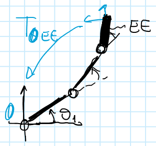


dalla cinematica posizione

- $T_{0EE}(q)$ $\qquad q = \begin{bmatrix}\theta_1\\ \theta_2\\ \theta_3\end{bmatrix}$

posa $\underset{6\times1}{x} = \begin{bmatrix} p^0 \\ \lambda \end{bmatrix} = \begin{bmatrix} [O_{EE}]_0 \\ \begin{bmatrix}\alpha\\ \beta\\ \varphi\end{bmatrix} \end{bmatrix}$ si ricava dalla $T_{0EE}(q)$

$$T_{0EE} = \begin{bmatrix} R_{0EE} & [O_{EE}]_0 \\ 0^T & 1 \end{bmatrix}$$

p^0

α, β, φ avendo dato la sequenza Eul
es. X - Y - Z $\qquad R_x(\alpha) R_y(\beta) R_z(\varphi)$
inv.

velocità $\qquad x = x(q) = x(q(t))$ $\qquad q(t)$

d/dt

$$\dot{x} = \begin{bmatrix} \dot{p}^0 \\ \dot{\lambda} \end{bmatrix} = J_{an}\, \dot{q}$$

$J_{an} \leftarrow \frac{\partial x}{\partial q}$

$\dot{p}^0 =$ vel $O_{EE} = v_{O_{EE}}$

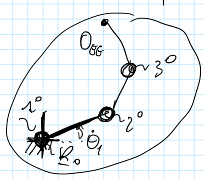
$v_{O_{EE}}$

FORM. FOND

$v_{O_{EE}}, \omega_{EE} \qquad P \in EE$

(A) $v_P = v_{O_{EE}} + \omega_{EE} \wedge \overrightarrow{O_{EE}P}$

$$v = \begin{bmatrix} \dot{p}^0 \\ \omega_{EE} \end{bmatrix} \quad (\dot{p}^0 = v_{O_{EE}})$$



$$v = J_g\, \dot{q}$$

$$J_g = \begin{bmatrix} R_0 \wedge \overrightarrow{O_0 O_{EE}} & | & | \\ R_0 & | & | \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \vdots \end{bmatrix}$$

1° $\quad$ 2° $\quad$ 3°

$6 \times n_{gdl}$

colonna i-esima, giunto i

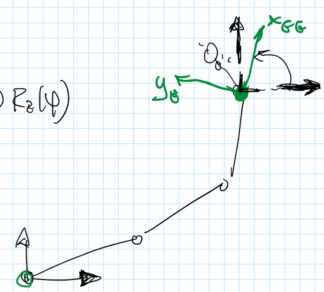
$$P \rightarrow \begin{bmatrix} R_{i-1} \\ 0 \end{bmatrix} \qquad R \rightarrow \begin{bmatrix} R_{i-1} \wedge \overrightarrow{O_{i-1} O_{EE}} \\ R_{i-1} \end{bmatrix}$$

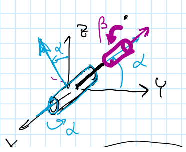
Relazione J_{an} $\quad J_g$

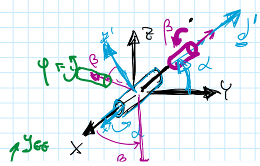
$$\dot{x} = J_{an}\, \dot{q} = \begin{bmatrix} \dot{p}^0 \\ \dot{\lambda} \end{bmatrix} \qquad v = J_g\, \dot{q} = \begin{bmatrix} \dot{p}^0 \\ \omega \end{bmatrix}$$

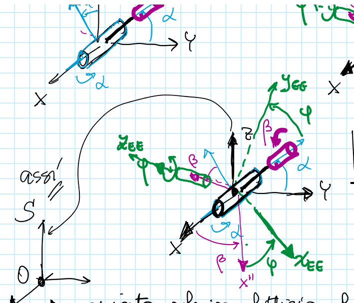
$[3\times n] = \dot{p}^0 = [3\times n]$

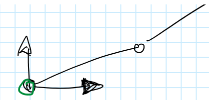
$$\lambda = \begin{bmatrix} \alpha \\ \beta \\ \varphi \end{bmatrix} \quad \text{sequenza, es. } X - Y - Z \qquad R_x(\alpha) R_y(\beta) R_z(\varphi)$$





