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Optimal Sampling for Uncertainty-of-Information Minimization in a Remote Monitoring System

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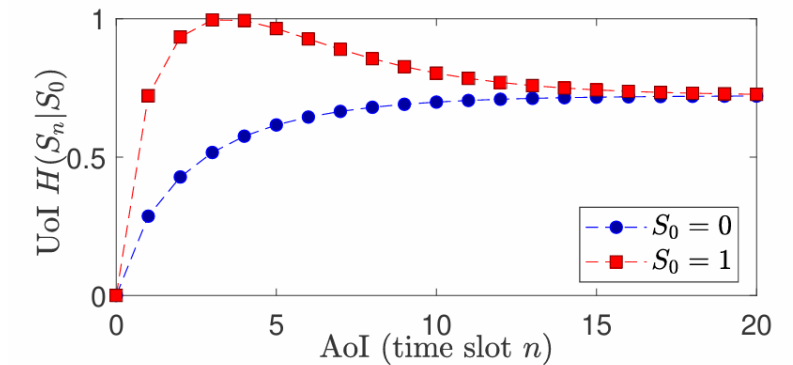
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Introduction

- Why focus on **Uncertainty-of-Information** [1],[2]?
 - Non-monotonic relationship with AoI, unlike some of other information freshness metrics which are non-decrease functions of AoI [3],[4]
 - UoI's dependence on AoI varies with the latest observed state.
- UoI reflects the different rates of information quality evolution caused by different values of the last observation



UoI vs. AoI and the latest observed state S_0 (Fig. 1)

UoI: Uncertainty-of-Information AoI: Age-of-Information

[1] G. Chen, S. C. Liew, and Y. Shao, "Uncertainty-of-information scheduling: A restless multi-armed bandit framework," *IEEE Trans. Inf. Theory*, vol. 68, no. 9, pp. 6151–6173, Sep. 2022.

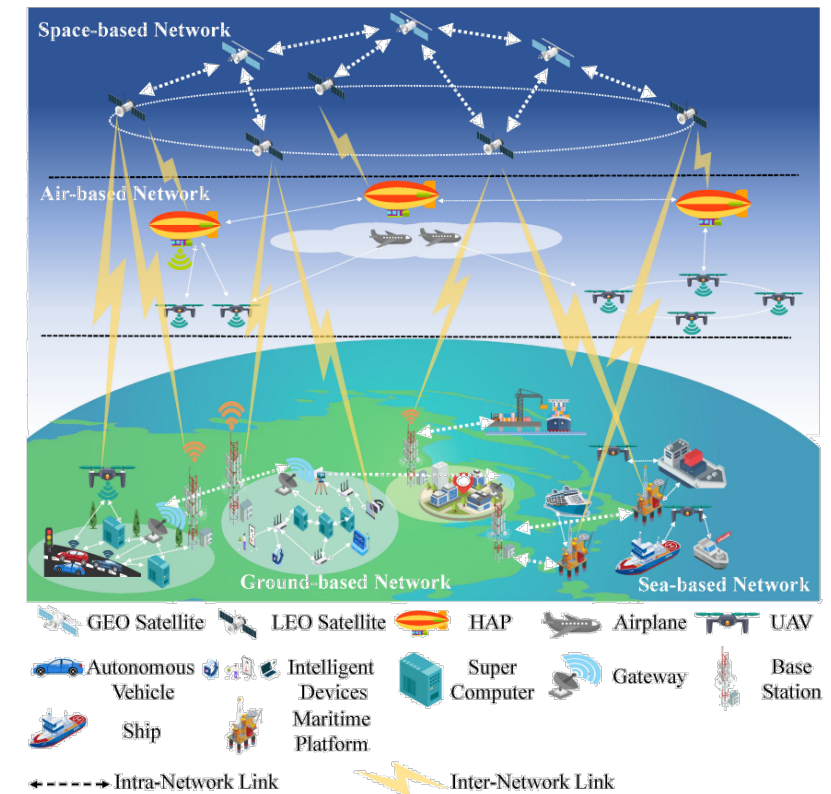
[2] G. Chen and S. C. Liew, "An index policy for minimizing the uncertainty-of-information of Markov sources," *IEEE Trans. Inf. Theory*, vol. 70, no. 1, pp. 698–721, Jan. 2023.

[3] Y. Sun and B. Cyr, "Sampling for data freshness optimization: Nonlinear age functions," *J. Commun. Networks*, vol. 21, no. 3, pp. 204–219, 2019.

