11. Lezione 2 novembre

lunedì 2 novembre 2020 10:41

Trasform. omogenee e gdl

- posizione di un corpo → terna L → T_{SL} matr. omogenea definisce posizione (posa) L rispetto terna fissa S.

$$T_{SL} = \left[\begin{array}{c|c} R_{SL} & [O_L]_S \\ \hline 0^T & 1 \end{array}\right]$$

R_{SL} → ORIENTATA L RISP S

$[O_L]_S$ → POSIZIONE DI UN PUNTO (O_L) DEL CORPO RISP. S

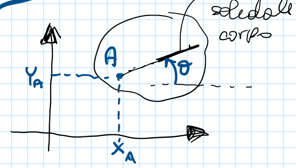
- gdl CORPO RIGIDO 3D

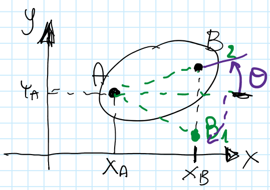
6 GDL (NUM MIN PAR. NEC E SUFF A DEF. POSIZIONE) GDL

$$\underbrace{6\ \text{GDL}}_{3\ \text{DEF. POSIZ. PUNTO}\ +\ 3\ \text{ORIENT.}}$$

→ NEL 2D 3 GDL $\begin{cases} 2 \times \text{PUNTO} \\ 1\ \text{ANGOLO ORIENTATO} \end{cases}$

x_A, y_A, θ ← ANG. ORIENTATO!





PROVA GDL CON COORD 2 PUNTI

$x_A, y_A + x_B / y_B$??

$\underbrace{x_A, y_A, x_B}_{3}$ → 3 C.L. SONO "BUONI"?

NO!

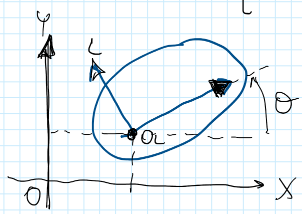
$\overline{AB} = l \qquad (x_B - x_A)^2 + (y_B - y_A)^2 = l$

↳ y_B → 2 soluz.

→ COORD. PUNTO + ANGOLO ORIENT. RIF. FISSO

$$T_{SL} = \left[\begin{array}{cc|c} \multicolumn{2}{c|}{R_z(\theta)} & [O_L]_S \\ \hline 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{cc|c} c\theta & -s\theta & x_{OL} \\ s\theta & c\theta & y_{OL} \\ \hline 0 & 0 & 1 \end{array}\right]$$

6 EL.



$\underbrace{(x_{OL}, y_{OL}, \theta)}_{3}$ → T_{SL}

? CON 3 ELEM.

- NOTA T_{SL} POSSO CON 3 SUOI ELEM TROVARE x_{OL}, y_{OL}, θ?

$\underbrace{(x_{OL}, y_{OL})}_{2} + \begin{cases} T_{11} \\ T_{22} \\ T_{12} \\ T_{21} \end{cases}$ → NON POSIZIONE UNIVOCA

- T_{SL} RAPPRESENTAZIONI NON MINIME

LEGATA A R_{SL}

↳ DATA R_{SL} E' POSSIBILE RISALIRE A θ?

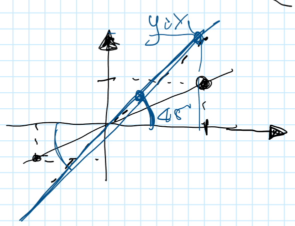
- SI MA NON CON 1 SOLO ELEMENTO

2D

$\theta \longrightarrow R$ RAPPR. NON MINIMA

$\theta \longleftarrow \text{atan2}(\underbrace{X, Y}_{\text{2 INPUT}})$

es. R_{11}, R_{21}



- $\text{atan}(1) \to 45°$, $225°$ 2 POSSIBILITÀ
- $\text{atan2}(3, 3) \to 45°$

CASO 3D

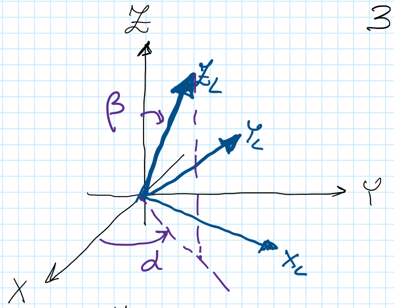
$$T_{SL} = \begin{bmatrix} \overset{3\times3}{R_{SL}} & \overset{3}{[O_L]_S} \\ \underline{0}^T & 1 \end{bmatrix}$$