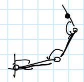






$$\boxed{\underline{\omega} = \underline{i}\,\dot\alpha + \underline{j}'\,\dot\beta + \underline{k}_{EE}\,\dot\varphi}$$

vettoriali



giunto sferico fittizio che orienta EE

$$\underline{\omega} = \dot\alpha\,\underline{r}_1 + \dot\beta\,\underline{r}_2 + \dot\varphi\,\underline{r}_3$$

dip. dalla sequenza

$\underline{i} = \begin{bmatrix}1\\0\\0\end{bmatrix}$ $\quad \underline{j}' \rightarrow R_x(\alpha) = [\underline{i}' \; \underline{j}' \; \underline{k}']$ 2ª colonna

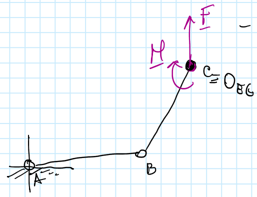
$\underline{k}_{EE}$ - $T_{0\,EE}$ o $R_x(\alpha)\,R_y(\beta) \rightarrow$ 3ª colonne

$$\underline{\omega} = \underbrace{\begin{bmatrix}\underline{r}_1 & \underline{r}_2 & \underline{r}_3\end{bmatrix}}_{A_G}\begin{bmatrix}\dot\alpha\\\dot\beta\\\dot\varphi\end{bmatrix}_{\dot{\underline{x}}}$$

an $\rightarrow$ geom

$$J_g = \begin{bmatrix} I \\ A_G \end{bmatrix} J_a$$

Analogia cinetostatica $\longleftrightarrow$ PLV



- azioni scambiate con EXT dall' EE

$\downarrow$

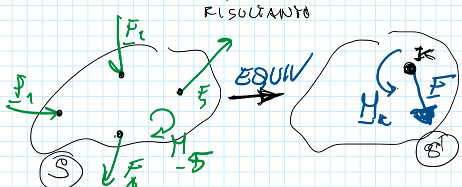
sistema di forze può essere sempre ricondotto

$(O_{EE}, \underline{F}) + \underline{M}_{O_{EE}} = \underline{M}$

RISULTANTE

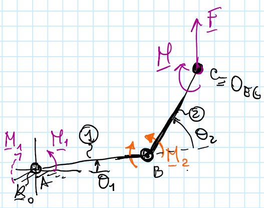
RICORDA DEF. SIST. EQUIV.

- $\underline{R} = \underline{R}'$
- $\underline{M}_A = \underline{M}_A'$



$$\underline{F} = \underline{F}_1 + \underline{F}_2 + \underline{F}_3 + \underline{F}_4$$

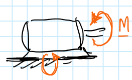
$$\underline{M}_K = \underline{M}_S + \vec{KP}_1 \wedge \underline{F}_1 + \vec{KP}_2 \wedge \underline{F}_2 + \dots$$



Equilibrio nella cond. data attraverso azioni ai giunti

R $\rightarrow$ coppia

P $\rightarrow$ forza



$\underline{M}_1 = M_1\,\underline{k}_0$

$\underline{M}_2 = M_2\,\underline{k}_1$

? $M_1 - M_2$ per equilibrio $(\underline{F}, \underline{M})_{EE}$

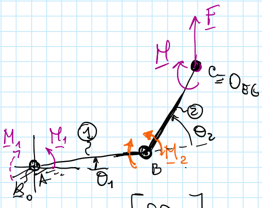
$\underline{\tau} = \begin{bmatrix}M_1\\M_2\end{bmatrix}$ vettore coppie ai giunti (torque) $n \times 1$

$\underline{\gamma} = \begin{bmatrix}\underline{F}\\\underline{M}\end{bmatrix}$ 6×1

PLV — CNS affinché una config $\bar{C}$ sia di equilibrio è che $\delta L^{(a)} = 0 \quad \forall$ SP

(bilateroli, privi di attrito o puro rotolamento) è che $\delta\mathcal{L}^{(a)} = 0 \quad \forall\, \delta P$

(bilateroli, privi di attrito o puro rotolamento)



