University of Nottingham
Department of Mechanical, Materials and Manufacturing Engineering

MMME3086 (Computer Modelling Techniques)

MMME3086 Coursework submission template (NM) 2022/23

(Edit and Submit as PDF file)

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Module:	Computer Modelling Techniques
Coursework:	Numerical Methods (NM)

Task (A)

Student ID	Last digit of student ID	Resulting value of S_P
20260084	4	-100

A1

- Discretisation equation for 1st (leftmost) control volume:

$$\left(\frac{\lambda}{\delta x} - S_p \Delta x_B\right) T_1 + \left(-\frac{\lambda}{\delta x}\right) T_2 = S_c \Delta x_B + q_a$$

- Discretisation equation for nth (rightmost) control volume:

$$T_n = T_b$$

- Discretisation equation for internal control volume:

$$\left(-\frac{\lambda}{\delta x}\right) T_{i-1} + \left(2\frac{\lambda}{\delta x} - S_p \Delta x_B\right) T_i + \left(-\frac{\lambda}{\delta x}\right) T_{i+1} = S_c \Delta x_B$$

- Screen shot of matlab code for populating matrix **A** and vector **B**:

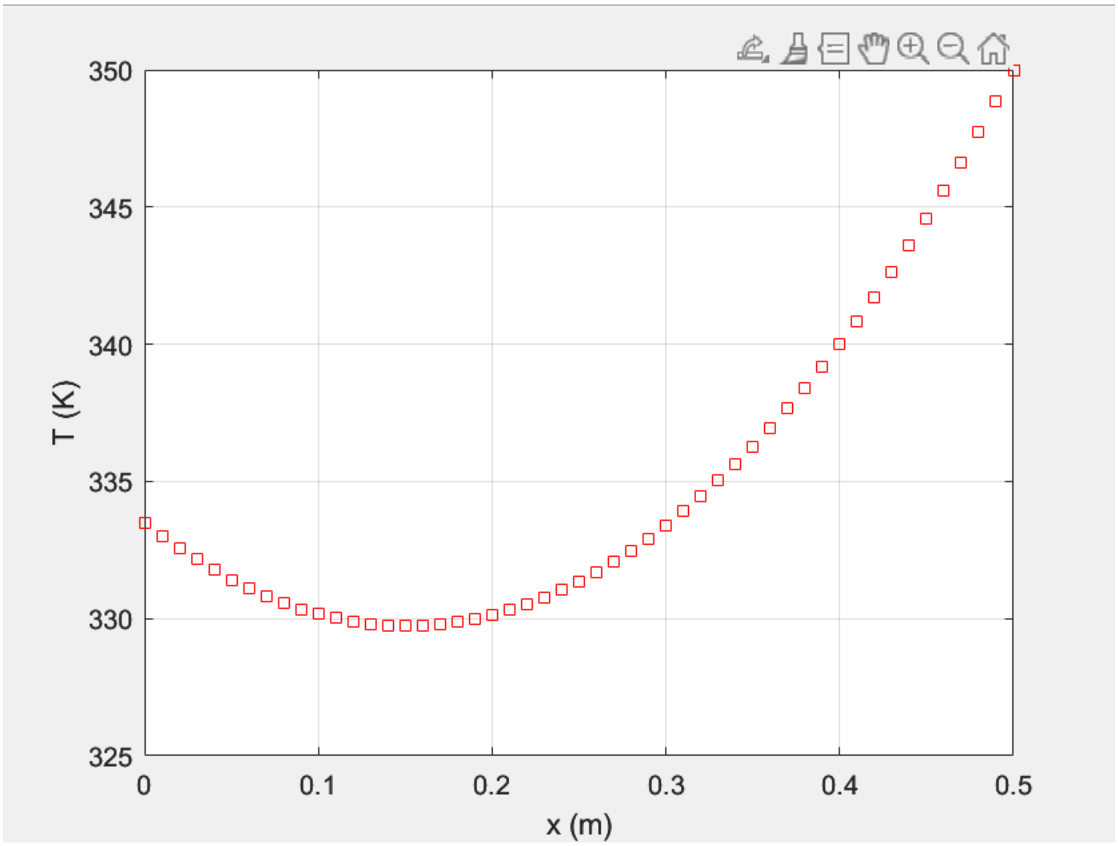
```
% BCs
A(1,1)=(lamda/dx)-Sp*DxB; A(1,2)=-lamda/dx; B(1)=Sc*DxB+qa;

A(n,n)=1; B(n)=Tb;

% Fill A and B
for i=2:n-1
    A(i,i-1)=-lamda/dx;
    A(i,i)=(2*lamda/dx)-Sp*Dx;
    A(i,i+1)=-lamda/dx;
    B(i)=Sc*Dx;
end
```

