Applied Signal Processing Laboratory

Assignment 4: Array Signal Processing

Francesco Laterza, 271087

06/06/2023

Contents

1 Introduction to antenna array signal processing

1.1 Exercise 1

In this exercise we are going to generate an array pattern of a ULA (Uniform Linear Array) using the Conventional Beamforming technique.

1.1.1 Parameters

The pattern is characterized by the number of antennas N and from the distance between the elements d. Initially we set $N = 8$ and $d = \lambda/2$.

We also choose as starting DoA (Direction of Arrival) $\theta_1 = 0°$

1.1.2 Pattern computation

To compute the pattern we first compute $\Psi = 2\pi\frac{d}{\lambda}\sin(\theta_1)$ in order to compute the vector of weights w as

$$w = \frac{1}{\sqrt{N}} \cdot \begin{bmatrix} 1 \\ e^{j\Psi} \\ \\ \vdots \\ e^{j(N-1)\Psi} \end{bmatrix}$$

We compute also the steering matrix V as

$$V(\theta_k) = \begin{bmatrix} 1 \\ e^{j\theta_k} \\ \\ \vdots \\ e^{j(N-1)\theta_k} \end{bmatrix}$$

It is possible to compute the pattern as a vector by matrix multiplication pattern$= w^H V$

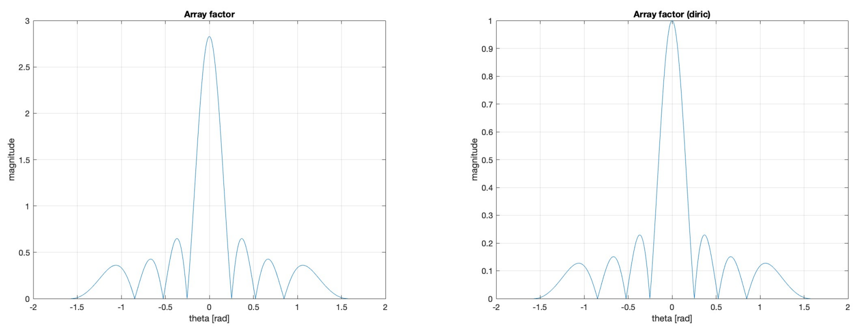

Figure 1: Pattern generated by a uniform linear array with 8 elements

In Figure 1 it is plotted the results of the computed pattern. On the left is shown the pattern computed as explained before, on the right is shown the pattern computed with the *diric* function with parameters Ψ and N. The only difference is that the diric function is normalized to have the maximum at 1, while the pattern we computed has the maximum at $\sqrt{N}$.

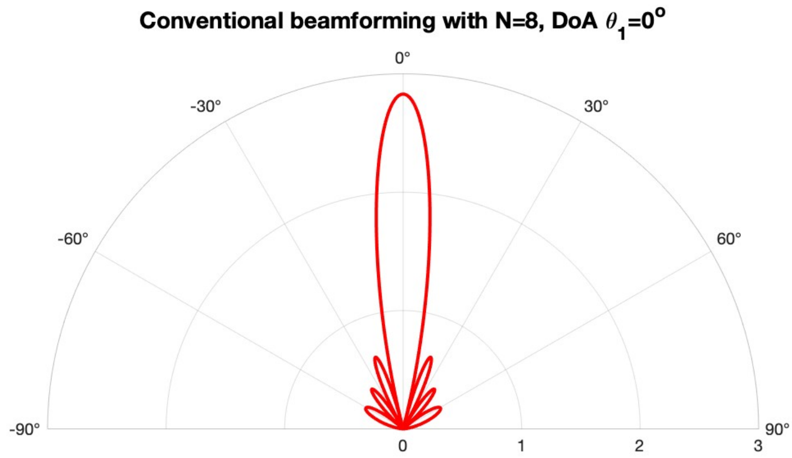

Figure 2: Pattern generated by a uniform linear array with 8 elements

In Figure 2 instead it is possible to observe the pattern plotted in a polar plot.