[4] J. P. Champati, M. Skoglund, M. Jansson, and J. Gross, "Detecting state transitions of a Markov source: Sampling frequency and age trade-off," *IEEE Trans. Commun.*, vol. 70, no. 5, pp. 3081–3095, Jun. 2022.

Introduction

- Motivation of a **Remote** Monitoring System
 - **Remote** here means random and large delays
 - **Random and large delays** are common in communication networks, for example, SAGSIN integrated network [3]
 - **Random and large delays** are caused by network loading, routing, retransmission and so on [4]
 - The optimization of UoI under random delay remains **largely unexplored**



An illustration of the SAGSIN. (Fig. 2 [from [3]])

SAGSIN: Space-Air-Ground-Sea Integrated Network

[3] S. Meng, S. Wu, J. Zhang, J. Cheng, H. Zhou, and Q. Zhang, "Semantics-empowered space-air-ground-sea integrated network: New paradigm, frameworks, and challenges," *IEEE Commun. Surv. Tutorials* (Early Access), 2024.

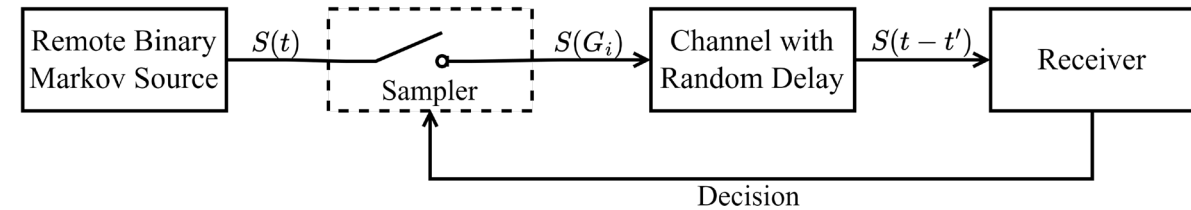
[4] W. Yao, L. Jiang, Q. H. Wu, J. Y. Wen, and S. J. Cheng, "Delay dependent stability analysis of the power system with a wide-area damping controller embedded," *IEEE Trans. Power Syst.*, vol. 26, no. 1, pp. 233–240, Dec. 2011.

System Model

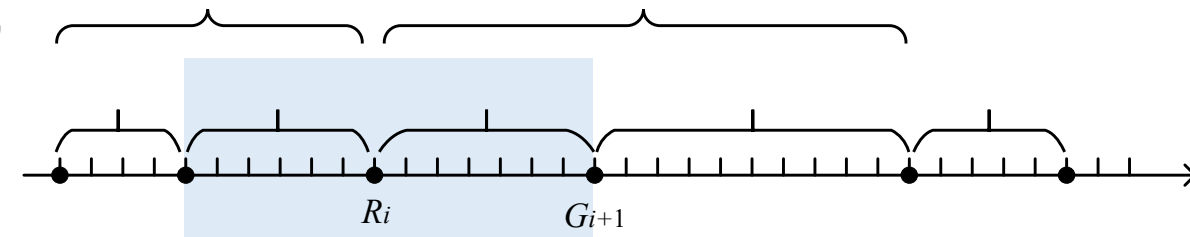
• A Discrete-time Remote Monitoring System

Sampling process (begin from the i -th packet)

1. **Sampler:** Sampling and sending at time G_i
2. **Channel:** Transmitting with random delay Y_i
3. **Receiver:** Receiving at time R_i , waiting for time Z_{i+1} ,
and sending decision of sampling at time G_{i+1}
4. **Sampler:** Receiving decision and sampling
(Return to step 1)



System model of the considered remote monitoring system (Fig. 3)



A sample evolution for two transmission periods (Fig. 4)

