as $p = 0.7$ and $q = 0.95$.

Algorithm 2: Bisec-index algorithm

Input: $l = 0$, $u = 1$, tolerance $\epsilon > 0$

while $u - l \geq \epsilon$ **do**
- $\beta_\psi := (l + u)/2$;
- $c := f(\beta_\psi) = f_1(\beta_\psi) - \beta_\psi f_2(\beta_\psi)$;
- **if** $c > 0$ **then**
  - $l := \beta_\psi$;
- **else**
  - $u := \beta_\psi$;

Output: $\bar{p}_{\text{nopt}} = \beta_\psi$

Fig.3], the k-step belief state evolutions with $p + q < 1$ and with $p+q > 1$ are quite different, so we operate both of them. First, we evaluate the following four sampling policies:

1. *UoI-optimal:* The policy is given by Theorem 1.
2. *Index-based:* The policy is given by Theorem 2.
3. *Zero wait:* An update is transmitted once the previous update is received, *i.e.*, $Z_i = 0$ for $\forall i \in \mathbb{N}$. This policy achieves the minimum delay and maximum throughput.
4. *AoI-optimal:* The AoI-optimal policy decides waiting time Z_i by [25, Theorem 4] and [25, Algorithm 2].

Fig. 3 shows the four policies comparison in terms of average UoI and average AoI, when the binary Markov source evaluates with the probability $p+q < 1$. The left panel shows that the average UoI obtained by the index policy is very close to the UoI-optimal one, compared to which Zero wait and AoI-optimal policy performs not well as $\mathbb{E}[Y_i]$ increases. But the right panel reveals that AoI-optimal policy consistently achieves the lowest AoI. This implies that the desired goal the receiver tends to achieve leads to different result.

In Fig. 4, we compare the four policies performance for the evaluated case that $p + q > 1$. The left panel shows a similar trend to the left panel of Fig. 3, except for the sub-optimal index-based policy. A watershed phenomenon occurs for the index-based policy: When $y \leq 7$, the index-based policy provides the same sampling strategy as zero wait policy, which performs worse than optimal policy; when $y > 7$, the index-based policy performs as well as the optimal policy. The reason for this phenomenon is that the index-based threshold policy is sub-optimal, deciding whether to sample or not by the comparison between the index and the constant value $\bar{p}_{\text{nopt}}$. Therefore, this sub-optimal policy can not always take future circumstances into account if the value of $\mathbb{E}(Y_i)$ is small, thus ignores the oscillation of $H(\omega)$ when $p + q > 1$. The index is always less than $\bar{p}_{\text{nopt}}$ if $y \leq 6$, causing the zero wait policy. Otherwise, if $y > 6$, the index outweighs $\bar{p}_{\text{nopt}}$ so the policy is optimal. The right panel demonstrates the lowest AoI is obtained by AoI-optimal policy as well.

The performance gains of UoI-optimal and index-based policies are close to the best average AoI, making them better choices when the system aims to optimize both AoI and UoI simultaneously. Moreover, Both of the pictures show that when $\mathbb{E}[Y_i]$ is large enough, the result of index policy is the same as that of the UoI-optimal policy, consistent with the theoretical induction we proposed before. But for what exact value $\mathbb{E}[Y_i]$ is meaning "large enough", is still an open issue.

VI. Conclusion

In this paper, we have used UoI as a state-aware metric to estimate the value of information in a remote monitoring system. First we have put forward an optimal policy to minimize the time-average expected sum-UoI by two-layered *bisec-RVI* algorithm. Based on the properties of belief state, we have further provided a sub-optimal index-based sampling policy owning lower computing complexity than the optimal one. The good performance of the sampling policies have been demonstrated by numerical simulations. Both of the proposed sampling policies outperform zero wait policy and AoI-optimal policy. Moreover, the performance of the sub-optimal policy approaches to that of the optimal policy, particularly under large delay. In the future work, it may be worthwhile to investigate the specific value of $\mathbb{E}[Y_i]$ that leads to the sub-optimal policy being identical to the optimal policy.

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