A2

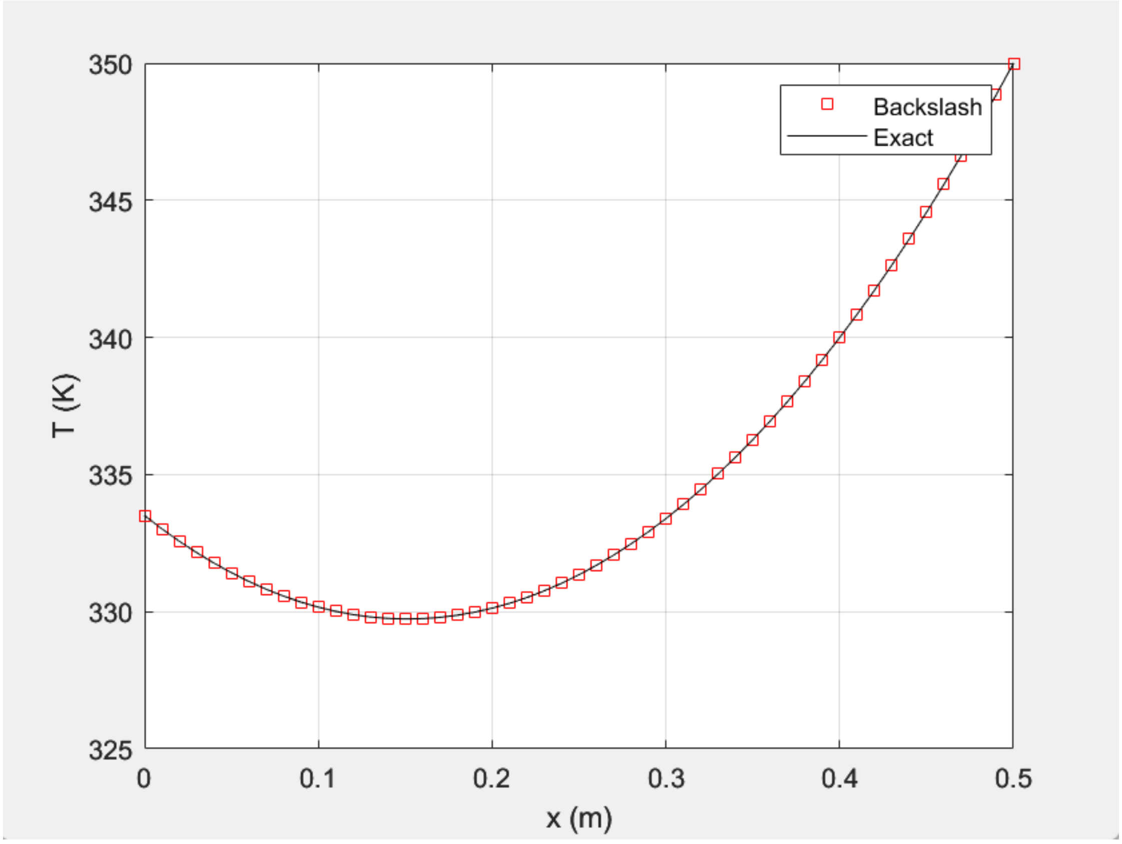
- Matlab plot of $T(x)$:



$\boldsymbol{T(x = L/2)}$	331.346

A3

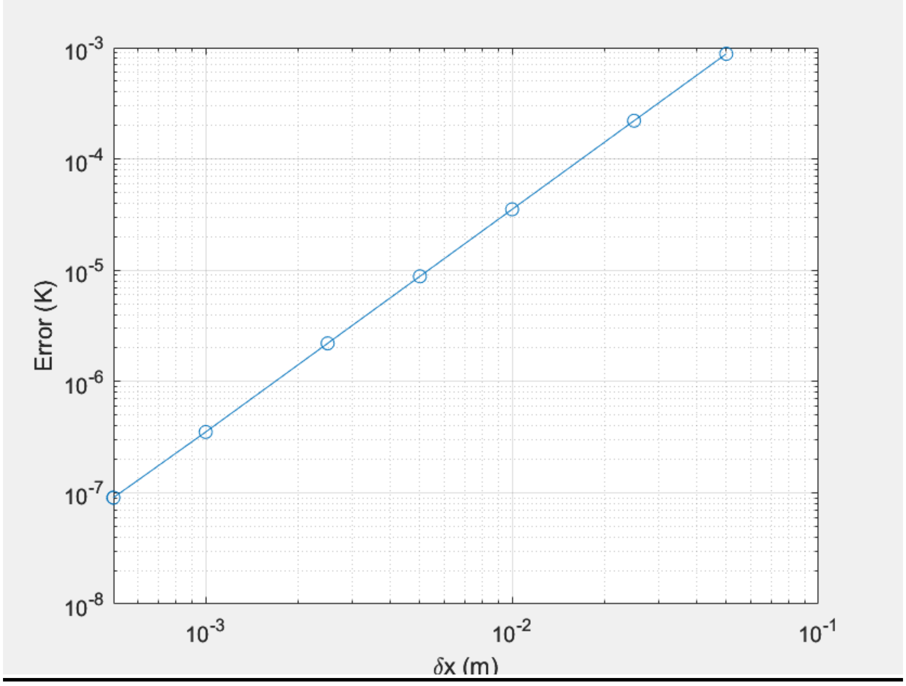
- Matlab plot of numerical and theoretical $T(x)$; you can make one single plot with A2.



Definition of the error	Value of the error
$Error = mean\lvert T - T_{teo}\rvert$	3.5259×10^{-5}

A4

Matlab log-log plot of error vs grid spacing:



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Convergence order of the error	2

- Comments (max 100 words):

The convergence order k can be found by plotting a graph of $error\ vs\ \delta x$ and using the shape to find the order of the relationship $error{\sim}\delta x^k$.

As seen in the graph A4, the error decreases by a factor of 10^4 when δx is decreased by 10^2, giving a relationship of $error{\sim}\delta x^2$, and therefore a convergence order of 2

Task (B)

