BOUNCER REPORT

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Contents

Section 1

Introduction

The aim of this report is to model the motion of a steel ball rolling down a PVC tube, projecting through the air, and bouncing twice. The main value to be calculated is d_t, the horizontal distance between the end of the tube and the second bounce. I will calculate this using three separate methods, each with differing assumptions and levels of accuracy, with methods 2 and 3 building off the basic method 1. I will then submit the 3 values of d_t to be analysed in the real-life experiment.

Methods

The same experimental setup will be used in each of the three methods, using the same starting parameters. Angle (**θ**) = 42˚ and h_2 = 0.82m. Figure 1 shows the basic setup.

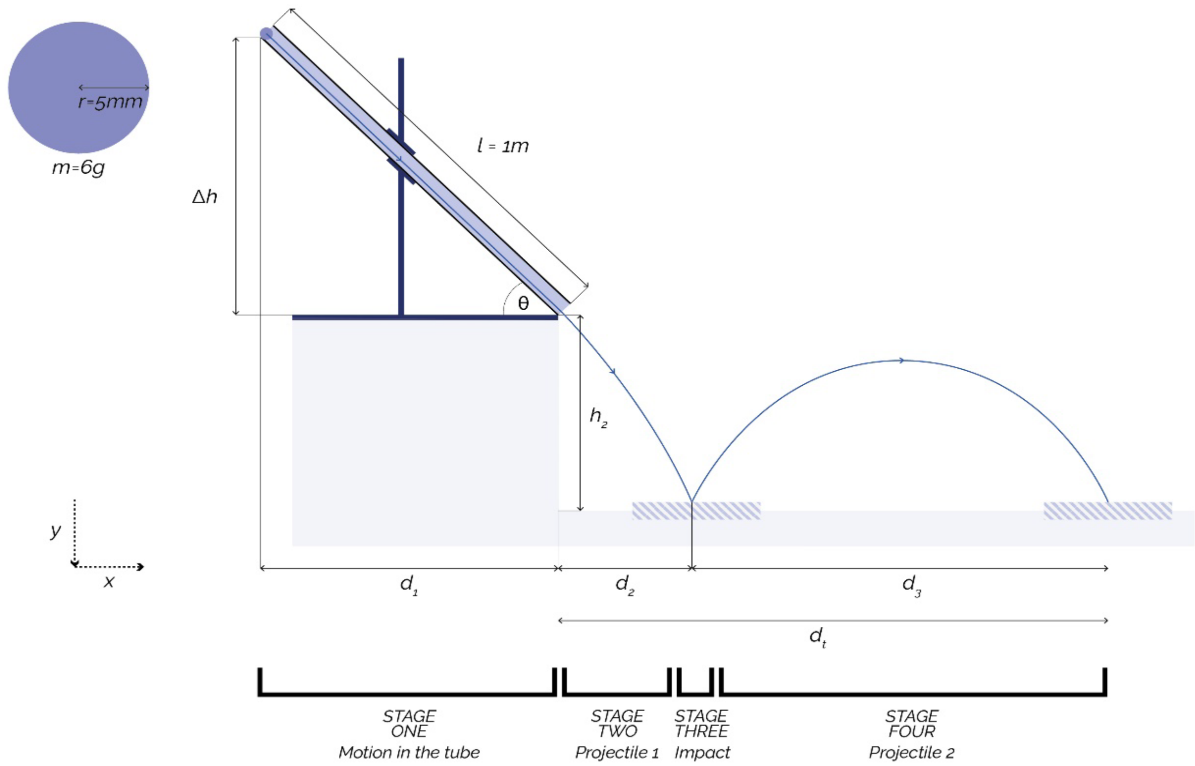

Figure 1 – Experimental Setup

Method 1 – Ideal

I will be assuming no loss of energy throughout this method, so I predict that it will be least accurate. I will be using g = 9.8ms^{-2} as this will be the simplest method, so it would be fitting to use a less precise value of g. Please see appendix for MATLAB code.

STAGE 1 : Motion in the tube

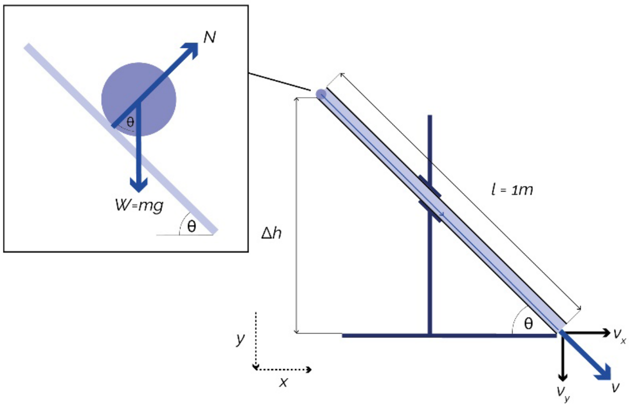

Figure 2

Using the principle of conservation of energy. I am assuming the ball is at rest at the top of the tube, so the only energy it possesses at the start is GPE. The entire change in GPE will all be transferred into kinetic energy by the time the ball reaches the bottom of the tube, as I am assuming there is no air resistance or friction with the surface of the tube (as shown in figure 2).

$$\Delta E_{GPE} = E_{KE}$$

$$where \ldots \quad \Delta E_{GPE} = mg\Delta h \;\; and \;\; E_{KE} = \frac{1}{2}mv^2$$

$$g\Delta h = \frac{1}{2}v^2$$

Finding the change in height, using trigonometry.

$$\Delta h = h_1 = lsin\theta = sin\theta$$

Inputting the change in height into the energy equation and solving for final velocity of the ball bearing at the end of the tube.

$$gsin\theta = \frac{1}{2}(v)^2$$

$$v = \sqrt{2gsin\theta}$$

$$velocity\ when\ exiting\ the\ tube = \sqrt{2(9.8)\sin(42)} = 3.6214 \;\; \boldsymbol{ms^{-1}}$$

Resolving exiting velocity into x (horizontal) and y (vertical) components.

$$v_x = vcos\theta = \left(\sqrt{2(9.8)\sin(42)}\right)cos42 = 2.6913\, \boldsymbol{ms^{-1}}$$

$$v_y = vsin\theta = \left(\sqrt{2(9.8)\sin(42)}\right)sin42 = 2.4232\, \boldsymbol{ms^{-1}}$$

STAGE 2 : Projectile Motion 1

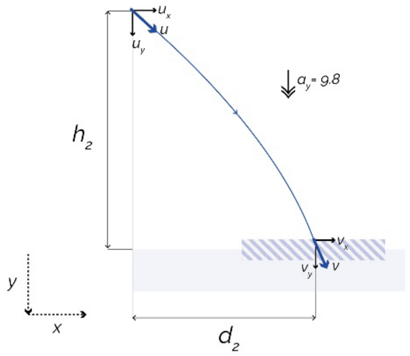

Figure 3

1. Calculating final velocity at the end of the partial parabola.

   Calculating final *vertical* velocity at the end of this stage (v_y), see figure 3, using SUVAT. I am assuming that there is no air resistance, so the vertical acceleration will be exactly 9.8ms^{-2}.

