Two-mode rotation-symmetric bosonic code error bias — Project proposal TRA200

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I. BACKGROUND

Quantum computing architectures are broadly categorized into discrete variable (DV) and continuous variable (CV) systems. CV quantum computing, in particular, leverages the infinite-dimensional state space of bosonic modes — such as the quantized energy states of a harmonic oscillator — to encode quantum information. A straightforward physical qubit can be formed using Fock states, for instance, by identifying the zero-photon state $|0\rangle$ and and the one-photon state $|1\rangle$ as the computational basis. However, such physical encodings are highly susceptible to ubiquitous errors such as photon loss and dephasing. To achieve fault tolerance, quantum error correction is used to define robust logical qubits ($|0\rangle_L$ and $|1\rangle_L$) that are protected from these physical errors. Among the various approaches, the two-mode rotation-symmetric bosonic (RSB) code [1] has emerged as a promising candidate. The two-mode binomial RSB code is shown below

$$|1\rangle_L = \frac{1}{\sqrt{2}}\hat{U}_{BS}(\theta,\phi)\left(|0\rangle + |2N\rangle\right)\otimes|N\rangle \tag{1a}$$

$$|0\rangle_L = \hat{U}_{BS}(\theta,\phi)\,|N\rangle \otimes \frac{1}{\sqrt{2}}\left(|0\rangle + |2N\rangle\right), \tag{1b}$$

where $\hat{U}_{SB}(\theta,\phi)$ is a beam-splitter and N the symmetry order. These logical qubits have shown promising results against dephasing and loss [1]. Previous work (e.g., by the Chalmers group) have showed the superiority of two-mode RSB codes over single-mode ones under dephasing noise, and comparable performance under single-photon losses.

The effect of errors on the encoded logical state can be shown using the logical channel $\mathcal{N}_{log}$ applied to the density matrix $\hat{\rho}_L = |\psi\rangle_L\, {}_L\langle\psi|$, where $|\psi\rangle_L = \alpha\,|0\rangle_L + \beta\,|1\rangle_L$ is the ideal encoded state,

$$\begin{aligned}\mathcal{N}_{log}(\rho_L) = {} & (1-p_x-p_y-p_z)\rho_L \\ & + p_x X_L\rho X_L + p_y Y_L\rho_L Y_L + p_z Z_L\rho_L Z_L,\end{aligned} \tag{2}$$

where p_x, p_y, p_z give the probabilities for a x-flip, y-flip, and z-flip respectively. The values of these probabilities affect the dynamics of the qubits. If $p_x \gg p_y + p_z$, the logical qubits would be biased toward bit-flips, and creating error correcting algorithms for specifically that error would reduce the amount of needed resources as compared to non-biased noise.

II. TASK

Investigate the two-mode RSB codes by deriving their logical channels $\mathcal{N}_{log}$, for photon loss and dephasing.

This will be done analytically, and numerically with QuTiP-based [2] simulations of two-mode RSB codes with physically reasonable truncations.

III. METHOD

1. Conduct a literature review to develop a solid understanding of RSB codes [3] and contemporary methods for deriving their logical channels.

2. Apply this knowledge to single-mode RSB codes for photon-loss channel and dephasing channel. Determine the noise biases. This is done to gain understanding needed for the two-mode case.

3. Apply this knowledge to the binomial two-mode RSB code. Begin with the photon-loss channel, first setting $\theta = 0$, $\phi = 0$, and then considering the optimal angles $\theta \in \{\frac{\pi}{4}, \frac{3\pi}{4}\}$, $\phi = \frac{\pi}{2N}$ [1], with $N = 2$.