1.1.3 Effect of changing the number of elements

We select a different value of N and compute again the pattern. We will use $[2, 12, 20, 50]$ as values for N.

In Figure 3 it can be seen how changing the number of elements changes the pattern generated. The more elements we have, the more the pattern will be directive. Also the maximum in each computed pattern coincide with the $\sqrt{N}$.

1.1.4 Effect of changing the distance between the elements

We select a different value for d/λ and compute again the pattern. We will use $[0.25, 0.5, 1, 2]$ as values for d/λ.

In Figure 4 it can be seen how changing the distance between the elements changes the pattern generated. It can be notice that increasing the distance, the directivity increases. But for values greater than 0.5 aliasing is present, so other grating lobes appear. For this reason we can count the number of maxima and their position for each case.

For case 1 and 2 we have only 1 maximum at 0°. For case 3 we have 3 maxima at [-90°, 0°, 90°]. For case 4 we have 5 maxima at [-90°, -30°, 0°, 30°, 90°].

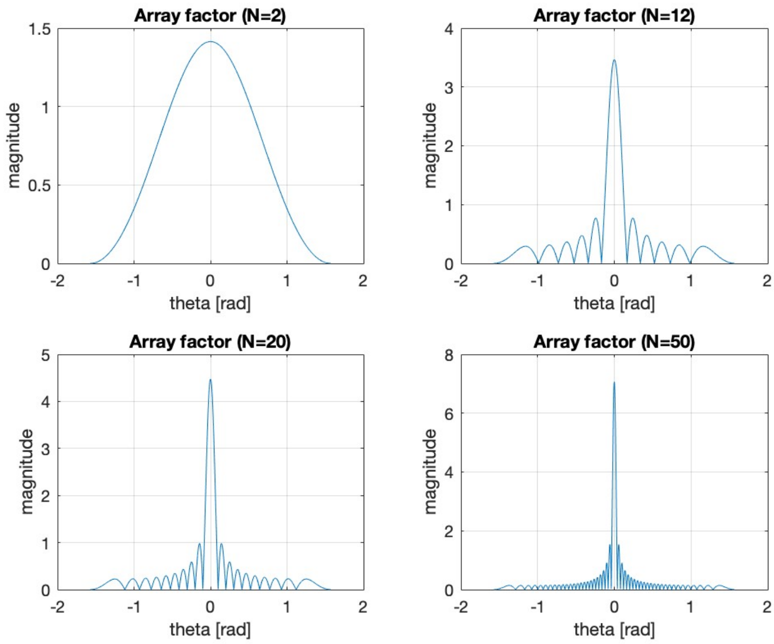

Figure 3: Pattern generated by a uniform linear array for different values of N

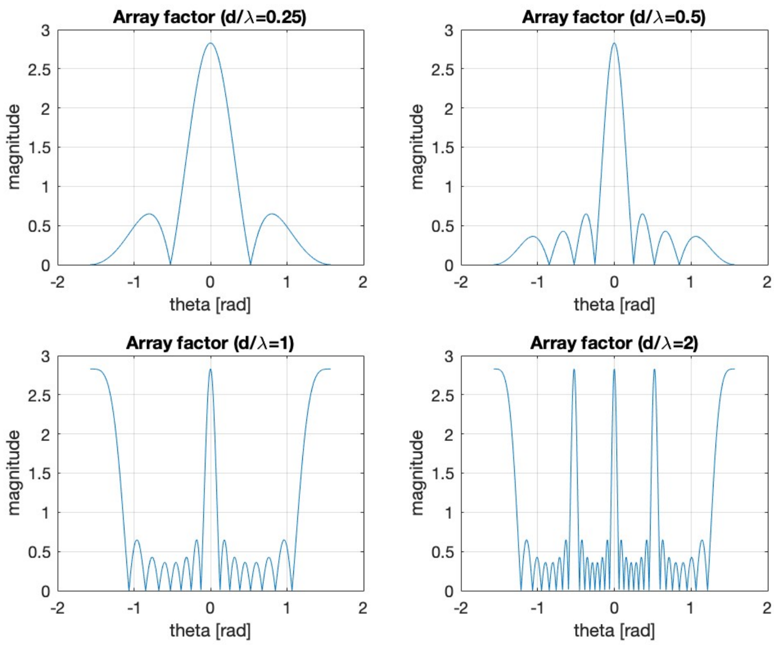

Figure 4: Pattern generated by a uniform linear array for different values of d/λ

1.1.5 Nulls

We can compute the position of the nulls and check for what value of d/λ we have the best directivity without grating lobes. This is obtained when the last null is at 90°.