$s_y = h_2 = 0.82\,m \qquad u_y = 2.4232\,ms^{-1} \qquad v_y = ? \qquad a_y = 9.8\,ms^{-2} \qquad t = ?$

$$using \ldots \ (v_y)^2 = (u_y)^2 = 2a_y s_y$$

$$v_y = \sqrt{(2.4232)^2 + 2(9.8)(0.82)} = 4.6844\,\boldsymbol{ms}^{-1}$$

   Calculating final *horizontal* velocity at the end of this stage (v_x), using SUVAT. I am assuming that there is no air resistance, so the horizontal acceleration will be zero.

$s_x = ? \qquad u_x = 2.6913\,ms^{-1} \qquad v_x = ? \qquad a_x = 0 \qquad t = ?$

$$using \ldots \ v_x = u_x + a_x t$$

$$v_x = 2.6913 + 0 = 2.6913\,\boldsymbol{ms}^{-1}$$

2. Calculating d_2 using both horizontal and vertical SUVAT.

   Using vertical motion to find time, t

$s_y = h_2 = 0.82\,m \qquad u_y = 2.4232\,ms^{-1} \qquad v_y = 4.6844\,ms^{-1} \qquad a_y = 9.8\,ms^{-2} \qquad t = ?$

$$using \ldots \ v_y = u_y + a_y t$$

$$t = \frac{4.6844 - 2.4232}{9.8} = 0.2307s$$

   Using the value of t to calculate *horizontal displacement.*

$s_x = d_2 = ? \qquad u_x = 2.6913\,ms^{-1} \qquad v_x = 2.6913\,ms^{-1} a_x = 0 \qquad t = 0.2307s$

$$using \ldots \ s_x = u_x t + \frac{1}{2} a_x t^2$$

$$s_x = d_2 = (2.6913)(0.2307) + 0 = 0.6209\,\boldsymbol{m}$$

STAGE 3 : Impact

In this method I am assuming no energy loss due to the first impact with the ground. In other words, I am taking the coefficient of restitution (e) as equal to 1 so it is a perfectly elastic collision. The only thing that changes is the direction of vertical velocity, see figure 4.

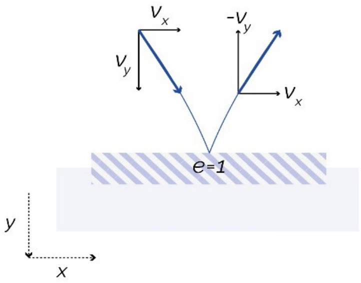

Figure 4

$$v_y = 4.6844\,ms^{-1} \rightarrow -4.6844\,\boldsymbol{ms}^{-1}\ (now\ vertically\ upwards)$$

$$v_x = 2.6913\,\boldsymbol{ms}^{-1}\ (stays\ the\ same)$$

STAGE 4 : Projectile Motion 2

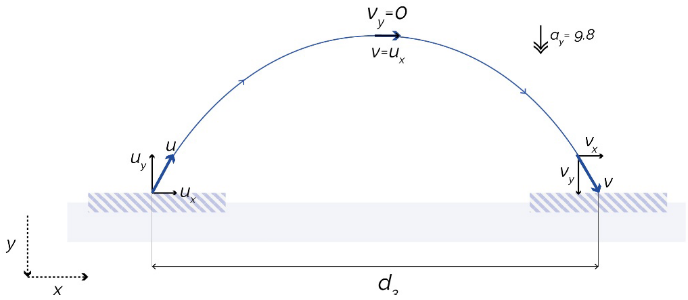

Figure 5

Calculating d_3 using both horizontal and vertical SUVAT, see figure 5. I am assuming no air resistance, so vertically, acceleration is exactly 9.8 ms^{-2} and horizontally, acceleration is zero.

Using vertical motion to calculate time, t.

$$s_y = ? \quad u_y = -4.6844\,ms^{-1} \quad v_y = 0\,ms^{-1} \quad a_y = 9.8\,ms^{-2} \quad t = ?$$

$$using\ \ldots\quad v_y = u_y + a_y t$$

$$t = \frac{0 - (-4.6844)}{9.8} = 0.4780s$$

Using the value of t to calculate horizontal displacement.

$$s_x = d_3 \quad u_x = 2.6913\,ms^{-1} \quad v_x = ? \quad a_x = 0 \quad t = 2(0.4780)s$$

$$using\ \ldots\quad s_x = u_x t + \frac{1}{2} a_x t^2$$

$$s_x = d_3 = (2.6913)(2(0.4780)) + 0 = 2.5729\,\boldsymbol{m}$$

Value for d_t

$$d_t = d_2 + d_3$$

$$d_t = 0.6209 + 2.5729 = 3.1938m = 319.4\boldsymbol{cm}\ (1\boldsymbol{dp})$$

Method 2 – Considering slipping friction & energy loss

This method should be more accurate than method 1, as I will be assuming more energy loss. I will be using g as 9.81ms^{-2}, a higher degree of accuracy than method 1. Please see appendix for MATLAB code.

STAGE 1 : Motion in the tube

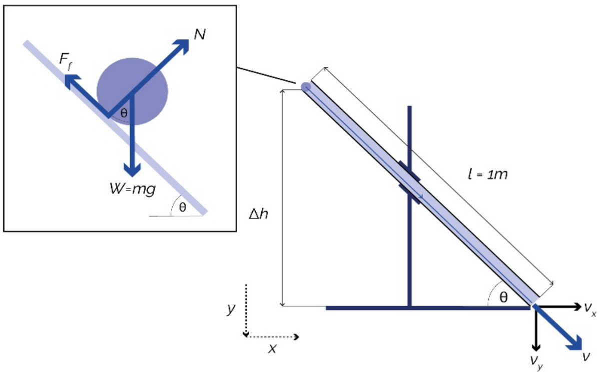

Figure 6

This time I am assuming that there will be some energy loss due to friction in the tube, using the principle of conservation of energy.

$$\Delta E_{GPE} = E_{KE} + E_f$$

$$where \ldots \quad \Delta E_{GPE} = mg\Delta h \;\; and \;\; E_{KE} = \frac{1}{2}mv^2 \; and \; E_f \; must \; be \; calculated$$

Calculating the work done by friction using the coefficient of kinetic/dynamic friction (sliding, dry) of plastic against metal (μ) as 0.2 (Coefficient of friction, Rolling resistance and Aerodynamics, n.d.)

