Optimal Asset Allocation in index funds

Let's go back to the year 2005 and assume you had got a fund from your grandparents, value of DKK 100.000, that you could use at the earliest at the year 2019 (for your educational purposes only 😊). We want to look into some realistic alternatives for what you could have done with the DKK 100.000.

Let's say you wanted to follow an index-based investment strategy. In this assignment, you are asked to compare your strategy's performance with a number of benchmarks. You may only choose from a positive list of XX assets. Use weekly data from 1998 to 2019.

You need to report how your strategy would have performed from 2005 (where you got the money) and onwards, if you were to go back in time and started investing according to your suggestion. You are also required to document the strategy and compare with a few alternative strategies.

Steps:

1- Scenario generation (10%)

You want to see whether a CVaR minimizing strategy provides good results on your data set. So you decide to implement a CVaR model. For that reason you need to generate scenarios. You decide to use data from 1998 to 2005 to generate your first set of scenarios. You use the bootstrap method to generate scenarios.

1.1 Use the first 7 years of returns to generate the first set of 250 scenarios. Each scenario should be 4 weeks long. In how many ways can you choose 4 out of 364? Use the random generator function in GAMS **uniformint(A,B)** to pick uniformly distributed random integer numbers in the interval A to B.
1.2 Generate a scenario with a length of 4 weeks by randomly picking 4 dates in the interval you are sampling from. Accumulate the 4 weeks' returns to make a 4-week scenario.
1.3 Repeat step 1.1 and 1.2 to generate 250 4-weekly scenarios. Save the scenario sets in a GDX file, so you can use them as you need them in the rest of this project. Call this scenario set FixedScenarios.gdx.
1.4 Calculate the first four moments of each asset both for the historic data (1998-2005) and for the scenarios and compare the values in a table – show it only for a few assets. Remember to turn the historical returns into a 4-week period as well.
1.5 Now roll the sampling period forward with four weeks and generate another 250 scenarios.
1.6 Repeat step 1.5 until you can't roll forward any longer due to the limits of the data set at hand.
1.7 Save all the generated scenario sets in a new GDX file that you call RollingScenarios.gdx.
1.8 Calculate the expected returns and CVaR values (in %) for the qual weight (1/N) strategy in each scenario set. Save the results in a new GDX file you call Targets.gdx.

2- Implementing the CVaR model (10%)

Use the first set of scenarios (start date in the beginning of 2005) and for that implement the Mean/CVaR model (using a lambda formulation). Find the efficient frontier (mean return as a function of CVaR) using 10 optimal solutions starting from the solution which minimized CVaR and going forward in equal increases in

CVaR until you reach the 10th solution which is only maximizing average return. Document the model, your start solution (portfolio mix) as well as the values of mean return and CVaR for the 10 solutions. Draw the efficient frontier and also draw histograms over portfolio returns for each of the 10 solutions.

Recall the average return of the equal weight (1/N) strategy from question 1.8. Now reformulate the CVaR model once maximizing expected return and once minimizing CVaR with proper constraints on CVaR and Expected return in the respective models. Depict these two portfolios on the efficient frontier.

For the backtest in the rest of this project, use the two latter formulations and compare their results with the equal weight strategy.

3- Updating the model to a portfolio revision model and backtesting (30%)

Now you start the first step of your back testing process. Assume that four weeks are gone, and you need to revise your existing portfolios after four weeks. Run the two CVaR models. Now notice that when you ran these models in the first period you had a starting budget. Now in the second period you don't have a starting budget but a starting portfolio. So you need to replace the budget constraint with portfolio revision constraints (hint: you need two constraints for the revision part. You should also use explicitly two new variables enabling the selling of assets from the existing portfolio or buying new assets into the portfolio. Obviously, it should also be possible to hold assets from the existing portfolio.).

Change the CVaR models so that they allow revising of existing solutions and document the model. Now add a variable transaction cost of 0.1% per amount sold or bought, though with a minimum of 20 DKK per transaction. Document this transaction cost structure.

Run the updated CVaR model for the first and second period again. Are there any revisions when you run the model for the second period? Report the revisions. Compare the CVaR and the expected returns as found by the CVaR model as well as the equal weight strategy you are using as benchmark.

Hint: In order not to make this project too complicated implement your equal weight strategy as a "set and forget" strategy. So you just start with the 1/N of the budget assigned to each of the XX assets and then let it develop through all the coming periods – no revision at all for this benchmark that is.

You should now extend the experiment using all the remaining scenario trees.

For each strategy draw the following graphs:

- The optimal portfolio mix (total of 100%), including all its revisions.
- Actual index price growth from following the strategy (ex-post), starting with an index of 100. In the same graph include the average, the worst and the best case as suggested by the scenarios (ex-ante).
- Compare the actual performance of the three strategies (two CVaR portfolios and the benchmark) in the same graph and comment your findings.

4- Rolling scenarios (10%)

Now re-run the complete backtest (question 3) again, but this time using the rolling scenarios. Compare the new results with the old ones. Comment your findings. Explain explicitly how you can see the effect of using the rolling scenarios from your results.

5- New strategy allowing leveraging (10%)

Now run your CVaR models once again, this time with a different twist. The twist is that we allow for leveraging up to a factor of 3, i.e. you can borrow up to 200% of your budget or initial portfolio value with a borrowing annual rate of 2%. Use the scenario sets that you prefer.

Document your formulation of this model. Run the model for all the periods and compare its performance with that of the CVaR model without possibility of leveraging. Update the graphs that you used for documenting the backtest under question 3.