4. Repeat the analysis for the dephasing channel.

5. Investigate the impact of varying the symmetry order N.

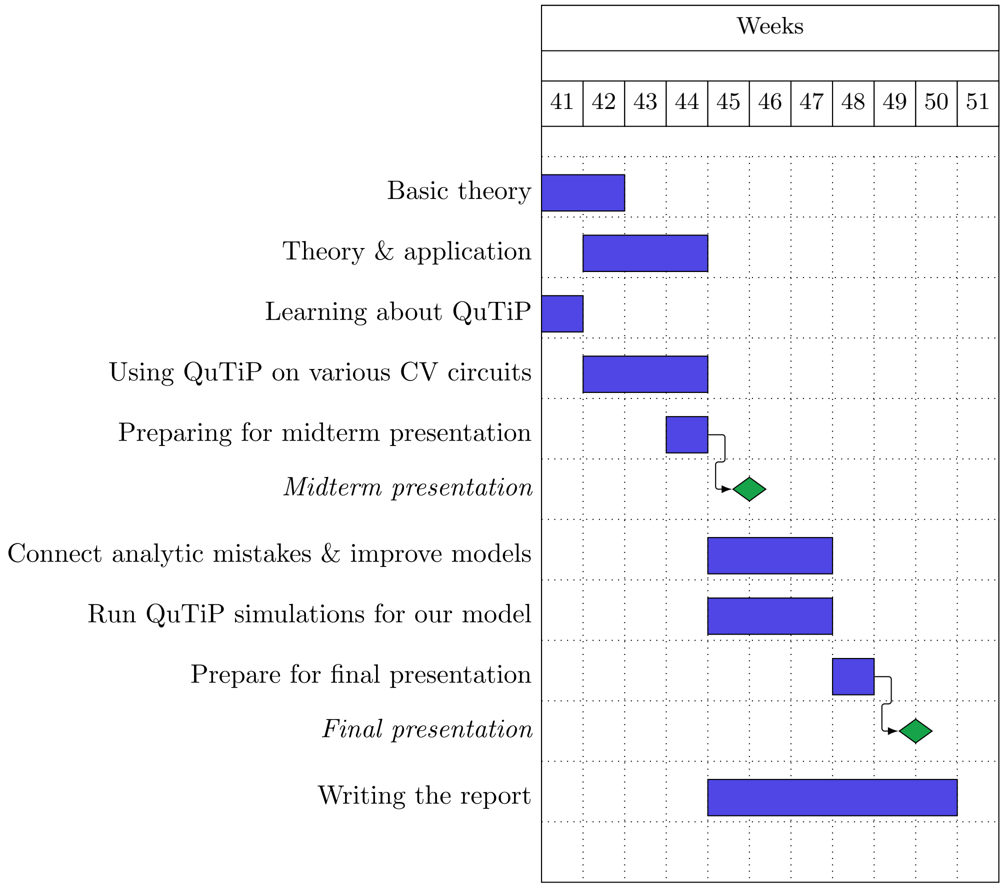
6. Derive the logical channels for other two-mode RSB codes (e.g., cat codes).

7. Explore the concatenation of different RSB codes with the repetition code.

8. Perform QuTiP-based simulations of QAOA with the investigated codes (Optional).

Numerical calculations (QuTip-based) will be done in the later part on each RSB code.

IV. PRELIMINARY TIME SCHEDULE



[1] R. G. Ahmed, A. Udupa, and G. Ferrini, Multimode rotationally symmetric bosonic codes from group-theoretic construction, arXiv preprint (2025), arXiv:2508.20647 [quant-ph].

[2] N. Lambert, E. Giguère, P. Menczel, B. Li, P. Hopf, G. Suárez, M. Gali, J. Lishman, R. Gadhvi, R. Agarwal, A. Galicia, N. Shammah, P. Nation, J. R. Johansson, S. Ahmed, S. Cross, A. Pitchford, and F. Nori, Qutip 5: The quantum toolbox in python, ??? (2024), arXiv:2412.04705 [quant-ph].

[3] A. L. Grimsmo, J. Combes, and B. Q. Baragiola, Quantum computing with rotation-symmetric bosonic codes, arXiv preprint **arXiv:1901.08071** (2019), preprint.