1. Calculating frictional force on the ball bearing as it slides

$$friction\,force = F_f = \mu N$$
$$using \ldots N = mgcos(\theta)$$
$$F_f = \mu(mgcos(\theta))$$

2. Calculating work done from frictional force

$$work\ done = force \times distance\ along\ action\ line\ of\ the\ force$$
$$E_f = F_f \times l$$
$$E_f = \mu(mgcos(\theta))$$

Inputting the change in height (calculated in method 1) and work done by friction into the energy equation and solving for final velocity of the ball bearing at the end of the tube.

$$mgsin\theta = \frac{1}{2}m(v)^2 + \mu(mgcos(\theta))$$

$$v = \sqrt{2\left(gsin\theta - \mu\left(gcos(\theta)\right)\right)}$$

$$velocity\ when\ exiting\ the\ tube = \sqrt{2((9.81)\sin(42) - 0.2(9.81\cos(42)))} = 3.1957\boldsymbol{ms^{-1}}$$

Resolving exiting velocity into x (horizontal) and y (vertical) components.

$$x = vcos\theta = \left(\sqrt{2((9.81)\sin(42) - 0.2(9.81\cos(42)))}\right)cos42 = 2.3748\ \boldsymbol{ms^{-1}}$$

$$v_y = vsin\theta = \left(\sqrt{2((9.81)\sin(42) - 0.2(9.81\cos(42)))}\right)sin42 = 2.1383\ \boldsymbol{ms^{-1}}$$

STAGE 2 : Projectile Motion 1

This stage will be largely the same as method 1, just with a more accurate value of g and smaller starting velocity, see figure 7.

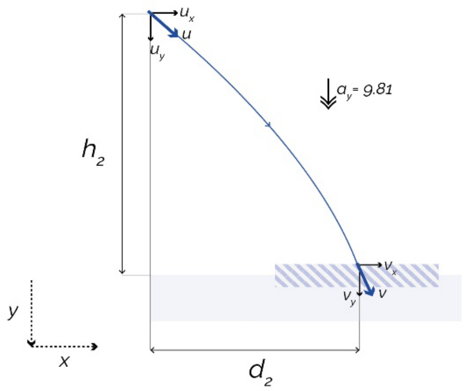

Figure 7

1. Calculating final velocity at the end of the partial parabola.

   Calculating final *vertical* velocity at the end of this stage (v_y), using SUVAT. I am assuming that there is no air resistance, as it is negligible with a ball of small mass and diameter, so the vertical acceleration will be exactly 9.81ms^{-2}.

   $s_y = h_2 = 0.82\ m \quad u_y = 2.1383\ ms^{-1} \quad v_y = ? \quad a_y = 9.81\ ms^{-2} \quad t = ?$

   $$using\ ...\ (v_y)^2 = (u_y)^2 = 2a_y s_y$$

   $$v_y = \sqrt{(2.1383\ )^2 + 2(9.81)(0.82)} = 4.5454\ \boldsymbol{ms^{-1}}$$

   Calculating final *horizontal* velocity at the end of this stage (v_x), using SUVAT. I am assuming that there is no air resistance, so the horizontal acceleration will be zero.

   $s_x = ? \quad u_x = 2.3748\ ms^{-1} v_x = ? \quad a_x = 0 \quad t = ?$

   $$using\ ...\ v_x = u_x + a_x t$$

   $$v_x = 2.3748 + 0 = 2.3748\ \boldsymbol{ms^{-1}}$$

2. Calculating d_2 using both horizontal and vertical SUVAT.

Using vertical motion to find time, t

$$s_y = h_2 = 0.82\,m \quad u_y = 2.1383\,ms^{-1} \quad v_y = 4.5454\,ms^{-1} \quad a_y = 9.81\,ms^{-2} \quad t = ?$$

$$using \ldots\ v_y = u_y + a_y t$$

$$t = \frac{4.5454 - 2.1383}{9.81} = 0.2454s$$

Using the value of t to calculate *horizontal displacement*.

$$s_x = d_2 = ? \quad u_x = 2.3748\,ms^{-1} \quad v_x = 2.3748\,ms^{-1} \quad a_x = 0 \quad t = 0.2454s$$

$$using \ldots\ s_x = u_x t + \frac{1}{2} a_x t^2$$

$$s_x = d_2 = (2.3748)(0.2454) + 0 = 0.5828\,\boldsymbol{m}$$

STAGE 3 : Impact

I will assume energy loss due to the impact, with a coefficient of restitution (e) of (exactly) 0.6557, meaning this is a plastic-elastic collision. I will assume that this will only effect vertical velocity (perpendicular to plane of contact of the ball and the ground), see figure 8. I am also assuming that the earth does not move as a result of the collision.

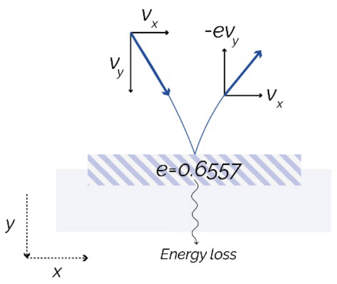

Figure 8

$$using \ldots\ e = \frac{-(velocity\ after\ collision)}{velocity\ before\ collision}$$

$$v_y = -(0.6557)(4.5454) = -2.9804\boldsymbol{ms^{-1}}\ (a\ smaller\ magnitude\ vertically\ upwards)$$

$$v_x = 2.3748\,\boldsymbol{ms^{-1}}\ (stays\ the\ same)$$

STAGE 4 : Projectile Motion 2

This stage will be largely the same as method 1, just with a more accurate value of g and smaller starting velocity, see figure 9.

