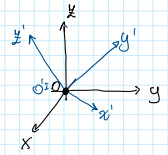
6. Lezione 15 ottobre

giovedì 15 ottobre 2020 08:44

CONCETTI IMPORTANTI IN QUESTA LEZIONE

- Formule per cambiamento sdr con spostamento origine, punti e vettori
- Composizione di cambiamenti sdr
- Coordinate omogenee
- Matrici di trasformazione omogenea

Cambiamento di sistemi di riferimento



• SDR CON STESSA ORIGINE

$$[P]_S = R_{SS'}[P]_{S'}$$

matrice rotazione

$$R_{SS'} = \left[[i']_S \; [j']_S \; [k']_S \right]$$

$$\begin{cases} R_{SS'}^T R_{SS'} = I \\ \det R_{SS'} = 1 \end{cases}$$

coordinate vettori posizione

$$[P]_S = [\overrightarrow{OP}]_S$$

$$\underline{p} = \overrightarrow{OP}$$

$$P - O = \overrightarrow{OP}$$

$$P = O + \overrightarrow{OP}$$

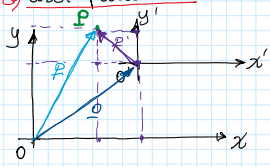
SISTEMI DI RIF. CON DIVERSA ORIGINE

$$S = \{O; x, y, z\}$$

$$S' = \{O'; x', y', z'\}$$

- $\underline{p}$ vett. posiz. S
- $\underline{p}'$ vett. pos. S'

2) assi paralleli

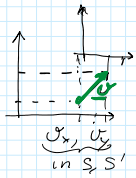


$$\underline{p} \neq \underline{p}' \quad \nabla$$

$$\underline{p} = \underline{p}' + \underline{o} \longleftrightarrow \overrightarrow{OP} = \overrightarrow{O'P} + \overrightarrow{OO'}$$

in componenti in S

$$[\underline{p}]_S = [\underline{o}]_S + [\underline{p}']_S = [\underline{p}']_{S'}$$



v_x, v_y in S, S'

2D e 3D

(*) $[\underline{p}]_S = [\underline{o}]_S + [\underline{p}']_{S'}$ vett. posizione

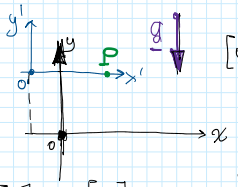
- $[P]_S = [O']_S + [P]_{S'}$ coord. punti

$$P = O' + \overrightarrow{O'P}$$

segm. orient.

$$[P]_S = [O']_S + [\overrightarrow{O'P}]_S$$

$$[\overrightarrow{O'P}]_{S'}$$



$$[O']_S = (-1, 2)$$

$$[P]_{S'} = (3, 0)$$

$$\rightarrow [P]_S = \begin{bmatrix} -1 \\ 2 \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

coord. punto SDR partenza

coord. orig. terna partenza nel SDR di arrivo

$$[\underline{g}]_S = [\underline{g}]_{S'}$$

$$[P]_S = [\overrightarrow{OP}]_S = [\underline{p}]_S$$

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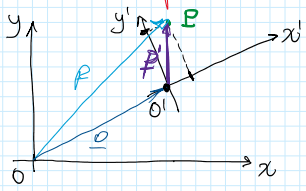
b) assi non paralleli



$$\underline{p} = \underline{o} + \underline{p}'$$

$$[\underline{p}]_S = [\underline{o}]_S + [\underline{p}']_S$$

ma ora $[\underline{p}']_S \neq [\underline{p}']_{S'}$

× matrice rotazione

$$\underline{p}' = p'_x \underline{i} + p'_y \underline{j} + p'_z \underline{k} = p'_{x'} \underline{i}' + p'_{y'} \underline{j}' + p'_{z'} \underline{k}'$$

$$\rightarrow \boxed{[\underline{p}']_S = R_{SS'} [\underline{p}']_{S'}}$$

vett. posizioni

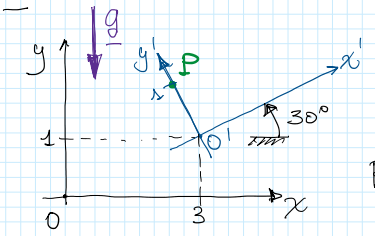
$$\boxed{[\underline{p}]_S = [\underline{o}]_S + R_{SS'} [\underline{p}']_{S'}}$$

