The Fibonacci-Prime-Spiral & Zeckendorf's Theorem

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This will be a quick overview of *a way* to **exactly** reproduce the number square on Page 15. Some of the concepts are well known, like the prime and the Fibonacci sequence, some maybe not so well known like Zeckendorf's theorem and the Fibonacci-Base. The web has many resources on these items so we will only summarize here.

The Primes: {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59…. 7817 }

The Fibonacci sequence: { 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987 }

(7817 is the 988th prime.)

Zeckendorf's theorem: en.wikipedia.org/wiki/Zeckendorf%27s_theorem

Zeckendorf's theorem states that every positive integer can be represented uniquely as the sum of one or more distinct Fibonacci numbers in such a way that the sum does not include any two consecutive Fibonacci numbers. Zeckendorf's theorem has two parts:

Existence: *every positive integer n has a Zeckendorf representation.*

Uniqueness: *no positive integer n has two different Zeckendorf representations.*

This means that every integer has a unique representation as a sum of Fibonacci numbers a 'Fibonacci-Sum-Representation' (FSR). This unique representation can also be expressed as the index (position) of the FSR in Fibonacci sequence, or their 'Fibonacci-Sum-Index-Representation' (FSIR). When we do this we are tacitly working in something known as the 'Fibonacci Base' number system (similar to binary, or decimal but instead of having powers of 2 or 10 we have Fibonacci numbers). Below is given example decimal , FSR, FSIR and The Fibonacci-Base representations of the first few integers:

Decimal	/ Fibonacci-Sum /	Fibonacci-Sum-Index	/ Fibonacci-Base
0	{0}	{1}	0
1	{1}	{2}	1
2	{2}	{3}	10
3	{3}	{4}	100

4	{1, 3}	{2, 4}	101
5	{5}	{5}	1000
6	{1, 5}	{2, 5}	1001
7	{2, 5}	{3, 5}	1010
8	{8}	{6}	10000
9	{1, 8}	{2, 6}	10001
10	{2, 8}	{3, 6}	10010
11	{3, 8}	{4, 6}	10100
12	{1, 3, 8}	{2, 4, 6}	10101
13	{13}	{7}	100000
14	{1, 13}	{2, 7}	100001
15	{2, 13}	{3, 7}	100010
16	{3, 13}	{4, 7}	100100
17	{1, 3, 13}	{2, 4, 7}	100101
18	{5, 13}	{5, 7}	101000
19	{1, 5, 13}	{2, 5, 7}	101001
20	{2, 5, 13}	{3, 5, 7}	101010
21	{21}	{8}	1000000

We can use this table to go from a number in the decimal system to a number in the ‘Zeckendorf system.’ We can also do the reverse, mapping from the Fibonacci-Sum-Index and/or Fibonacci-Base representation to decimal. Consider the decimal representation of the following (carefully chosen) numbers:

{1}	0	0
{2}	1	1
{3}	10	2
{4}	100	3
{5}	1000	5
{6}	10000	8
{7}	100000	13
{8}	1000000	21

We are counting in ‘*powers of Fibonacci*’ numbers. With the above we can now recreate the ‘Number Square’. Find the decimal representation of the first 16 powers of the *‘Fibonacci Base’* ie. 0, 1, 10, 100, 1000 to 10^16 in the ‘Fibonacci Base’:

A = 0, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987

Then take the prime that has the positional index for each A, with index 0 giving the first element (or simply take each A+1 ' th prime). These are:

B = 2, 3, 5, 7, 13, 23, 43, 79, 149, 263, 463, 829, 1481, 2593, 4507, 7817

Then take the Absolute value of | 3301 - B |
to give:

C = 3299, 3298, 3296, 3294, 3288, 3278, 3258, 3222, 3152, 3038, 2838, 2472, 1820, 708, 1206, 4516

If we arrange C in spiral (a form closely associated with the Fibonacci numbers) the square is reproduced.

*Comments, questions, suggestions, omissions etc ? please try #cicadasolvers

MSGA

Last Edit: Dec 10, 2016 at 1:32pm by mortlach

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