SP spost virtuali, infinites. compatibili con i vincoli

$$\delta\mathcal{L}^{(a)} = \underline{F} \cdot \delta O_{EE} + \underline{M} \cdot \delta \underline{\varepsilon}_2 + \underline{M}_1 \cdot \delta \underline{\varepsilon}_1 + \underline{M}_2 \cdot \delta\underline{\varepsilon}_2 - \underline{M}_2 \cdot \delta\underline{\varepsilon}_1 = 0$$

$$\delta \underline{q} = \begin{bmatrix} \delta\theta_1 \\ \delta\theta_2 \end{bmatrix}$$

- $\delta\underline{\varepsilon}_1 = \delta\theta_1\, \underline{k}_0 \Rightarrow \underline{\omega}_1\, \delta t$
- $\delta\underline{\varepsilon}_2 = (\delta\theta_1\, \underline{k}_0) + \delta\theta_2\, \underline{k}_1 \Rightarrow \underline{\omega}_2\, \delta t$
- $\delta O_{EE} \Rightarrow \underline{v}_{O_{EE}}\, \delta t$

$\underline{M}_1 \cdot \delta\underline{\varepsilon}_1 = M_1 \underline{k}_0 \cdot \delta\theta_1 \underline{k}_0 = M_1\, \delta\theta_1$

$\underline{M}_2 \cdot (\delta\underline{\varepsilon}_2 - \delta\underline{\varepsilon}_1) = \underline{M}_2 \cdot \delta\theta_2\, \underline{k}_1 = M_2 \underline{k}_1 \cdot \delta\theta_2 \underline{k}_2 = M_2\, \delta\theta_2$

$$\underbrace{M_1 \quad}_{\underline{\tau}}\ \delta\theta_1 + \underbrace{M_2\, \delta\theta_2}_{\delta\underline{q}} \quad \to \quad \underline{\tau} \cdot \delta\underline{q}$$

$$\left.\begin{aligned} \underline{F} \cdot \delta O_{EE} &= \underline{F} \cdot \underline{v}_{O_{EE}}\, \delta t \\ \underline{M} \cdot \delta\underline{\varepsilon}_2 &= \underline{M} \cdot \underline{\omega}_2\, \delta t \end{aligned}\right\} \to \underbrace{\begin{bmatrix} \underline{v}_{O_{EE}} & \underline{\omega}_2 \end{bmatrix}}_{(J_g \dot{\underline{q}})^T} \overbrace{\begin{bmatrix} \underline{F} \\ \underline{M} \end{bmatrix}}^{\underline{\gamma}} \delta t$$

$$\underline{\tau} \cdot \delta\underline{q} + (J_g \dot{\underline{q}})^T \underline{\gamma}\, \delta t = 0$$

$$\dot{\underline{q}}\, \delta t = \delta\underline{q}$$

$$\underline{\tau} \cdot \delta\underline{q} + (J_g\, \delta\underline{q})^T \underline{\gamma} = 0$$

$\underline{\tau} \cdot \delta\underline{q} = \delta\underline{q} \cdot \underline{\tau} \to \delta\underline{q}^T \underline{\tau}$

$$\delta\underline{q}^T (\underline{\tau} + J_g^T \underline{\gamma}) = 0$$

$\delta\underline{q}^T \underline{\tau} = -(J_g\, \delta\underline{q})^T \underline{\gamma}$

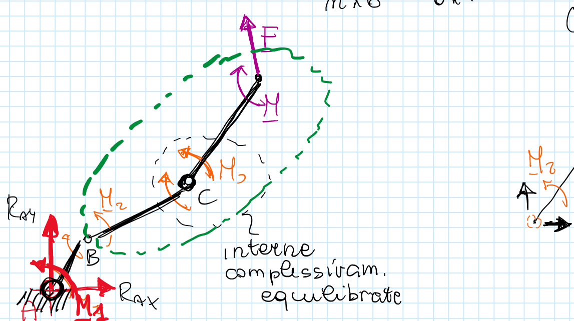
$\delta\underline{q}^T \underline{\tau} = -\delta\underline{q}^T J_g^T\, \underline{\gamma}$

$\forall\, \delta P \to \forall\, \delta\underline{q}$

$$\boxed{\underline{\tau} = -J_g^T \underline{\gamma}}$$

$$\underbrace{\underline{\tau}}_{n \times 1} = -\underbrace{J_g^T}_{n \times 6}\ \underbrace{\underline{\gamma}}_{6 \times 1}$$