forma generale per cambiamento SDR

coord punti $[P]_S = [O']_S + R_{SS'}\,[P]_{S'}$ ✳

$[O']_S$ ← orig. terna partenza

$R_{SS'}$ ← versori $\underline{i}', \underline{j}', \underline{k}'$



$[P]_{S'} = (0, 1)$

$[O']_S = (3, 1)$

$R_{SS'}_{2\times2} = \begin{bmatrix} c30° & -s30° \\ \sin30° & c30° \end{bmatrix}$

$$[P]_S = \begin{bmatrix} 3 \\ 1 \end{bmatrix} + \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{5}{2} \\ \frac{2+\sqrt{3}}{2} \end{bmatrix}$$

$$[\underline{g}]_S = \begin{bmatrix} 0 \\ -9.81 \end{bmatrix} \rightarrow \; ? \; [\underline{g}]_{S'}$$

ATTENZIONE: x i vettori non ci vuole $[O']_S$

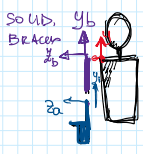
$$[\underline{g}]_{S'} = R_{S'S}\,[\underline{g}]_S = R_{SS'}^T\,[\underline{g}]_S$$

$$[\underline{g}]_{S'} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} 0 \\ -9.81 \end{bmatrix} = \begin{bmatrix} -\frac{9.81}{2} \\ -\frac{\sqrt{3}}{2}\,9.81 \end{bmatrix}$$

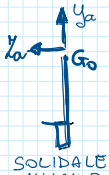
Cambiamento SDR per punti e vettori

$$\begin{cases} \text{VETT. POSIZIONE PUNTI} & [P]_S = [O']_S + R_{SS'}\,[P]_{S'} \\ \text{VETTORI} & [\underline{v}]_S = R_{SS'}\,[\underline{v}]_{S'} \end{cases}$$

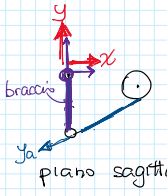
ESERCIZIO MATLAB



piano frontale



SOLIDALE AVAMBR.



piano sagittale

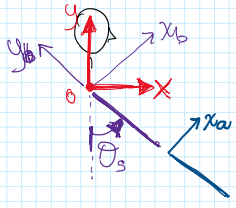
3 SIST. DI RIFERIMENTO

- FISSO $S = \{O; \underset{\text{avant}}{x}, \underset{\text{verticale}}{y}, \underset{\text{verso destra}}{z}\}$ (O: spalla)

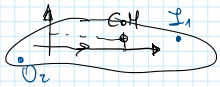
SOLID. BRACCIO $S_b = \{O; x_b, y_b, z_b\}$

terne corpo (mobili)

SOLID. AVAMBR $S_a = \{G_0; x_a, y_a, z_a\}$



→ Terne corpo utili perché dati anatomici gener. espr. terne corpo e rimar. costanti



$[CoM]_{Body} = cost$

+ Riportare punti corpi terna S (da terne corpo)

- braccio camb. SDR $S_L \rightarrow S$ ↷

+ Riportare punti corpi terna S (da terne corpo)

- braccio

camb. SDR $S_b \to S$ → STESSA ORIGINE (spalla)

$$[P_b]_S = R_{S S_b} [P_b]_{S_b}$$

- avambraccio

camb. SDR $S_a \to S$

$$[P_a]_S = [G_o]_S + R_{S S_a} [P_a]_{S_a}$$ caso generale

CONFIGUR. STUDIO

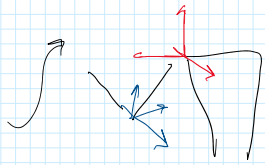
flex gomito $= 0 = \theta_g$

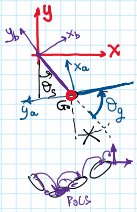
flex spalla θ_s

$$R_{S S_b} = \begin{bmatrix} c\theta_s & -s\theta_s \\ s\theta_s & c\theta_s \end{bmatrix}$$ FLEX = ROTAZ. ATTORNO $\hat{k}$