The resulting value of d/λ is 0.5 as expected.

1.1.6 FNBW

At last, we set $\theta_1 = 30°$ and compute the $FNBW$ as the span in degrees of the grating lobe. The result is 97.1808°.

Now we set $\theta_1 = 60°$. The minimum N that allows the first null to be less than 90° is 4.

1.2 Exercise 2

In this exercise we are going to generate an array pattern of a ULA using the MVDR Capon Beamforming technique. We set the starting parameters as before and we define a vector of 5 DoA of the disturbances [20°, -40°, 60°, -75°, 80°].

1.2.1 Pattern computation

First we compute the vector of weights w. In order to compute it we define the matrix $A(\theta)$ formed by the steering vector of the UE plus disturbances. Now we can compute the covariance matrix $R_y = A(\theta)A(\theta)^H + \sigma_n I$, where $\sigma_n = 10^{-5}$.

Finally we can compute $w = \frac{R_y^{-1} a(\theta_1)}{a(\theta_1)^H R_y^{-1} a(\theta_1)}$, where $a(\theta_1)$ is the steering vector of UE.

The pattern is computed by matrix multiplication as $pattern = w^H V$, where V is the matrix of steering vector.

In Figure 5 it can be seen the polar plot of the computed pattern. It his clearly visible how the pattern is zero in the direction of the disturbances.

1.2.2 Reducing first interferer distance

Now we reduce the first interference distance to 10° and then 5° to observe how the pattern change

In Figure 6 is plotted the pattern. The main lobe is not anymore on the direction of the UE. Since the interference is closer the lobe has to be shifted to keep magnitude 1 in the direction of UE.

1.2.3 Increasing excessively the interferers

Lastly we increase the number of interferers to 31. The MVDR cannot handle such a value.

In Figure 7 it can be observed that the pattern is the same as in the case of the conventional beamforming since there are too many interferences.

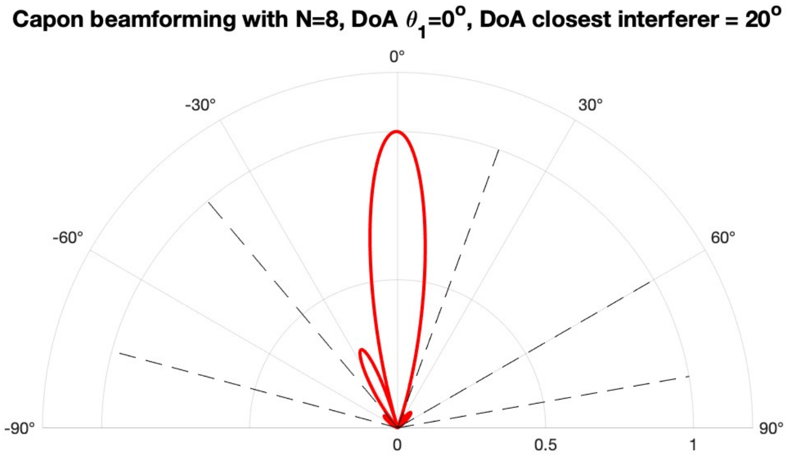

Figure 5: Pattern generated by a uniform linear array with Capon beamforming with closest interferer at 20°