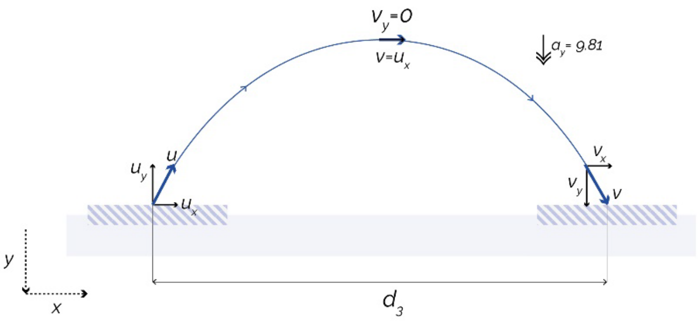

Figure 9

Calculating d_3 using both horizontal and vertical SUVAT. I am assuming there is negligible air resistance, so vertically, acceleration is exactly 9.81 ms^{-2} and horizontally, acceleration is zero.

Using vertical motion to calculate time, t.

$s_y = ?$ $\quad u_y = -2.9804\,ms^{-1}$ $\quad v_y = 0\,ms^{-1}$ $a_y = 9.81\,ms^{-2}$ $\quad t$
$= ?$

$$using \ldots \quad v_y = u_y + a_y t$$

$$t = \frac{0-(-2.9804)}{9.81} = 0.3038s$$

Using the value of t to calculate horizontal displacement.

$s_x = d_3$ $\quad u_x = 2.3748\,ms^{-1}$ $\quad v_x$
$= ?$ $\quad a_x = 0$ $\quad t = 2(0.3038)s$

$$using \ldots \quad s_x = u_x t + \frac{1}{2}a_x t^2$$

$$s_x = d_3 = (2.3748)\big(2(0.3038)\big) + 0 = 1.4429\,\boldsymbol{m}$$

Value for d_t

$$d_t = d_2 + d_3$$

$$d_t = 0.5828 + 1.4429 = 2.0257m = 202.6\boldsymbol{cm}\ (1\boldsymbol{dp})$$

Method 3 – Considering rotation & energy loss

This method should be the most accurate, as I will be using a model that is most likely to reflect reality. The ball will be rolling in the tube rather than slipping, as seen if figure 10. I will be using g as 9.80665 ms^{-2} (Fundamental Physical Constants, 2018), a higher degree of accuracy than method 1 and method 2. Please see the appendix for MATLAB code.

STAGE 1 : Motion in the tube

Using the principle of conservation of energy, assuming that all gravitational potential energy is transferred to the sum of liner and rotational kinetic energy, as the ball is rolling in the tube.

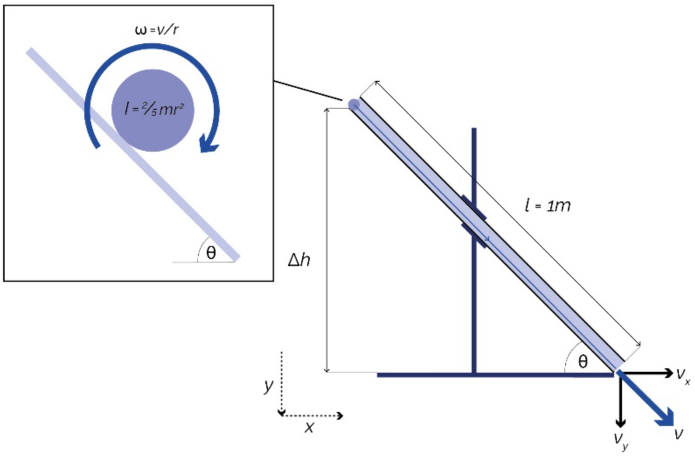

Figure 10

$$\Delta E_{GPE} = E_{KE(linear)} + E_{KE(rotational)}$$

$$where \ldots \quad \Delta E_{GPE} = mg\Delta h \quad and \quad E_{KE(linear)} = \frac{1}{2}mv^2 \quad and \quad E_{KE(rotational)} \; must\ be\ calculated$$

Calculating rotational kinetic energy, using moment of inertia and angular velocity (assuming that the ball is rolling without slipping)

$$E_{KE(rotational)} = \frac{1}{2}I\omega^2\mathrm{m}$$

$$using \ldots \quad I = \frac{2}{5}mr^2 \quad and \quad \omega = \frac{v}{r}$$

$$E_{KE(rotational)} = \frac{1}{5}mv^2$$

Inputting the value of rotational kinetic energy and the change in height (calculated in method 1) into the original energy equation, simplifying and finding the value of the linear velocity (v) of the centre of mass at the bottom of the tube.

$$mgsin\theta = \frac{1}{2}mv^2 + \frac{1}{5}mv^2$$

$$v = \sqrt{\frac{10gsin\theta}{7}}$$

$$v = \sqrt{\frac{10(9.80665)(sin42)}{7}} = 3.06173\ \boldsymbol{ms^{-1}}$$

Resolving exiting velocity into x (horizontal) and y (vertical) components.

$$v_x = vcos\theta = \left(\sqrt{\frac{10(9.80665)(sin42)}{7}}\right)cos42 = 2.27531\boldsymbol{ms}^{-1}$$

$$v_y = vsin\theta = \left(\sqrt{\frac{10(9.80665)(sin42)}{7}}\right)sin42 = 2.04870\ \boldsymbol{ms}^{-1}$$

STAGE 2 : Projectile Motion 1

This stage will be largely the same as method 1 and 2, just with a more accurate value of g and different starting velocity, see figure 11.

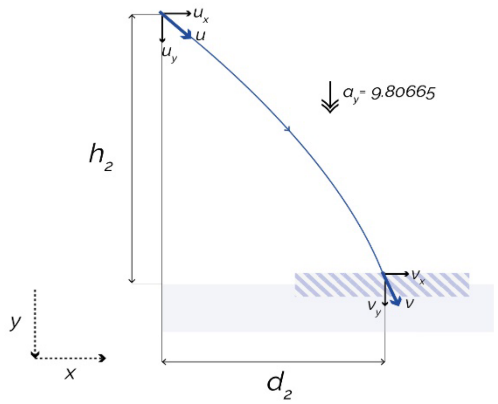

Figure 11

1. Calculating final velocity at the end of the partial parabola.

   Calculating final *vertical* velocity at the end of this stage (v_y), using SUVAT. I am assuming that there is no air resistance, as it is negligible with a ball of diameter 1cm, so the vertical acceleration will be exactly 9.80665 ms^{-2}.

$$s_y = h_2 = 0.82\ m \qquad u_y = 2.04870\ ms^{-1} \qquad v_y = ? \qquad a_y = 9.80665\ ms^{-2} \qquad t = ?$$

$$using\ ...\ \ (v_y)^2 = \ (u_y)^2 = 2a_y s_y$$

$$v_y = \ \sqrt{(2.04870)^2 + 2(9.80665)(0.82)} = 4.50334\ \boldsymbol{ms}^{-1}$$