System Model

• Binary Markov Source

A discrete-time binary Markov process

with a state at time t denoted by $S(t) \in \{0, 1\}$

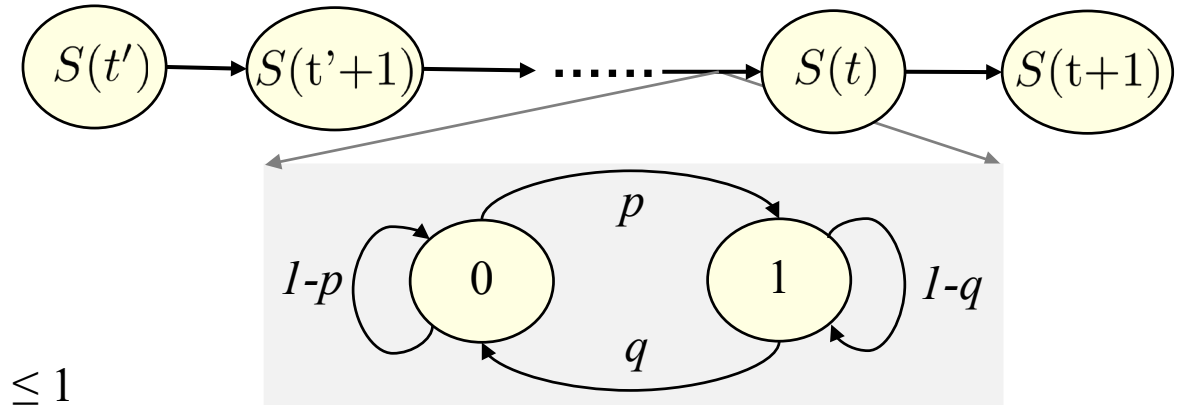
• One-step transition matrix P

$$\begin{array}{cc} & \begin{array}{cc} \text{State 0} & \text{State 1} \end{array} \\ \begin{array}{c} \text{State 0} \\ \text{State 1} \end{array} & \begin{bmatrix} 1-p & p \\ q & 1-q \end{bmatrix} \end{array} \quad (1) \quad \text{where } 0 < p \leq q < 1, p+q \leq 1$$

• n -step transition matrix [5, Appendix A]

$$\mathbf{P}^n = \begin{bmatrix} 1-p^{(n)} & p^{(n)} \\ q^{(n)} & 1-q^{(n)} \end{bmatrix} \quad (2) \quad \mathbf{P}^n = \begin{bmatrix} 1-p^{(n)} & p^{(n)} \\ q^{(n)} & 1-q^{(n)} \end{bmatrix}$$

where $p^{(n)} \triangleq \frac{p-p(1-p-q)^n}{p+q}$, $q^{(n)} \triangleq \frac{q-q(1-p-q)^n}{p+q}$, $n \in \mathbb{N}^+$



Binary Markov process (Fig. 4)

Problem Formulation

• Definition of UoI

• Original definition [4]

$$U(t) = - \sum_{i \in \{0,1\}} P[S(t) = i | S(t')] \log_2 P[S(t) = i | S(t')] \quad (4)$$

where $t' \triangleq \max_i \{G_i : R_i \leq t\}$ is the time stamp of the most recently received update

• A piecewise function

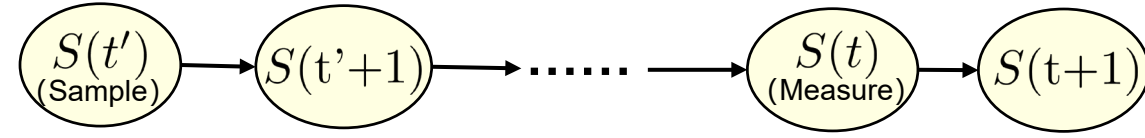
$$U(t) = - \sum_{i \in \{0,1\}} P[S(t) = i | S(G_i)] \log_2 P[S(t) = i | S(G_i)], \quad (5)$$

With (2)

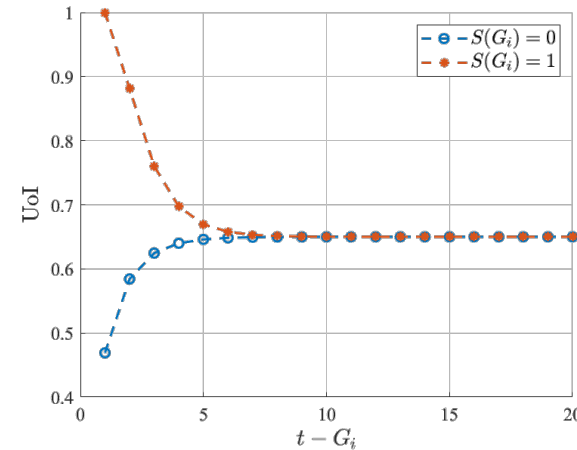
if $R_i \leq t < R_{i+1}, \forall i \in \mathbb{N}$.