6 GDL $\to$ $[O_L]_S$, (3 ANG. EUL.

- $R_{SL} \longleftrightarrow \underbrace{\text{ANG. EULERO}}$

3 ROTAZ. ELEM

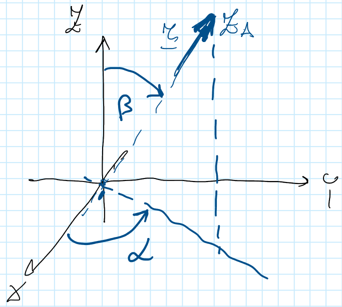
$S \longrightarrow L$ (ROTAZ. FISICHE) (R_{SL})



RICORDA

$R(\underline{k}, \theta)$ — $[R(\underline{k},\theta)]_A = R_{\underline{k}}(\theta)$; EUL. RODR.

- $[R(\underline{k},\theta)]_S = R_{SA}\,[R(\underline{k},\theta)]_A\,R_{SA}^T$



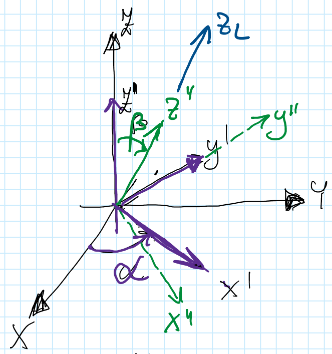
$\underbrace{R_{SS'}}_{R_Z(\alpha)}$ $\quad y' \perp Z, Z_L$

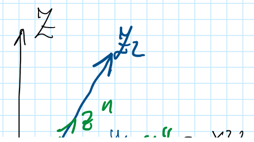
$R_{S'S''} = R_y(\beta)$

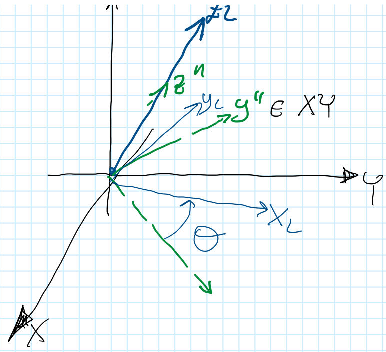
CON 2 ANGOLI ABBIAMO POSIZIONATO Z_L

3 ANGOLO ROTAZ. ATTORNO Z_L x PORTARE $x'' \to x_L$ E $y'' \to y_L$

$R_{S''L} = R_Z(\theta)$







$R_{S''L} = R_Z(\theta)$

COME ROTAZ. FISICA

$S \to S' \to S'' \to L$

COME CAMB. SIST. RIF

$L \xrightarrow{R_{S''L}} S'' \xrightarrow{R_{S'S''}} S' \xrightarrow{R_{SS'}} S$

$R_{SL} = R_Z(\alpha)\, R_y(\beta)\, R_Z(\theta)$

3 ANGOLI
(EULERO)

• ANGOLI **EULERO** $Z-Y-Z$ o $3-2-3$

$X-Z-X$ o $1-3-1$

... ...

• ANGOLI **CARDANO** $Z-Y-X$ assi nominalm. diversi

$Y-X-Z$

RPY { Roll - Rollio; Pitch - Beccheggio; Yaw - Imbardata

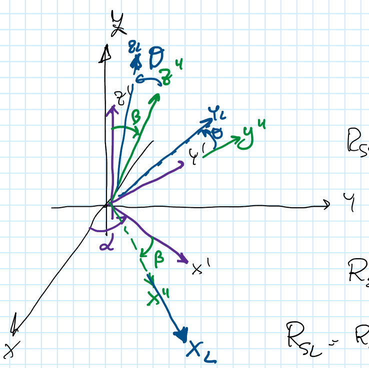
RPY assi fissi ??