B1

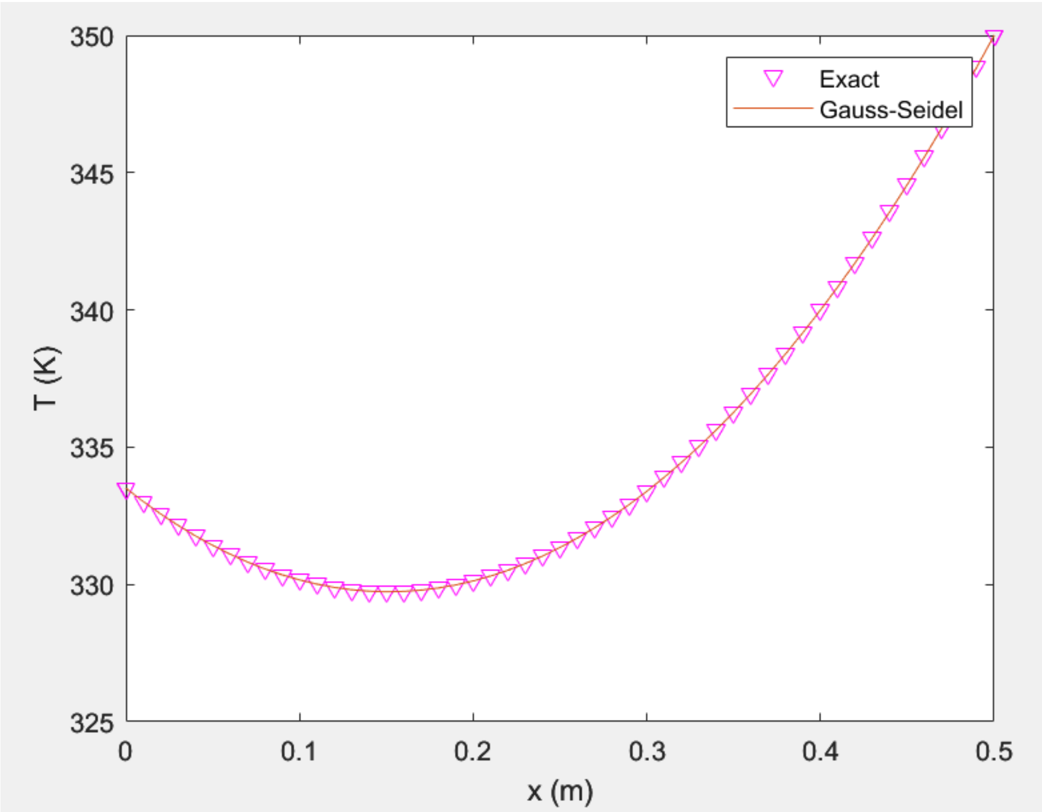
- Screen shot of Matlab code for Gauss-Seidel method:

```
% Using Gauss-Seidel Method
Imax=15000;
tol=1e-10;
T_ast=B(n)*ones(n,1); %Initial guess

[T_gs,res_gs,m_gs]=GaussSeidel(A,B,T_ast,tol,Imax);
```

```
%Gauss-Seidel function
function [x,res,m]=GaussSeidel(A,B,x,tol,Imax)
m=0;
n=numel(x);
res=sum(abs(B-A*x))/sum(abs(diag(A).*x));

while (res>tol & m<Imax)
    m=m+1;
    for i=1:n
        x(i)=x(i)+((B(i)/A(i,i)-(A(i,:)/A(i,i))*x));
    end
    res(m)=sum(abs(B-A*x))/sum(abs(diag(A).*x));
end
```

B2

- Matlab plot of numerical and theoretical $T(x)$.