   Calculating final *horizontal* velocity at the end of this stage (v_x), using SUVAT. I am assuming that there is no air resistance, so the horizontal acceleration will be zero.

$$s_x =? \qquad u_x = 2.27531\ ms^{-1} \qquad v_x = ? \qquad a_x = 0 \qquad t = ?$$

$$using\ ...\ \ v_x = u_x + a_x t$$

$$v_x = 2.27531 + 0 = \ 2.27531\ \boldsymbol{ms}^{-1}$$

2. Calculating d_2 using both horizontal and vertical SUVAT.

Using vertical motion to find time, t

$$s_y = h_2 = 0.82\, m \quad u_y = 2.04870\, ms^{-1} \quad v_y = 4.50334\, ms^{-1} \quad a_y = 9.80665\, ms^{-2} \quad t = ?$$

$$using \ldots \; v_y = u_y + a_y t$$

$$t = \frac{4.50334 - 2.04870}{9.80665} = 0.25030s$$

Using the value of t to calculate *horizontal displacement*.

$$s_x = d_2 = ? \quad u_x = 2.27531\, ms^{-1} \quad v_x = 2.27531\, ms^{-1} \quad a_x = 0 \quad t = 0.25030s$$

$$using \ldots \; s_x = u_x t + \frac{1}{2} a_x t^2$$

$$s_x = d_2 = (2.27531)(0.25030) + 0 = 0.56951\, \boldsymbol{m}$$

STAGE 3 : Impact

I will use the same method as method 2, with the same assumptions but a different starting velocity, see figure 12.

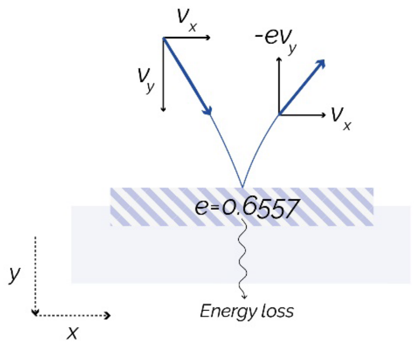

Figure 12

$$using \ldots \quad e = \frac{-(velocity\ after\ collision)}{velocity\ before\ collision}$$

$$v_y = -(0.6557)(4.50334) = -2.95284\boldsymbol{ms}^{-1} \quad (now\ smaller\ magnitude\ vertically\ upwards)$$

$$v_x = 2.27531\, \boldsymbol{ms}^{-1}\ (stays\ the\ same)$$

STAGE 4 : Projectile Motion 2

This stage will be largely the same as method 1 and 2, just with a more accurate value of g and smaller starting velocity, see figure 13. The same assumptions have been made.

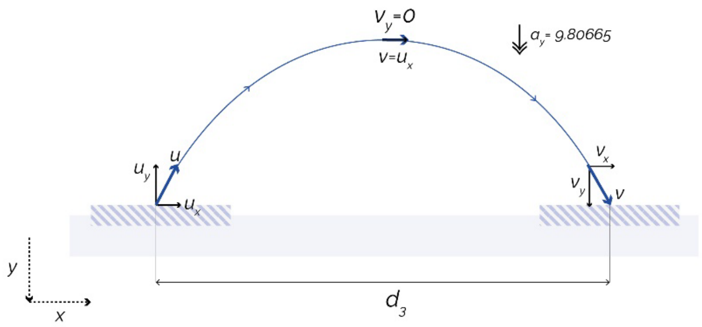

Figure 13

Calculating d_3 using both horizontal and vertical SUVAT.

Using vertical motion to calculate time, t.

$$s_y = ? \quad u_y = -2.95284\,ms^{-1} \quad v_y = 0\,ms^{-1} \quad a_y = 9.80665\,ms^{-2} \quad t = ?$$

$$using \ldots \quad v_y = u_y + a_y t$$

$$t = \frac{0 - (-2.95284)}{9.80665} = 0.30111s$$

Using the value of t to calculate horizontal displacement.

$$s_x = d_3 \quad u_x = 2.27531\ ms^{-1} \quad v_x = ? \quad a_x = 0 \quad t = 2(0.30111)s$$

$$using \ldots \quad s_x = u_x t + \frac{1}{2} a_x t^2$$

$$s_x = d_3 = (2.27531)\big(2(0.30111)\big) + 0 = 1.37024\,\boldsymbol{m}$$

Value for d_t

$$d_t = d_2 + d_3$$

$$d_t = 0.56951 + 1.37024 = 1.93975m = 194.0\,\boldsymbol{cm}(1\boldsymbol{dp})$$

Results

An overview of the results of all 3 methods, with an analysis of error before the experiment.

Table 1 –Results of all 3 methods

Method	Velocity at the end of stage 1 (ms^{-1})	Velocity at the end of stage 2 (ms^{-1})	Velocity at the end of stage 3 (ms^{-1})	Value of d_2 (m)	Value of d_3 (m)	Value of d_t (cm)
Method 1 – Ideal	v_x = 2.6913 v_y = 2.4232	v_x = 2.6913 v_y = 4.6844	v_x = 2.6913 v_y = -4.6844	0.6209	2.5729	319.4
Method 2 – Considering slipping friction & energy loss	v_x = 2.3748 v_y = 2.1383	v_x = 2.3748 v_y = 4.5454	v_x= 2.3748 v_y =-2.9804	0.5828	1.4429	202.6
Method 3 – Considering rotation & energy loss	v_x = 2.27531 v_y= 2.04870	v_x = 2.27531 v_y = 4.50334	v_x = 2.27531 v_y =-2.95284	0.56951	1.37024	194.0

Table 2 – Sensitivity Analysis

Variable associated with error	Range of value of variable	Lower bound would result in…	Upper bound would result in…
Coefficient of restitution (e) = 0.6557	0.6357 - 0.6757	Lower value for d_t	Higher value for d_t
Angle (**θ**) = 42°	41.5° - 42.5°	Method 1 – Lower value for d_t Method 2 & 3 – Higher value for d_t	Method 1 – Higher value for d_t Method 2 & 3 – Lower value for d_t
Length of tube (l) = 1m	0.995m – 1.005m	Lower value for d_t	Higher value for d_t
Height (h_2) = 0.82m	0.815m – 0.825m	Lower value for d_t	Higher value for d_t