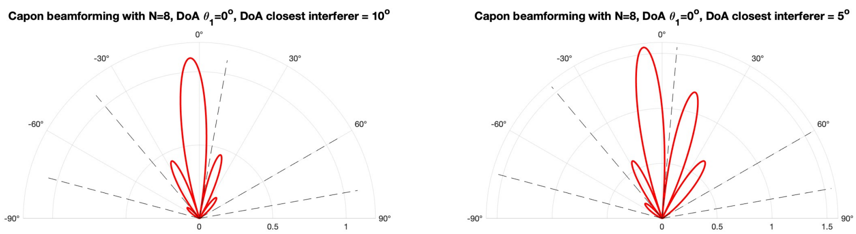

Figure 6: Pattern generated by a uniform linear array with Capon beamforming with closest interferer at 10° and 5°

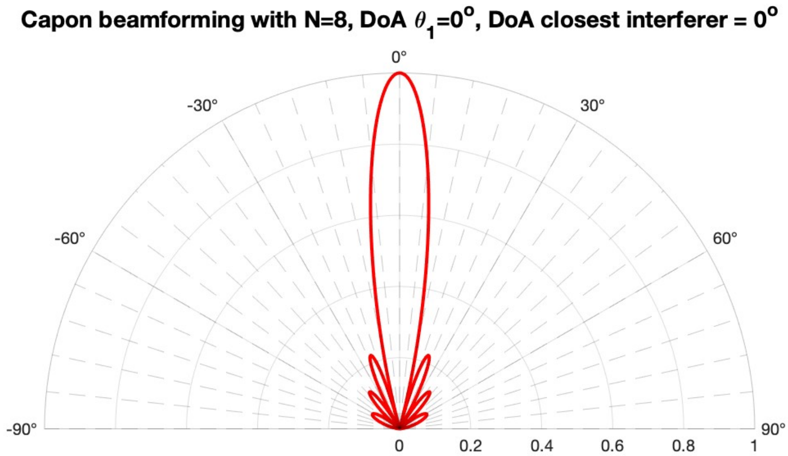

Figure 7: Pattern generated by a uniform linear array with Capon beamforming with 31 interferers

2 Linear and planar array processing

Notice: for the following exercises, other than the usual Matlab file, there will be available another file called *ex_4_{3/4}_complete*. The only difference is that the file ending in "complete" runs the script automatically for all the cases asked to be tested in the exercises, while in the other one the parameters have to be changed manually.

2.1 Exercise 3

In this exercise we are going to generate an array pattern of a UPA (Uniform Planar Array) using the Conventional Beamforming technique.

2.1.1 Pattern generation

In order to generate the pattern for a UPA, we can apply the same method of the ULA case. Though the array in the ULA is 1-dimensional, instead in the UPA is 2-dimensional.

In the computation of the pattern nothing changes. We can compute the steering vectors independently for the 2 dimensions. The final respose will be the Kronecker product between the two vectors.

The initial parameters to be set this time are: the number of elements for each dimension N_z and N_y, the DoA elevation angel and azimuth θ and ϕ and a flag *directive* to chose if the antenna elements are isotropic or directive.

2.1.2 Directive Element

Let's generate the pattern of a single directive element with directivity function

$$d(\theta, \phi) = 0.25 \cdot [1 - \cos(2\theta)] \cdot [1 + \cos(\phi)]$$

in order to do it is enough to set N_z and N_y both to 1.