EUL assi locali ??

$Z-Y-X$ $R = R_Z(\alpha)\, R_Y(\beta)\, R_z(\theta)$

loc

FISSI



$R_{SS'} = R_Z(\alpha)$

$R_{S'S''} = R_y(\beta)$

$R_{S''L} = R_X(\theta)$

$R_{SL} = R_Z(\alpha)\, R_y(\beta)\, R_x(\theta)$

DOVE α, β, θ DIPENDONO DALLA SEQUENZA SCELTA

$\forall$ SEQUENZA { ~~Z-X-Z~~, Y-Z-Y, Z-Y-X, ⋮ (12) } → 3 ANGOLI

data $\boxed{R_{SL}}$ → ANGOLI EUL. DIVERSI DIPEND. DA SEQUENZA

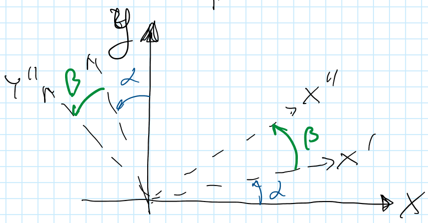
(*) Teor. Eulero 3 ANGOLI

rappresentazione minima orientam.

• $\underset{\alpha}{Z} - \underset{\beta}{Z} - X$ NO DUE ASSI OMONIMI CONSECUTIVI

• $Z - Z - X$ OMONIMI CONSECUTIVI

$\dot{\alpha} \quad \dot{\beta}$



$$R_z(\alpha+\beta) = R_z(\alpha)\,R_z(\beta)$$

$(\underline{r}, \theta)$ inv. naturali / asse - angolo $\to$ rappresent. minima? (3 SCALARI)

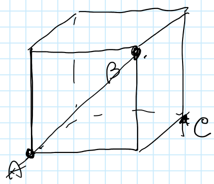
$\begin{cases} \underline{r} \to \text{versore} \to 3 \text{ scal.} \quad r_x^2 + r_y^2 + r_z^2 = 1 \\ \theta \to 1 \text{ scal.} \end{cases}$

4 param. $\to$ $\theta, \ (r_x, r_y, r_z)$

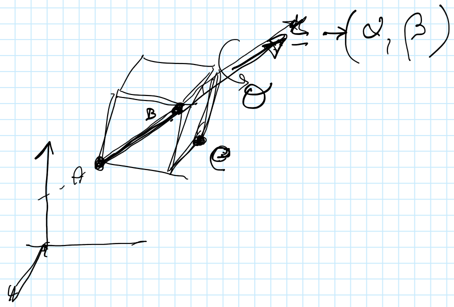
2 ?

$r_z = \pm\sqrt{1 - r_x^2 - r_y^2}$

• NON MINIMA

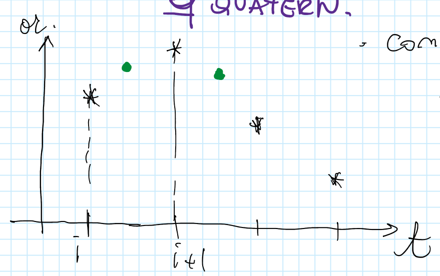


3 PUNTI (NON ALLINEATI)

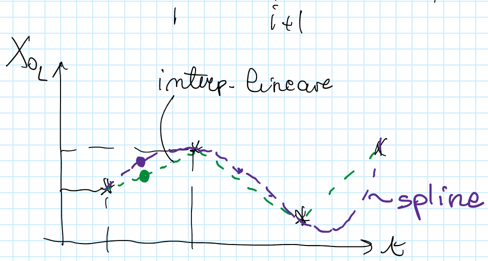


ORIENTAMENTO C.R.G.
- R — **NON MINIMA** — agevole x conti
- $(\underline{r}, \theta)$ — **NON MIN.** — fisicam. + evidente
- ANG. EUL. — **MINIMA**, ARBITRARIETÀ
- 4 QUATERN.