To calculate the lower and upper bound for each method, I inputted the different values of the variables into my MATLAB code. I calculated percentage uncertainty for each method using the formula below:

$$percentage\ uncertainty = \frac{\frac{1}{2}(upper\ bound - lower\ bound)}{final\ value\ of\ d_t} \times 100$$

Table 3 – Error and uncertainty

Method	Lower Bound		Upper Bound		Uncertainty
	Values used	Value of d_t	Values used	Value of d_t	
Method 1 – Ideal	θ = 41.5 l = 0.995 h_2 = 0.815 e = 1	3.1803 m	θ = 42.5 l = 1.005 h_2 = 0.825 e = 1	3.2068 m	0.41 % or ±1.33 cm
Method 2 – considering slipping friction & energy loss	θ = 42.5 l = 0.995 h_2 = 0.815 e = 0.6357	1.9701 m	θ = 41.5 l = 1.005 h_2 = 0.825 e = 0.6757	2.0811 m	0.027 % or ±5.55 cm
Method 3 – considering rotation & energy loss	θ = 42.5 l = 0.995 h_2 = 0.815 e = 0.6357	1.8774 m	θ = 41.5 l = 1.005 h_2 = 0.825 e = 0.6757	2.0027 m	0.032 % or ±6.26 cm

Although I submitted each of the values for my 3 methods, I believe that my most accurate value will be 194 cm, as this was calculated using the most accurate method (method 3). I predict that the true value is 194 cm ± 6.26cm, as seen on the far right of figure 14.

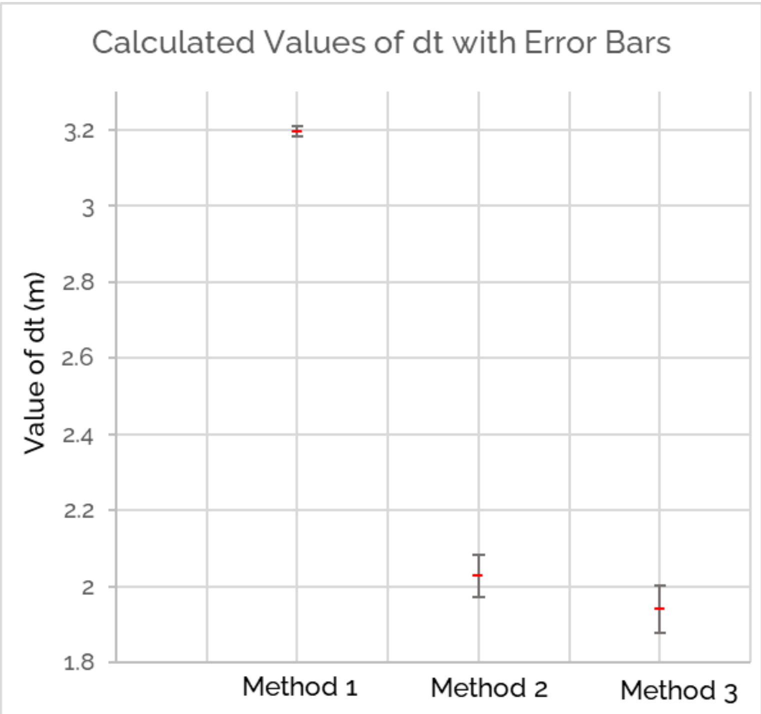

Figure 14

Section 2

This section will detail the results of the experiment, with an analysis of how accurate our predicted values were along with a discussion of sources of error both in the experiment and in our prediction methods.

Table 4-Values submitted by all group members

Group member	Prediction 1 (cm)	Prediction 2 (cm)	Prediction 3 (cm)
Tee	194.0	189.8	198.2
Tharany	194.0	189.9	198.2
Ciara	194.0	319.4 *	202.6
Bert	194.0	189.9	198.2
Zyque	194.0	183.2	205.1

* This value was not used in the experiment as it was an outlier. 194 cm was used twice instead.

All members of the group agreed that 194 cm would be the most accurate prediction, as it is the median and mode value.