$$U(t) = \begin{cases} H(p^{(t-G_i)}), & \text{if } S(G_i) = 0, R_i \leq t < R_{i+1}, \forall i \in \mathbb{N} \\ H(q^{(t-G_i)}), & \text{if } S(G_i) = 1, R_i \leq t < R_{i+1}, \forall i \in \mathbb{N}. \end{cases} \quad (6)$$

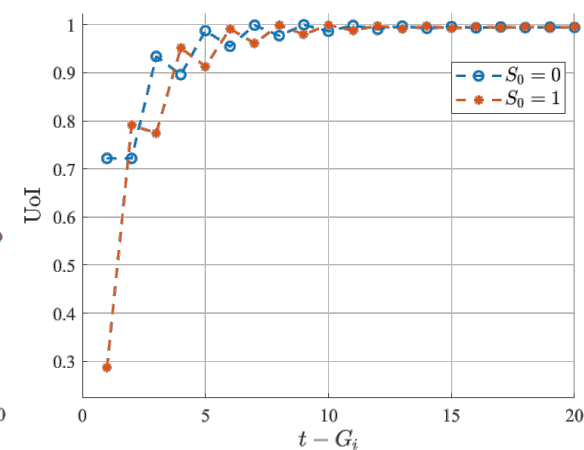
Aol indicates



Binary Markov process to explain (4) (Fig. 5)



$p=0.1, q=0.5$ (Fig. 6.a)



$p=0.8, q=0.95$ (Fig. 6.b)

UoI v.s. Aol based on (6), for $R_i \leq t < R_{i+1}$ (Fig. 6)



Problem Formulation

• Definition of Uol

• Original definition ^[4]

$$U(t) = - \sum_{i \in \{0,1\}} P[S(t) = i | S(t')] \log_2 P[S(t) = i | S(t')] \quad (4)$$

where $t' \triangleq \max_i \{G_i : R_i \leq t\}$ is the time stamp of the most recently received update

• A piecewise function

$$U(t) = - \sum_{i \in \{0,1\}} P[S(t) = i | S(G_i)] \log_2 P[S(t) = i | S(G_i)], \quad (5)$$

With (2)

if $R_i \leq t < R_{i+1}, \forall i \in \mathbb{N}$.

$$U(t) = \begin{cases} H(p^{(t-G_i)}), & \text{if } S(G_i) = 0, R_i \leq t < R_{i+1}, \forall i \in \mathbb{N} \\ H(q^{(t-G_i)}), & \text{if } S(G_i) = 1, R_i \leq t < R_{i+1}, \forall i \in \mathbb{N}. \end{cases} \quad (6)$$

Aol indicates

• Problem Formulation

• A sampling Policy (equivalence)

Time stamps of *sampling times* for each packet

Time stamps of *waiting times* for each packet

Because , given (Y_0, Y_1, \dots) ,

(G_1, G_2, \dots) depends only on (Z_1, Z_2, \dots)

• Goal



Solution

• Belief State

• Why introduce *Belief State*

The problem (7) \rightarrow a *POMDP* \rightarrow MDP(SMDP)

• Original Definition

The probability of $S(t)=1$ given the observation $S(t')$

$$\Omega(t) = P[S(t) = 1 | S(t')].$$

• A piecewise function

$$\Omega(t) = \begin{cases} p^{(t-G_i)}, & \text{if } S(G_i) = 0 \\ 1 - q^{(t-G_i)}, & \text{if } S(G_i) = 1. \end{cases} \quad (9)$$

• Lemma 1

Given $\Omega(t) = \omega \in \{p^{(n)}, 1 - q^{(n)}\}$, $\Omega(t + k)$ can be calculated by

$$\Omega(t + k) = \frac{p - p(1 - p - q)^k}{p + q} + \omega(1 - p - q)^k, \quad (10)$$

the right-hand side of which is denoted as $\tau^k(\omega)$

• Corollary 1

The *equilibrium belief state* of $\Omega(t)$ is

$$\omega^* \triangleq \lim_{k \rightarrow \infty} \tau^k(\omega) = \frac{p}{p + q}. \quad (11)$$



Solution

• Reformulation of Problem (7)

• Recall the goal

$$\inf_{\pi \in \Pi} \limsup_{T \rightarrow \infty} \frac{1}{T} \mathbb{E} \left[\sum_{t=0}^{T-1} U(t) \right] \quad (7)$$