$\Rightarrow$ COPPIE ATTUATORI (non si determinano le reazioni) $\searrow$ cardinali



EQ. CARD. $\Rightarrow \forall$ corpo sistema CNS

$\searrow$ sistema completo
$\searrow$ sotto sistemi } C.N.

equilibrio sistema completo

- $\underline{F} + \underline{R}_A = \underline{0}$

A) $\underline{M}_1 + \cancel{(\underline{M}_2 - \underline{M}_2)} + \cancel{(\underline{M}_3 - \underline{M}_3)} + \underline{M} + \overrightarrow{AO_{EE}} \wedge \underline{F} = \underline{0}$

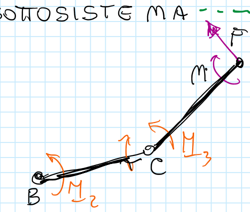
$\underline{M}_1 + \underline{M} + \overrightarrow{AO_{EE}} \wedge \underline{F} = \underline{0}$

A) $\underline{M}_1 + (\cancel{\underline{M}_2} - \cancel{\underline{M}_2}) + (\cancel{\underline{M}_3} - \cancel{\underline{M}_3}) + \underline{M} + \vec{AO}_{EE} \wedge \underline{F} = \underline{0}$

$$\underline{M}_1 + \underline{M} + \vec{AO}_{EE} \wedge \underline{F} = \underline{0}$$

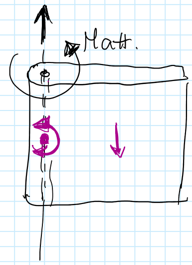
EQ. VETTORIALI

SOTTOSISTEMA



$$\underline{R}_{B_2} + \underline{F} = \underline{0}$$

B) $\underline{M}_2 + \vec{BO}_{EE} \wedge \underline{F} + \underline{M} = \underline{0}$



in un giunto R
1 comp. momento attuatori
2 " " reazioni

attuatori

$$\underline{M}_1 = M_1 \underline{k}_0 + \underline{M}_1^R$$

$$\underline{M}_2 = M_2 \underline{k}_2 + \underline{M}_2^R$$

• $\underline{M}_1 + \underline{M} + \vec{AO}_{EE} \wedge \underline{F} = \underline{0}$ $\quad (\cdot\, \underline{k}_0)$ direzione asse giunto

$$\underline{M}_1 \cdot \underline{k}_0 + \underline{M} \cdot \underline{k}_0 + \vec{AO}_{EE} \wedge \underline{F} \cdot \underline{k}_0 = 0$$

$$M_1 = -\left(\vec{AO}_{EE} \wedge \underline{F} \cdot \underline{k}_0 + \underline{k}_0 \cdot \underline{M}\right)$$

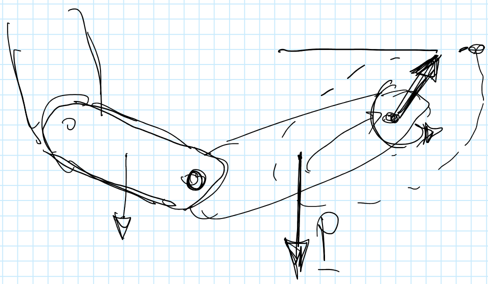
$$\underline{\tau} = -J_g^T \underline{\gamma}$$

giunto R $\Rightarrow \begin{bmatrix} \underline{k}_0 \wedge \vec{AO}_{EE} \\ \underline{k}_0 \end{bmatrix} \xrightarrow{\text{tras}} \begin{bmatrix} \underline{k}_0 \wedge \vec{AO}_{EE} & \underline{k}_0 \end{bmatrix}$

$$M_1 = -(J_g^T)_{1R}\, \underline{\gamma}$$

ricorda $\quad \vec{AO}_{EE} \wedge \underline{F} \cdot \underline{k}_0 = \underline{k}_0 \wedge \vec{AO}_{EE} \cdot \underline{F}$

$M_1 = -\begin{bmatrix} \underline{k}_0 \wedge \vec{AO}_{EE} & \underline{k}_0 \end{bmatrix} \begin{bmatrix} \underline{F} \\ \underline{M} \end{bmatrix}$ come PLV γ



J_g considerando sotto catene