Target Sheet

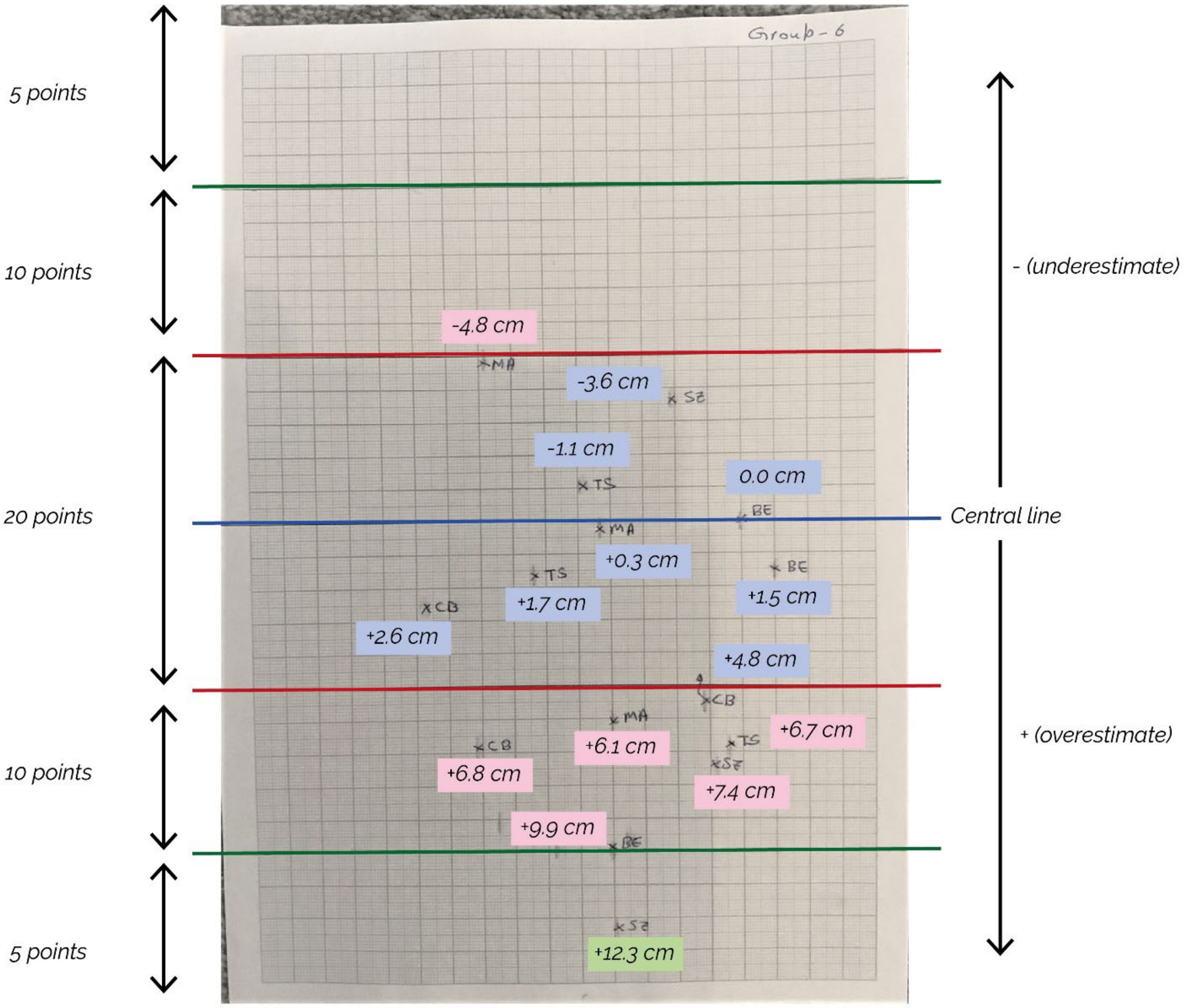

Figure 15

Table 5 – Average error calculations

Group Member	Error for each trial (cm)	Mean Error (cm)	Median Error (cm)
Tee	+ 6.7	+ 2.4	+ 1.7
	+ 1.7		
	- 1.1		
Tharany	- 4.8	+ 0.5	+ 0.3
	+ 6.1		
	+ 0.3		
Ciara	+ 2.6	+ 4.7	+ 4.8
	+ 6.8		
	+ 4.8		
Bert	+ 9.9	+ 3.8	+ 1.5
	+ 1.5		
	0.0		
Zyque	+ 12.3	+ 5.4	+ 7.4
	- 3.6		
	+ 7.4		
		Entire group mean: + 3.4	Entire group median: + 4.8

Both the mean and median for the entire group lies in the 20 points range, so, on average, all group members calculated a relatively accurate value of dt. It is also important to note that the average errors were positive, meaning as a group, we overestimated the value of dt. This is what was expected, as there were many sources of energy loss that we did not account for. There is an element of randomness in this experiment, as there was a large variation in error for the same values of dt, see figure 15.

Analysis of my results & error predictions

The error that I predicted was ± 6.26 cm for my method 3, my method with the highest error range. My mean and median error from the experiment fall within this range, fitting my prediction.

Discussion

The sources of error that we identified largely explain why our average values are an overestimate of the distance dt. The ball did not travel as far as we predicted because there were energy losses that we did not account for. This is most likely a combination of various sources of error which individually we decided were negligible in our calculations. We identified 7 main sources of error.

1. Air Resistance
   Like the rest of my group members, I assumed that the entire experiment was performed in a vacuum, with no drag or air resistance during the motion in the tube or outside of it. In reality, the ball will collide with air molecules, transferring energy to them as they gain velocity (or heat up). This happens due to the principle of conservation of momentum. This process is very complex to calculate as it varies with the velocity and shape of the ball. The effect of air resistance is slowing down the ball and decreasing the total horizontal distance that it can travel.

2. Friction in tube (assumption of rolling without slipping)
   When calculating the rotational kinetic energy, an assumption that must be made is that the ball is rolling without slipping. The motion of the ball may be a combination of rolling and slipping, with energy losses due to friction in the PVC tube. I was unable to find an accurate coefficient of friction for this specific type of motion between PVC and steel. If this were considered, some energy would have been lost through heating the tube by a very small amount and the value of dt would be smaller.

3. Value of g
   My group members and I used a variety of value of g. These values ranged from 9.8 ms^{-1} to 9.80665 ms^{-1}. This value was used many times in our calculations, so inaccuracies caused by the value chosen could end up being significant. There are minor changes in the value of g depending on location relative to the centre of mass of the earth.

4. Rotation during projectile motion and collision
   As the ball leaves the tube, it will still be rotating as it moves through the air. This could affect the aerodynamics of the ball and would also change the way that the ball bounces on the ground. The fact that it is rotating as it collides may mean that both horizontal and vertical velocity is affected by the collision. We have assumed that the ball moves perfectly in one smooth line as we are modelling the centre of mass. We also assumed that the plane of impact and plane of contact are exactly perpendicular, meaning only vertical velocity is affected.

5. Errors in experimental method

   Errors in setup (detailed in table 2 in section 1)
   When setting up the experiment, the starting parameters may not be exactly the values that we used in our calculations. This is likely due to parallax error and the precision of the measuring equipment used. The values that I am referring to are h2, l, theta and e.

   Error in measurement (ball landing in an area)
   The method of measuring where the ball landed involved printing using carbon paper onto the target sheet. The mark made by the ball covered an area rather than showing a specific point. The value recorded was an average, so therefore may not be exactly accurate. This is further explained in figure 16.

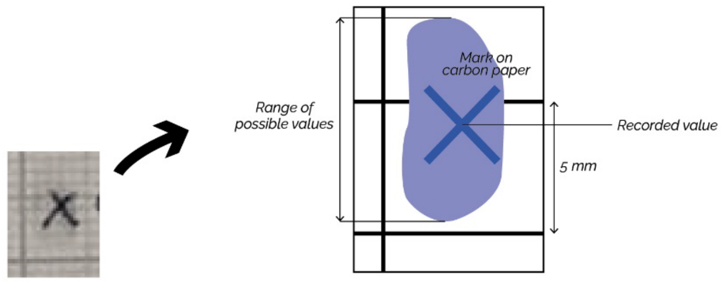

Figure 16

6. Rounding errors
   Using method 1 and 2, I rounded at each stage to 5 significant figures and using method 3 I rounded to 6 significant figures to increase accuracy. This will cause a slight positive or negative error and ideally, there would be no need for rounding at all. Even using my code in MATLAB, the program rounds the values of each of the variables each time they are calculated.

7. Path modelled vs path measured
   There is a slight difference between the path that we modelled and the path that was measured, as we were modelling the path of the centre of mass of the ball. This is explained visually in figure 17.