• Combing (6) and (9) we have $U(t) = H(\Omega(t))$

$$U(t) = \begin{cases} H(p^{(t-G_i)}), & \text{if } S(G_i) = 0, R_i \leq t < R_{i+1}, \forall i \in \mathbb{N} \\ H(q^{(t-G_i)}), & \text{if } S(G_i) = 1, R_i \leq t < R_{i+1}, \forall i \in \mathbb{N}. \end{cases} \quad (6)$$

$$\Omega(t) = \begin{cases} p^{(t-G_i)}, & \text{if } S(G_i) = 0 \\ 1 - q^{(t-G_i)}, & \text{if } S(G_i) = 1. \end{cases} \quad (9)$$

• Reformulation

$$(7) \rightarrow \bar{p}_{\text{opt}} = \inf_{\pi \in \Pi} \limsup_{T \rightarrow \infty} \frac{1}{T} \mathbb{E} \left[\sum_{t=0}^{T-1} H(\Omega(t)) \right] \quad (13)$$

where \bar{p}_{opt} is the optimum value of

• Analysis for problem (13)

An infinite-horizon average cost **SMDP** [5, Chapter 11]



Solution

• An Optimal Sampling Policy

• Recall the goal→SMDP

$$\bar{p}_{\text{opt}} = \inf_{\pi \in \Pi} \limsup_{T \rightarrow \infty} \frac{1}{T} \mathbb{E} \left[\sum_{t=0}^{T-1} H(\Omega(t)) \right] \quad (13)$$

• Bellman optimality equation

$$V^*(\omega) = \inf_{Z_{i+1} \in \mathbb{N}} \left\{ \mathbb{E} \left[\sum_{k=0}^{Z_{i+1}+Y_{i+1}-1} (H(\tau^k(\omega)) - \bar{p}_{\text{opt}}) \right] \right. \\ \left. + \mathbb{E} \left[\tau^{Z_{i+1}+Y_{i+1}}(\omega) V^*(1 - q^{(Y_{i+1})}) \right] \right. \\ \left. + (1 - \tau^{Z_{i+1}+Y_{i+1}}(\omega)) V^*(p^{(Y_{i+1})}) \right\}, \quad (18)$$

Depends on Z_{i+1} ,
hard to solve

where $V^*(\omega)$ is the relative value function

• Transformation of (13) [5, Chapter 11]

$$(13) = \inf_{\pi \in \Pi} \lim_{n \rightarrow \infty} \frac{\sum_{i=0}^{n-1} \mathbb{E} \left[\sum_{k=0}^{Z_{i+1}+Y_{i+1}-1} H(\tau^k(\omega)) \right]}{\sum_{i=0}^{n-1} \mathbb{E}[Z_{i+1} + Y_{i+1}]}$$

↓

$$g(\beta) \triangleq \inf_{\phi \in \Phi} \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=0}^{n-1} \mathbb{E} \left[\sum_{k=0}^{Z_{i+1}+Y_{i+1}-1} [H(\tau^k(\omega)) - \beta] \right]. \quad (33)$$

SMDP→MDP

• Two assertions [6]

(i) $\bar{p}_{\text{opt}} \leq \beta$ if and only if $g(\beta) \leq 0$

(ii) \bar{p}_{opt} is the unique root of $g(\beta) = 0$

[5] M. L. Puterman, *Markov Decision Processes: Discrete Stochastic Dynamic Programming*. Hoboken, NJ, USA: Wiley, 2005.

[6] W. Dinkelbach, "On nonlinear fractional programming," *Manag. Sci.*, vol. 13, no. 7, pp. 492–498, 1967.



Solution

• An Optimal Sampling Policy

- Recall the goal $\text{SMDP} \rightarrow \text{MDP} \rightarrow g(\beta) = 0$

$$g(\beta) \triangleq \inf_{\phi \in \Phi} \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=0}^{n-1} \mathbb{E} \left[\sum_{k=0}^{Z_{i+1}+Y_{i+1}-1} [H(\tau^k(\omega)) - \beta] \right] \quad (33)$$