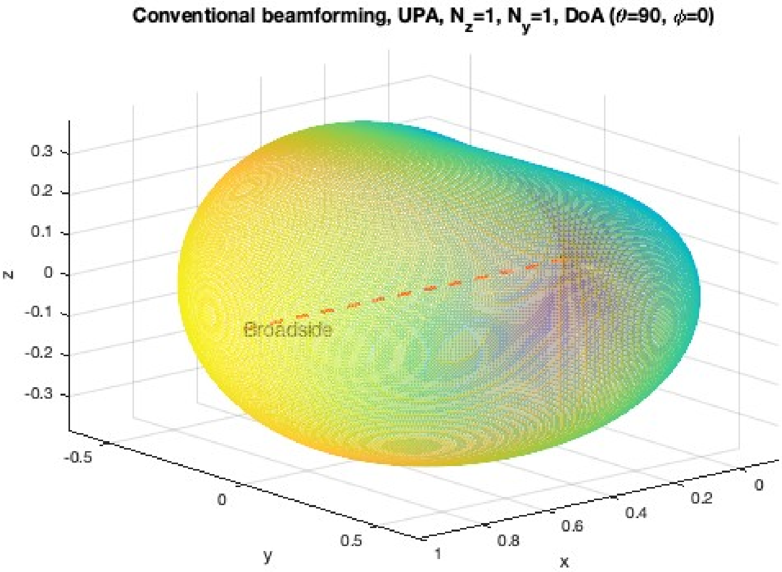

Figure 8: Pattern generated by a single directive element

In Figure 8 it is possible to visualize what the pattern of the single directive element looks like.

2.1.3 UPA pattern

Now we can generate the pattern for a UPA made up by 8 elements for each direction. We compute it both with isotropic and directive antenna elements. The DoA is set to broadside, thus (90°, 0°).

In Figure 9 it is plotted the computed pattern. It can be noticed how the directive elements generate a pattern that propagates only in the positive x axis, while the isotropic elements generate a pattern specular in the positive and negative x axis.

2.1.4 Different DoAs

Now we change the DoA to see the differences in the pattern. We test the following configurations: $(\theta, \phi) = (105°, 30°)$ and $(\theta, \phi) = (70°, -45°)$

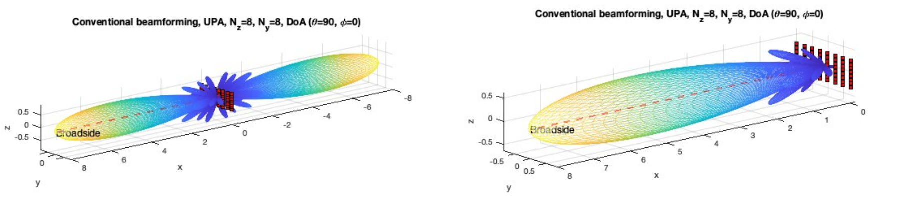

Figure 9: Pattern generated by a UPA with 8 horizontal elements and 8 vertical elements

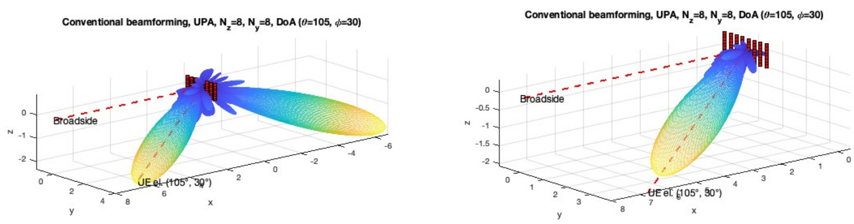

Figure 10: Pattern generated by a UPA with DoA (105°, 30°)

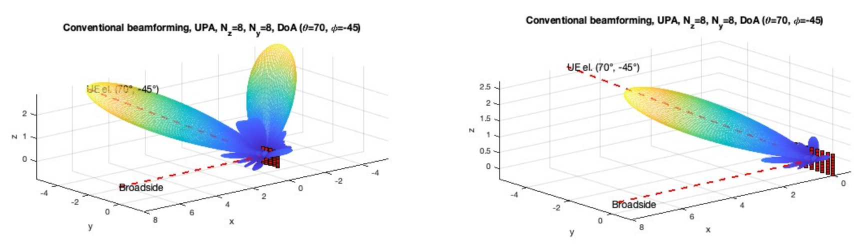

Figure 11: Pattern generated by a UPA with DoA (70°, -45°)

In Figure 10 the pattern with DoA equal to (105°, 30°) is plotted both for isotropic and directive antenna elements. The direction of the main lobe is now shifted from the broadside.

In Figure 11 we can see the result for another DoA, which is (70°, -45°).

Now we set another direction of arrival which differs more from the broadside. We first set 32 elements horizontally and 4 elements vertically. We select as DoA (100°, 60°).