$R_{S S_a} = R_{S S_b}$

$\theta_g = 0$ S_b — S_a assi // origini diverse





- ANGOLI - ORIENTAM. TRA SEGMENTI CORPOREI

ANG. RELATIVI INFATTI

θ_g NON SI MISURA DA y FISSO MA DA y_b ⇐

$S_a \to S_b \to S$ sequenza di Passaggi tra SDR risalendo catena

$$[P_a]_{S_b} = [G_o]_{S_b} + R_{S_b S_a} [P_a]_{S_a}$$

$[P_a]_{S_b}$: BRACCIO; $R_{S_b S_a} [P_a]_{S_a}$: AVAMB

$$+$$

$$[P_b]_S = R_{S S_b} [P_b]_{S_b}$$

$$[P_a]_S = R_{S S_b} [G_o]_{S_b} + R_{S S_a} [P_a]_{S_a}$$

$$[P_a]_S = \underbrace{R_{S S_b} [G_o]_{S_b}}_{[G_o]_S} + \underbrace{R_{S S_b} R_{S_b S_a}}_{R_{S S_a}(\theta_s, \theta_g)} [P_a]_{S_a}$$

COMPOSIZIONE $\begin{cases} \cdot R_{S S_b}(\theta_s) \\ \cdot R_{S_b S_a}(\theta_g) \end{cases}$

RUOTARE PUNTI INSIEME

$$[Punti]_S = R_{SS'} [Punti]_{S'}$$

(Punti: 3 × m)

$$[P]_S = [O']_S + R_{SS'} [P]_{S'}$$

$[O']_S$: 3×1 — $[P]_{S'}$: Punti, 3 × m — NON COMPAT.

COORD. OMOGENEE

2D 3×1 / 3D 4×1

$$\{P\}_S = \begin{Bmatrix} [\ ]_S \\ 1 \end{Bmatrix}$$

SI AGGIUNGE 1

$$\{\underline{v}\}_S = \begin{Bmatrix} [\underline{v}]_S \\ 0 \end{Bmatrix}$$

VETTORE

PUNTI O VETT. POSIZIONE

$$T_{SS'} = \begin{bmatrix} R_{SS'} & [O']_S \\ \underline{0}^T & 1 \end{bmatrix} = \begin{bmatrix} [\underline{i}']_S & [\underline{j}']_S & [\underline{k}']_S & [O]_S \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \{\underline{i}'\}_S & \{\underline{j}'\}_S & \{\underline{k}'\}_S & \{O'\}_S \end{bmatrix}$$

STRUTTURA

$$\begin{Bmatrix} [P]_S \\ 1 \end{Bmatrix} = \begin{bmatrix} R_{SS'} & [O']_S \\ \underline{0}^T & 1 \end{bmatrix} \begin{Bmatrix} [P]_{S'} \\ 1 \end{Bmatrix} = \begin{Bmatrix} R_{SS'}[P]_S + [O']_S \\ 1 \end{Bmatrix}$$

$0 \rightarrow R_{SS'}[\underline{v}]_{S'}$

RACCHIUDE TUTTE LE INFORMAZIONI + CAMBIAM. SDR

$$\{P\}_S = T_{SS'} \{P\}_{S'} \leftarrow \text{COORD. OMOGENEE}$$

$$[P]_S = R_{SS'} [P]_{S'}$$

MATRICE DI PUNTI 4 × m

- $\{Punti\}_S = T_{SS'} \{Punti\}_{S'}$ — 4×m = 4×4 · 4×m

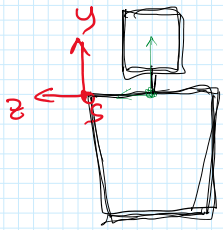
$$[AT \cdot \ T_{SS'}^{-1} \neq T_{SS'}^{T}]$$

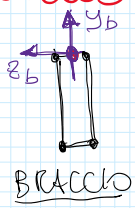
- $\{P_a\}_S = T_{SS_b} T_{S_b S_a} \{P_a\}_{S_a}$ — $T_{SS_b} = \theta_S$, $T_{S_b S_a} = \theta_g$

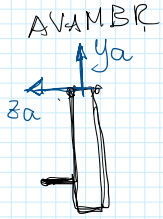
× COMPOSIZIONE + IMMEDIATA

Preparazione Matlab

Preparazione Matlab







1) identif. corpi, SDR solidali
2) scrivono Punti_corpo nella relativa terna corpo
3) matrici T di passaggio terne → GDL attribuiti al modello (movimenti da studiare)
4) ..... come x avambraccio ...