- Optimality equation

$$g(\beta) + \tilde{V}(\omega, \beta) = \inf_{Z_{i+1} \in \mathbb{N}} \{c(\omega, Z_{i+1}, \beta) + r(\omega, Z_{i+1}, \beta)\}, \quad (38)$$

$$g(\beta) = \inf_{Z_{i+1} \in \mathbb{N}} \{c(p, Z_{i+1}, \beta) + r(p, Z_{i+1}, \beta)\}, \quad (39)$$

where

$$c(\omega, Z_{i+1}, \beta) = \mathbb{E} \left[\sum_{k=0}^{Z_{i+1}+Y_{i+1}-1} (H(\tau^k(\omega)) - \beta) \right]$$

$$r(\omega, Z_{i+1}, \beta) = \mathbb{E} \left[\tau^{Z_{i+1}+Y_{i+1}}(\omega) \tilde{V}(1 - q^{(Y_{i+1})}, \beta) \right. \\ \left. + (1 - \tau^{Z_{i+1}+Y_{i+1}}(\omega)) \tilde{V}(p^{(Y_{i+1})}, \beta) \right]$$

- Algorithm 1

Algorithm 1: Bisec-RVI algorithm

Input: $l = 0, u = 1$, tolerance $\epsilon > 0$

```

1 while  $u - l \geq \epsilon$  do
2    $\beta := (l + u)/2$ ;
3   Run RVI to solve  $g(\beta)$  and  $\tilde{V}(\omega, \beta)$ ;
4   if  $g(\beta) > 0$  then
5      $l := \beta$ ;
6   else
7      $u := \beta$ ;

```

Inner layer

Outer layer

Output: $\bar{p}_{\text{opt}} = \beta$

Find $g(\beta) = 0$, Get the *optimal sampling policy* at the same time



Solution

• A Sub-optimal Index-based Policy

• Motivation for sub-optimal policy

1. *High computing complexity* caused by executing the RVI algorithm repeatedly
2. *Large delays* are common in communication networks, e.g., SAGSIN integrated network ^[3]

• Explanation for Large delays

The transmission process spends a long time, i.e., the value of $E[Y_i]$, $\forall i \in \mathbb{N}$ is large enough

• Core assumption

Constant state transition probabilities (Corollary 1)

$$P[\Omega(R_{i+1}) = 1 - q^{(Y_{i+1})} | \Omega(R_i)] = \tau^{Z_{i+1} + Y_{i+1}}(\omega) = \omega^* \quad (19)$$

• Corollary 1 (Recall)

The *equilibrium belief state* of $\Omega(t)$ is

$$\omega^* \triangleq \lim_{k \rightarrow \infty} \tau^k(\omega) = \frac{p}{p + q}. \quad (11)$$



Solution

• A Sub-optimal Index-based Policy

• Goal

$$\bar{p}_{\text{nopt}} = \inf_{\psi \in \Pi} \limsup_{T \rightarrow \infty} \frac{1}{T} \mathbb{E} \left[\sum_{t=0}^{T-1} H(\Omega(t)) \right] \quad (20)$$

• Bellman optimality equation

$$V_{\psi}(\omega) = \inf_{Z_{i+1} \in \mathbb{N}} \left\{ \mathbb{E} \left[\sum_{k=0}^{Z_{i+1}+Y_{i+1}-1} (H(\tau^k(\omega)) - \bar{p}_{\text{nopt}}) \right] \right\} \\ + \mathbb{E} \left[\omega^* V_{\psi}(1 - q^{(Y_{i+1})}) + (1 - \omega^*) V_{\psi}(p^{(Y_{i+1})}) \right], \quad (24)$$

Not depends on Z_{i+1} , easy to solve

for all $\omega \in \{p^{(n)}, 1 - q^{(n)}\}$, $n \in \mathbb{N}$, where $V_{\psi}^*(\omega)$ is the relative value function

• An index function

$$\eta(\omega) \triangleq \inf_{Z_i \in \mathbb{N}^+} \frac{1}{Z_i} \sum_{k=0}^{Z_i-1} \mathbb{E} [H(\tau^{k+Y_i}(\omega))] , \quad (21)$$

where $\omega \in \{p^{(n)}, 1 - q^{(n)}\}$, $n \in \mathbb{N}$

• An index-based policy

$$Z_{i+1}(\beta_{\psi}) = \min\{k \in \mathbb{N} : \eta(\Omega(t+k)) \geq \beta_{\psi}, t \geq R_i(\beta_{\psi})\}, \quad (22)$$

and β_{ψ} is the unique root of

$$\mathbb{E} \left[\sum_{R_i(\beta_{\psi})}^{R_{i+1}(\beta_{\psi})-1} H(\Omega(t)) \right] - \beta_{\psi} \mathbb{E} [R_{i+1}(\beta_{\psi}) - R_i(\beta_{\psi})] = 0 \quad (23)$$