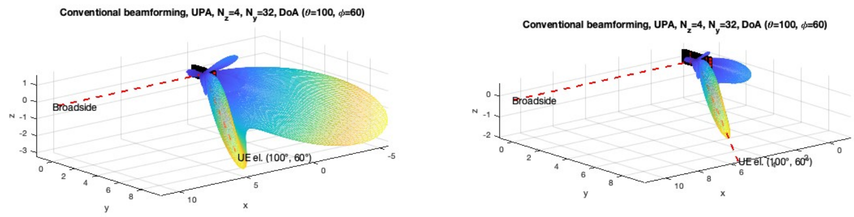

Figure 12: Pattern generated by a UPA with DoA (100°, 60°)

In Figure 12 we can see the plot of the computed pattern. Notice how if we get far enough from the broadside the main lobes enlarge. Settings N_y to 32 helps keeping the pattern directive in the y-axis, while in the z-axis it enlarges loosing directivity.

2.2 Exercise 4

In this exercise we are going to generate an array pattern of a UPA using the MVDR Capon Beamforming technique. We define a vector of 5 DoA of the disturbances θ = [86°, 85°, 80°, 100°, 105°] and ϕ = [4°, 20°, 5°, -15°, 15°].

2.2.1 Pattern computation

The pattern is computed as in Exercise 3, the difference is in the weights vector computation. Also the weights are computed as in Exercise 2, though the steering vectors, used in the computation of the covariance matrix, has to be computed for both dimensions and then the final response is given by the Kronecker product between them.

In Figure 13 the pattern is plotted. Notice how the main lobe is not perfectly directed in the UE direction, but shifts to avoid the closest interference and still get magnitude 1 at UE.

2.2.2 Directive elements

Now we compute again the pattern but using directive antenna elements

In Figure 14 instead the pattern computed using directive antenna elements is plotted. The propagation happens almost only in the positive x-direction.