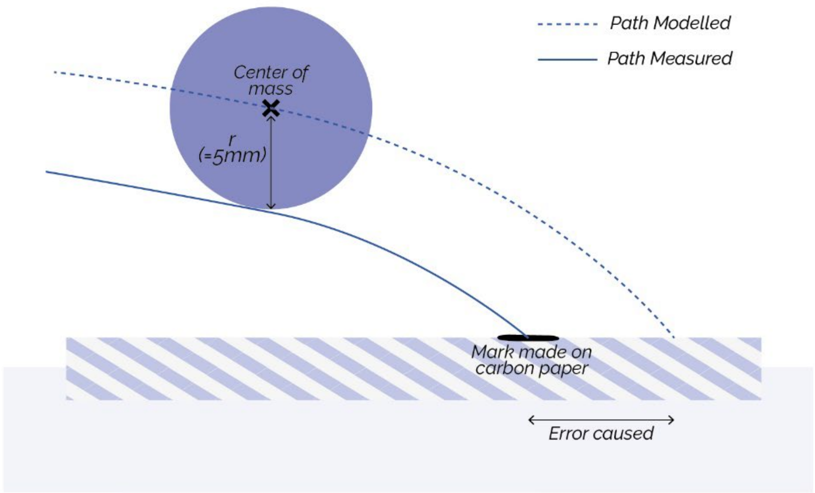

Figure 17

We also noticed that not all the points on the target sheet lie on one central straight line, meaning there was some motion in the z direction, a dimension that we did not include in our calculations. This change in path is random, hard to predict exactly, and changes between trials using the same parameters. This is explained visually in figured 18.

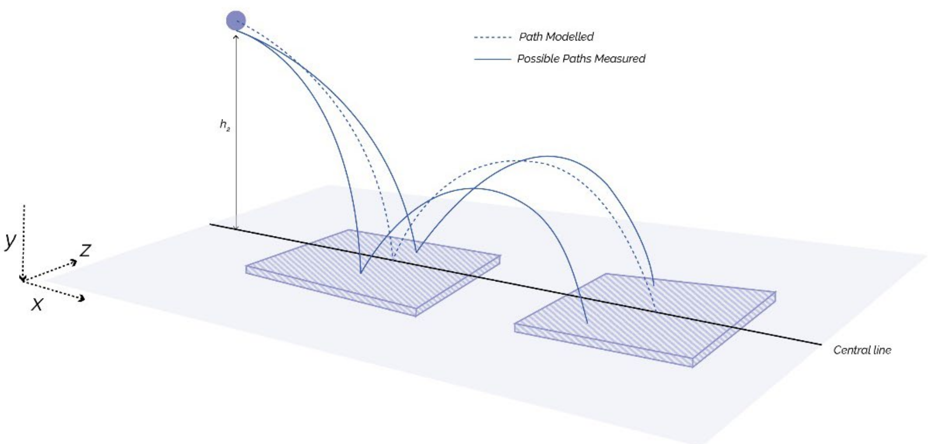

Figure 18

Analysis of my three methods

Method 1 – the value of 319.4cm was very inaccurate, so inaccurate that it was not used in the experiment. It was a very high overprediction because the method was oversimplified to not include any energy losses.

Method 2 – the value of 202.6 was much more accurate, with the use of a coefficient of restitution. However, because the ball was only assumed to be slipping down the plane, there was not any accounting for rotational energy. The value was an overprediction.

Method 3 – the value of 194.0 cm was the most accurate, as the method was more comprehensive; it included rotation and energy loss at the collision. There were still factors such as air resistance that were not accounted for, so this method is still an overprediction.

Conclusion

My group, on average, were able to model and predict the path of the bouncing ball with a high level of accuracy, with the majority of results being within 10 cm of the true value, even considering the ball's path changed between each trial. The sources of error were all individually quite insignificant and only ended up causing a slight overestimation of 3.4 cm, on average. If the calculations were to be done again, a model of air resistance should be included as well as compensation for the difference in path modelled versus the path measured.

Appendix

MATLAB code: Method 1

```
%----------------------------% VARIABLES %------------------------------%
theta = 42.5;
l = 1.005;
g = 9.8;
h2 = 0.825;

%----------------------------% STAGE 1 %------------------------------%
v1 = sqrt(2*g*l*sind(theta));
v1x = v1 * cosd(theta);
v1y = v1 * sind(theta);


%----------------------------% STAGE 2 %------------------------------%
v2y = sqrt((v1y^2) + (2*g*h2));
v2x = v1x;

t2 = (v2y - v1y)/g;
d2 = v2x * t2;

%----------------------------% STAGE 3 %------------------------------%
v3x = v2x;
v3y = -v2y;

%----------------------------% STAGE 4 %------------------------------%
t4 = (-v3y)/g;
d3 = (2*t4)*(v3x);

%---------------------------% FINDING Dt %------------------------------%
dt = d3 + d2
```

MATLAB code: Method 2

```
%----------------------------% VARIABLES %------------------------------%
theta = 41.5;
l = 1.005;
g = 9.81;
h2 = 0.825;
e = 0.6757;
friction = 0.2;

%-----------------------------% STAGE 1 %-------------------------------%
v1 = sqrt( 2*((g*l*sind(theta)) - (l*friction*g*cosd(theta))) );
v1x = v1 * cosd(theta);
v1y = v1 * sind(theta);


%-----------------------------% STAGE 2 %-------------------------------%
v2y = sqrt((v1y^2) + (2*g*h2));
v2x = v1x;

t2 = (v2y - v1y)/g;
d2 = v2x * t2;

%-----------------------------% STAGE 3 %-------------------------------%
v3x = v2x;
v3y = e*(-v2y);

%-----------------------------% STAGE 4 %-------------------------------%
t4 = (-v3y)/g;
d3 = (2*t4)*(v3x);

%----------------------------% FINDING Dt %-----------------------------%
dt = d3 + d2
```

MATLAB code: Method 3

```
%----------------------------% VARIABLES %------------------------------%
theta = 41.5;
l = 1.005;
g = 9.80665;
h2 = 0.825;
e = 0.6757;

%-----------------------------% STAGE 1 %-------------------------------%
v1 = sqrt((10/7)*(g*sind(42)));
v1x = v1 * cosd(theta);
v1y = v1 * sind(theta);


%-----------------------------% STAGE 2 %-------------------------------%
v2y = sqrt((v1y^2) + (2*g*h2));
v2x = v1x;

t2 = (v2y - v1y)/g;
d2 = v2x * t2;

%-----------------------------% STAGE 3 %-------------------------------%
v3x = v2x;
v3y = e*(-v2y);

%-----------------------------% STAGE 4 %-------------------------------%
t4 = (-v3y)/g;
d3 = (2*t4)*(v3x);

%----------------------------% FINDING Dt %-----------------------------%
dt = d3 + d2
```