β_{ψ} is exactly the optimum value of (20), i.e., $\beta_{\psi} = \bar{p}_{\text{nopt}}$



Solution

• A Sub-optimal Index-based Policy

• An index-based policy

$$Z_{i+1}(\beta_\psi) = \min\{k \in \mathbb{N} : \eta(\Omega(t+k)) \geq \beta_\psi, t \geq R_i(\beta_\psi)\},$$

$$(22)$$

and β_ψ is the unique root of

$$\mathbb{E} \left[\sum_{R_i(\beta_\psi)}^{R_{i+1}(\beta_\psi)-1} H(\Omega(t)) \right] - \beta_\psi \mathbb{E} [R_{i+1}(\beta_\psi) - R_i(\beta_\psi)] = 0$$

$$(23)$$

β_ψ is exactly the optimum value of (20), i.e., $\beta_\psi = \bar{p}_{\text{nopt}}$

• Notation

Rewrite (23) as

$$f(\beta_\psi) = f_1(\beta_\psi) - \beta_\psi f_2(\beta_\psi) = 0$$

$$(25)$$

• Algorithm 2

Algorithm 2: Bisec-index algorithm

Input: $l = 0, u = 1$, tolerance $\epsilon > 0$

```

1 while  $u - l \geq \epsilon$  do
2    $\beta_\psi := (l + u)/2$ ;
3    $c := f(\beta_\psi) = f_1(\beta_\psi) - \beta_\psi f_2(\beta_\psi)$ ;
4   if  $c > 0$  then
5      $l := \beta_\psi$ ;
6   else
7      $u := \beta_\psi$ ;

```

Output: $\bar{p}_{\text{nopt}} = \beta_\psi$

Find *the zero point of (25)*



Numerical Result

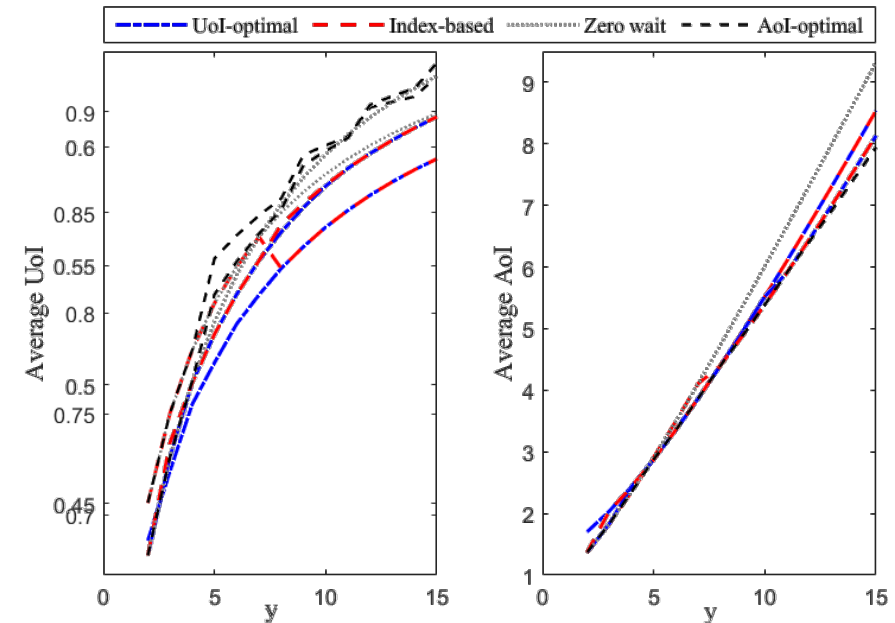
• Benchmark

- **Zero wait:** An update is transmitted once the previous update is received, maximizing throughput
- **Aol-optimal:** The Aol-optimal policy determines waiting time Z_i by [7, Theorem 4] and [7, Algorithm 2]

• Insight

1. The average UoI obtained by the index-based policy: The UoI-optimal policy does not always take future circumstances into account if the value of $F(Y_i)$ is small; thus ignores the oscillation of UoI when $p + q > 1$
2. The lowest Aol is obtained by Aol-optimal policy

• Results Analysis



The dynamics of the Markov source depicted as $p = 0.7$ and $q = 0.2$ (Fig. 9.b)

Average UoI and Average Aol v.s. y with i.i.d random delay, where $P[Y_i = 1] = 0.8$ and $P[Y_i = y] = 0.2$ (Fig. 9)