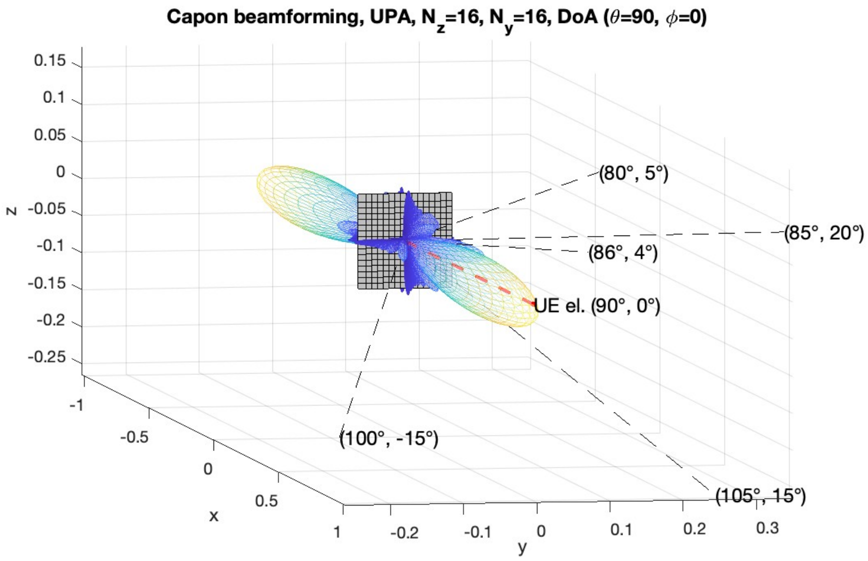

Figure 13: Pattern generated by a UPA with DoA broadside

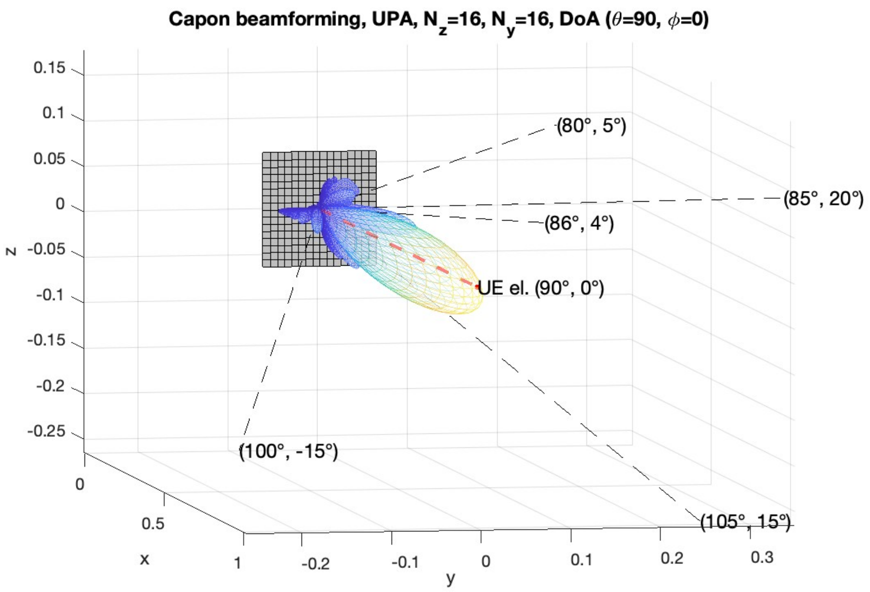

Figure 14: Pattern generated by a UPA with directive antenna elements and DoA broadside

2.2.3 Different DoA

We now reduce the distance of the first interferer from the UE to see the changes in the pattern. We set the closest interferer to (88°, 2°).

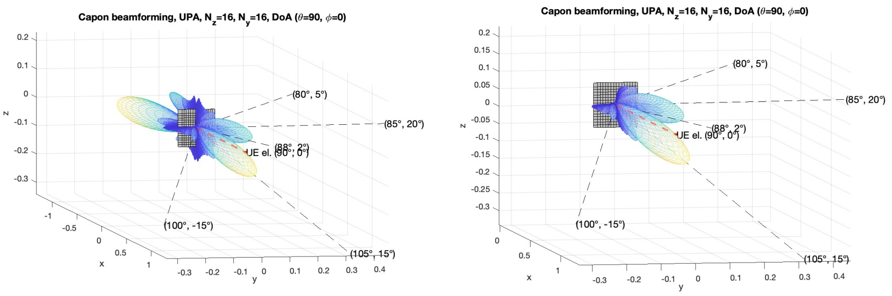

Figure 15: Pattern generated by a UPA with directive antenna elements and DoA broadside, closest interferer (88°, 2°)

In Figure 15 the results are plotted both in the case of isotropic and directive antenna elements. The pattern changes a lot to avoid the closest interference and still keep magnitude 1 at UE.

2.2.4 Increasing the variance of the interference

The exercise ask to increase the value of σ_n^2 from 10^{-5} to 1. Though in my script no difference was present. Thus, I increased the value of σ_n^2 up to 10^3.

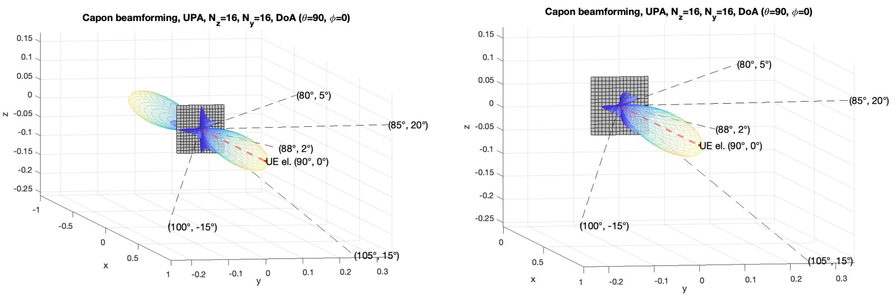

Figure 16: Pattern generated by a UPA with directive antenna elements and DoA broadside, closest interferer (88°, 2°)

In Figure 16 the pattern is plotted in the case $\sigma_n^2 = 10^3$. Notice that for a value this high the MVDR is not able to generate a pattern so the result is the same of the conventional beamforming.