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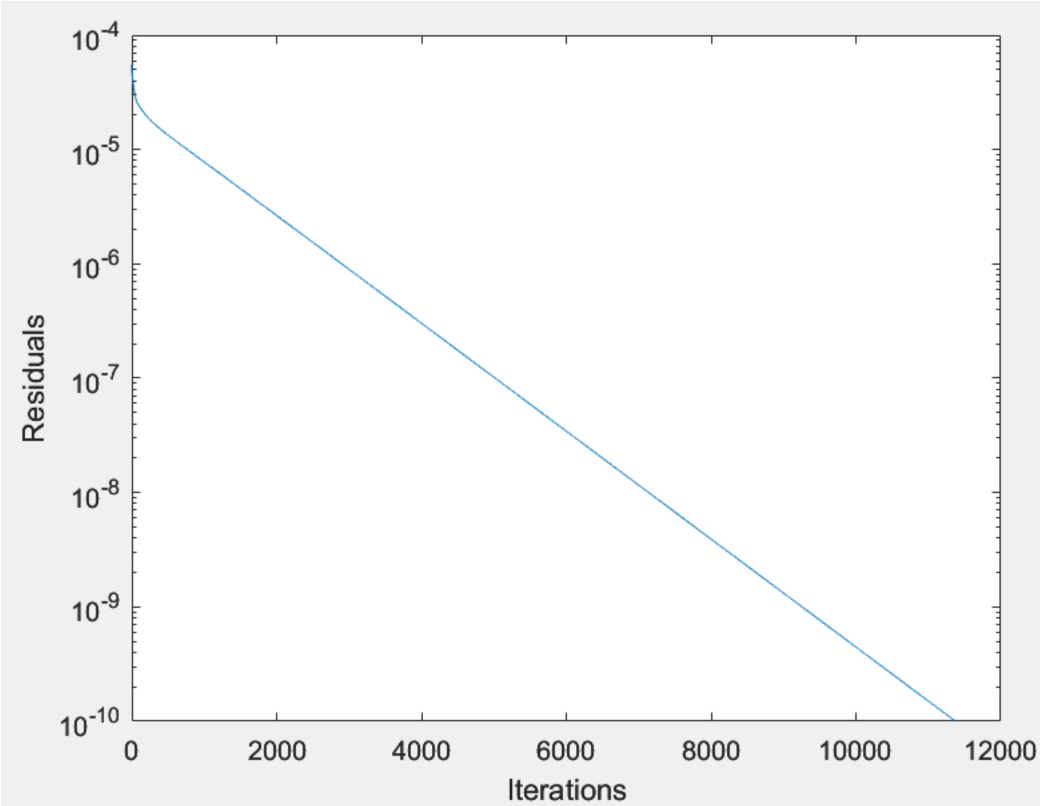
Definition of the error	Value of the error
$Error = mean\|T_{gs} - T_{teo}\|$	8.1630×10^{-5}

B3

- Matlab plot of residuals vs number of iterations graph in a semilogy scale



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Number of iterations	Definition of residuals	Residuals at convergence
11374	$res = \frac{\Sigma\lvert B-A\cdot T\rvert}{\Sigma\lvert diag(A)\cdot T\rvert}$	9.9894×10^{-11}