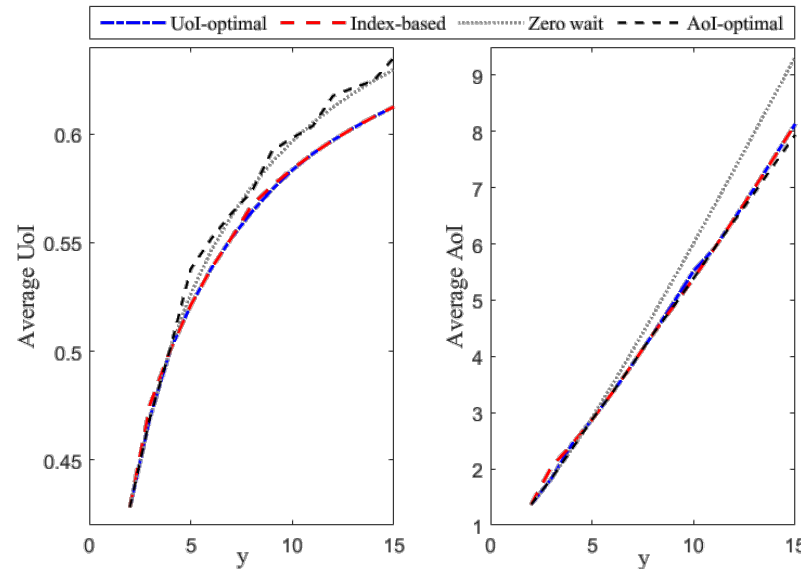


Numerical Result

● Insight

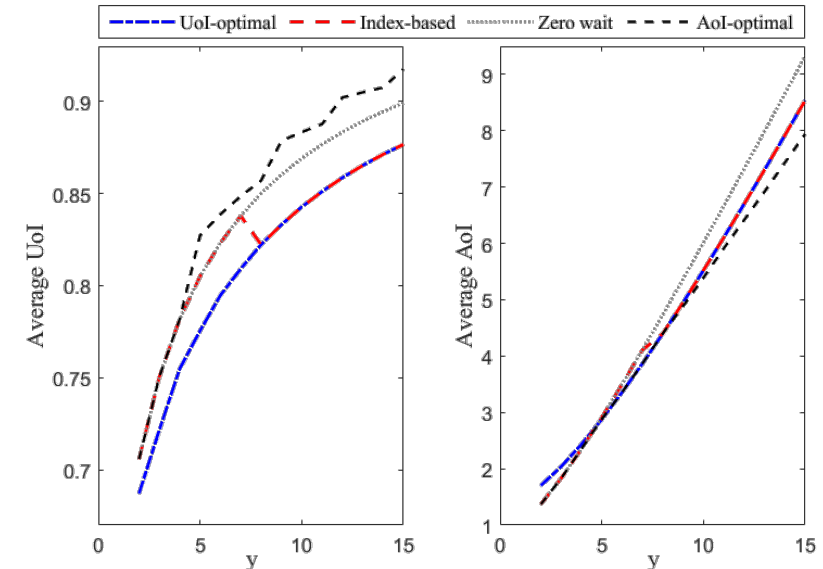
1. When $E(Y_i)$ is large enough, the result of *index policy* is the same as that of the *UoI-optimal policy*, consistent with the theoretical induction we proposed before
2. The performance *gains of UoI-optimal* and *index-based policies* are close to *the best average AoI*, making them better choices when the system aims to optimize both AoI and UoI simultaneously.

● Result Analysis



The dynamics of the Markov source depicted as $p = 0.05$ and $q = 0.2$ (Fig. 9.a)

Average UoI and Average AoI v.s. y with i.i.d random delay, where $P[Y_i = 1] = 0.8$ and $P[Y_i = y] = 0.2$ (Fig. 9)



The dynamics of the Markov source depicted as $p = 0.7$ and $q = 0.95$ (Fig. 9.b)



Conclusion

- **Brief summary of results**
 - Using Uol to estimate the value of information in a remote monitoring system
 - **An optimal sampling policy**, to minimize the time-average expected sum-Uol by two-layered *bisec-RVI* algorithm
 - **A sub-optimal index-based sampling policy**, owning lower computing complexity than the optimal one
 - **Both of the proposed sampling policies** outperform the zero wait policy and the Aol-optimal policy



VLEO Satellite Constellation Design for Regional Coverage of Aviation and Marine Users

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THANKS!