* = sper.
• interp.



• configuraz. discrete in n punti



$R_i \xrightarrow[\text{LINEARM.}]{} R_{i+1}$

$R_{11_i} \to R_{11_{i+1}}$

interp. lineari elem x elem ?

proprietà R ???

$$(\alpha, \beta, \theta)_i \xrightarrow{LINEAR} (\alpha, \beta, \theta)_{i+1}$$

$\alpha_i \to \alpha_{i+1}$

$\beta_i \to \beta_{i+1}$

$\theta_i \to \theta_{i+1}$

α, β, θ $\quad$ Z, Y, Z

$R_{EUL}(\alpha, \beta, \theta)$

$R_{EUL_i}(\alpha_i, \beta_i, \theta_i)$, $R_{EUL_{i+1}}(\alpha_{i+1}, \beta_{i+1}, \theta_{i+1})$

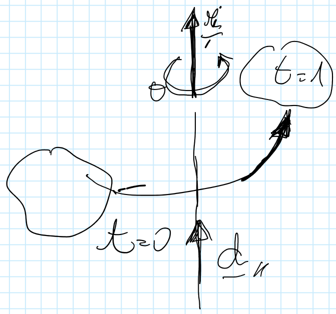
$R_{EUL_{interp}}(\alpha_{int}, \beta_{interp}, \theta_{inter}) \to$ ROTAZ! proprietà garantite

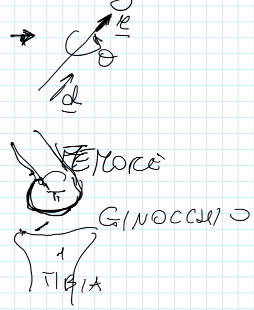
- $R_{EUL}(\alpha, \beta, \theta) = R_Z(\alpha)\, R_Y(\beta)\, R_Z(\theta)$

- <u>ASSE - ANGOLO</u>

$\forall$ sequenza di rotazione è riconducibile ad un'unica rotazione $(\underline{r}, \theta)$ $\boxed{\text{SCREW - AXIS}}$

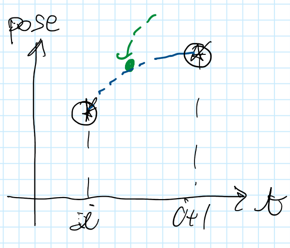
Un qualsiasi spostam. rigido è riconducibile ad un moto a vite $\to$





- <u>QUATERNIONE</u> (UNITARIO)
  deriva da asse - angolo

  $\underline{q}$ $\quad$ SLERP



- <u>interpol.</u> angoli Eulero
  configuraz. interpolata dipende dalla sequenza di Eulero che scelgo

  (α, β, θ) - YZY $\to$
  - ZYZ $\to$
  XYZ $\to$
  configuraz. interp. diverse

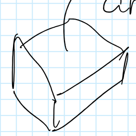
? quale sequenza migliore × avere soluz. interpolate realistiche

→ **quaternioni** × movimento umano e non solo

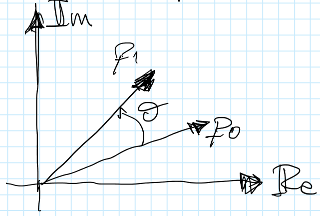
$(\underline{r}, \theta)$ × Hamilton (IMU) Xsens

output sito

BOX × analisi movimento



numeri complessi



- $e^{i\theta} = \cos\theta + i\sin\theta$
- Rotazione vista nel piano complesso

q = $\underset{\text{reale}}{x}$ + 3 unità immagin. i j k

$$\underbrace{\cos\theta/2}_{\text{Reale}} + \underbrace{\underline{r}\ \sin\theta/2}_{\text{Immaginaria}}$$

4 elem. scalari

$$q_i \xrightarrow{\text{SLERP}} q_{i+1}$$