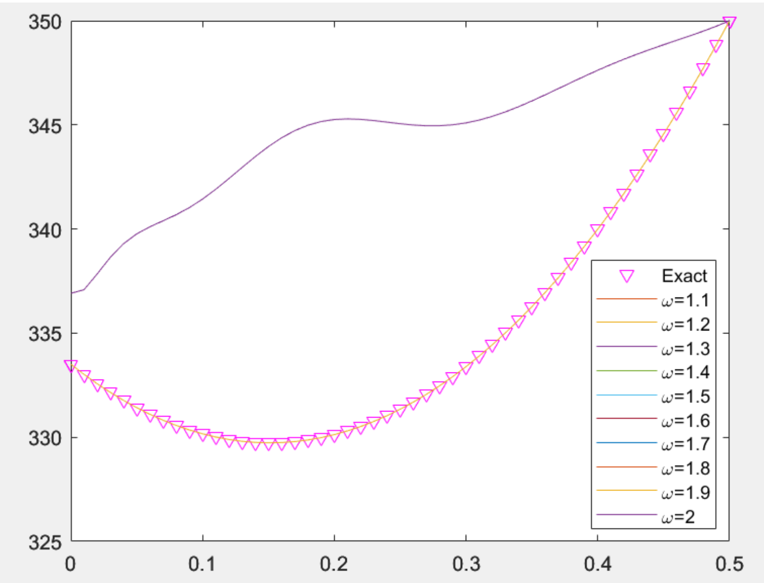
B4

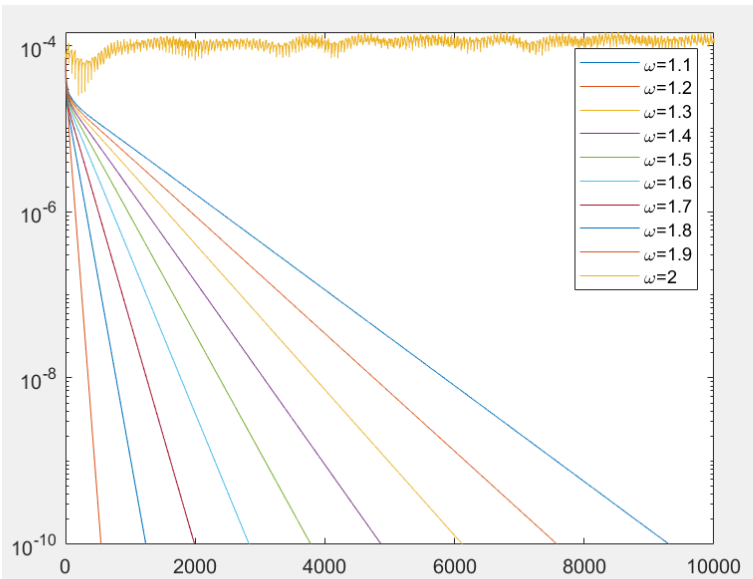
- Screen shot of matlab code for Gauss-Seidel with over-relaxation:

```
w=(1.1:0.1:2);
for i=1:numel(w)
    [T_gs,res_gs,m_gs]=GaussSeidelw(A,B,T_ast,tol,Imax,w(i));
    figure(1)
    plot(x0,T_gs)
    hold on
    figure(2)
    semilogy(res_gs)
    hold on
end
```

```
%Gauss-Seidel function w/ over-relaxation
function [x,res,m]=GaussSeidelw(A,B,x,tol,Imax,omega)
m=0;
n=numel(x);
res=sum(abs(B-A*x))/sum(abs(diag(A).*x));

while (res>tol & m<Imax)
    m=m+1;
    for i=1:n
        x(i)=x(i)+omega*((B(i)/A(i,i)-(A(i,:)/A(i,i))*x));
    end
    res(m)=sum(abs(B-A*x))/sum(abs(diag(A).*x));
end
```

- Matlab plot of solution and residuals vs number of iterations graph for $\omega = 1.1, 1.2, \ldots, 2$





Omega	Successful (solution matches theory)?	N. of iterations	Best (Yes/No)?
$\omega = 1.1$	Yes	9304	No
$\omega = 1.2$	Yes	7579	No
$\omega = 1.3$	Yes	6118	No
$\omega = 1.4$	Yes	4867	No
$\omega = 1.5$	Yes	3781	No
$\omega = 1.6$	Yes	2831	No
$\omega = 1.7$	Yes	1990	No
$\omega = 1.8$	Yes	1240	No
$\omega = 1.9$	Yes	548	Yes
$\omega = 2$	No	N/A	No

Task (C)

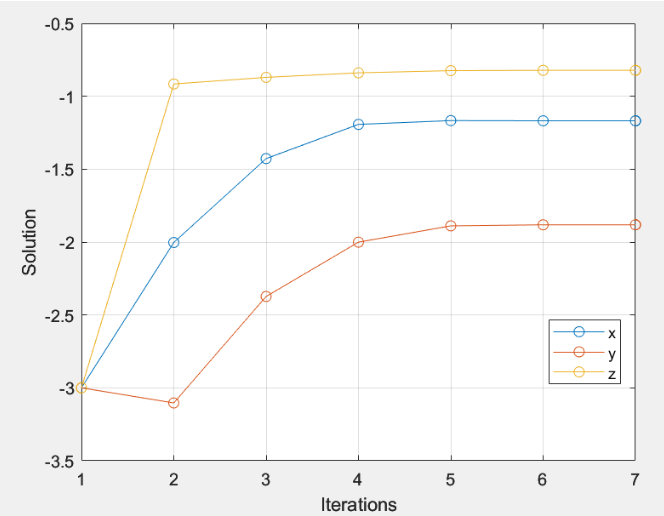
C1

$$J = \begin{bmatrix} e^y & xe^y & 1 \\ -3x^2 & z & y \\ y^2z & 2xyz & xy^2 \end{bmatrix}, F = \begin{bmatrix} xe^y + z + 1 \\ yz - x^3 - \pi \\ xy^2z - 3.4 \end{bmatrix}$$

