Midterm 1

No books, notes, or other aids are allowed. **Show all your work for full credit.**

1. (10 points) Write down the coefficient matrix and augmented matrix of the following linear system.

$$\begin{aligned} 4x_1 \qquad\quad + x_3 &= 2 \\ 2x_1 - 8x_2 \qquad &= 0 \\ x_1 + 2x_2 - x_3 &= 7 \end{aligned}$$

2. (20 points) For the following statements decide whether they are true or false. Explain your reasoning.

(a). Matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ is in echelon form.

(b). Matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ is in reduced echelon form.

(c). Matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ has two leading entries.

(d). Every solution of $Ax = b$ can be written as a linear combination of solutions of $Ax = 0$.

3. (10 points) Write the following matrix in reduced echelon form. Show all steps required.

$$\begin{bmatrix} 1 & 2 & 2 & 1 \\ 2 & 0 & 4 & 2 \\ -2 & 1 & 0 & 4 \end{bmatrix}.$$

4. (10 points) Write down the free variable(s) of the following matrix equations.
   (a). $Ax = 0$ where

$$A = \begin{bmatrix} 1 & 1 & 3 & 2 & 5 \\ 0 & 0 & 4 & 1 & 2 \\ 0 & 0 & 0 & 0 & 8 \end{bmatrix}.$$

   (b). $Ax = b$ where

$$A = \begin{bmatrix} 1 & 1 & 3 & 2 & 5 & 1 \\ 0 & 0 & 4 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 & 8 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}.$$

5. (10 points) Compute the following or state that the problem is undefined:
   (a) $\begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

(b) $\begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 2 & 3 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$.

6. (20 points) Write the solution of the following matrix equation in parametric vector form:
(a). $Ax = 0$ where

$$A = \begin{bmatrix} 1 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

(b). $Ax = b$ where

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}.$$

7. (15 points) Do the following vectors span $\mathbb{R}^3$? Are they linearly independent or linearly dependent?

$$\begin{bmatrix}1\\0\\2\end{bmatrix}, \begin{bmatrix}0\\3\\0\end{bmatrix}, \begin{bmatrix}1\\1\\2\end{bmatrix}, \begin{bmatrix}1\\2\\4\end{bmatrix}.$$

8. (5 points) Find all possible numbers of h so that the columns of $Ax = b$ has solution where

$$A = \begin{bmatrix}1 & 1\\1 & h\\1 & 1\end{bmatrix}, \quad b = \begin{bmatrix}3\\1\\h^2-h+3\end{bmatrix}.$$