C2

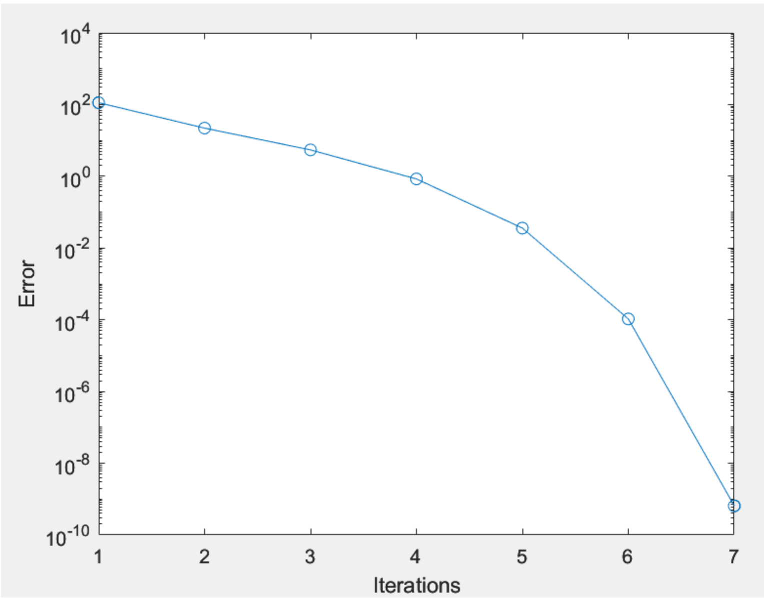
```
maxI=1000;
tol=1e-8;
c=4;
x=-3; y=-3; z=-3;%Initial guess

error=abs(x*exp(y)+z+1)+abs(y*z-x^3-pi())+abs(x*y^2*z-(3+(c/10)));
i=1;

while error(i)>tol & i<maxI
    J(1,1)=exp(y(i)); %du/dx
    J(1,2)=x(i)*exp(y(i)); %du/dy
    J(1,3)=1; %du/dz
    J(2,1)=-3*x(i)^2; %dv/dx
    J(2,2)=z(i); %dv/dy
    J(2,3)=y(i); %dv/dz
    J(3,1)=y(i)^2*z(i); %dw/dx
    J(3,2)=2*x(i)*y(i)*z(i); %dw/dy
    J(3,3)=x(i)*y(i)^2; %dw/dz

    F(1,1)=x(i)*exp(y(i))+z(i)+1;   % u(x(i),y(i),z(i))
    F(2,1)=y(i)*z(i)-x(i)^3-pi();   % v(x(i),y(i),z(i))
    F(3,1)=x(i)*y(i)^2*z(i)-(3+(c/10));    % w(x(i),y(i),z(i))

     X(1,1)=x(i); X(2,1)=y(i); X(3,1)=z(i);

     X=J\(-F+J*X);
     x(i+1)=X(1); y(i+1)=X(2); z(i+1)=X(3);  %New guesses

     F(1)=x(i+1)*exp(y(i+1))+z(i+1)+1;
     F(2)=y(i+1)*z(i+1)-x(i+1)^3-pi();
     F(3)=x(i+1)*y(i+1)^2*z(i+1)-(3+(c/10));

     error(i+1)=sum(abs(F));
     i=i+1;
 end
figure
plot(x,'o-'); grid on; hold on
plot(y,'o-'); plot(z,'o-')
legend('x','y','z')

figure
semilogy(error,'o-')
xlabel('Iterations'); ylabel('Error')
```

C3



Converged values of the solution	Use 5 significant digits
x	-1.1684
y	-1.8815
z	-0.82199

C4



Error at convergence	Number of iterations
6.4690×10^{-10}	7