浙江大学爱丁堡大学联合学院

ZJU-UoE Institute

Comparing multiple means

ADS 2, Lecture 13

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So, you know how to do a t-test

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But what if you want to compare more than two means?

Learning Objectives

After this week, you should be able to ...

- Design and interpret a simulation-based hypothesis test
- Use a simulation-based test to compare more than two means
- Discuss limitations of t-tests
- Discuss problems around multiple testing

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More than 1 predictive variable

- Effect of diet *and* exercise on health

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More than 2 populations

- Effect of different drugs on recovery from injury
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- ...

More than 1 predictive variable

- Effect of diet *and* exercise on health
- Effect of genetic background *and* drugs on stress levels
- Differences in height by gender *and* birth province
- ...

OK, we can't just run a t-test here

Why?

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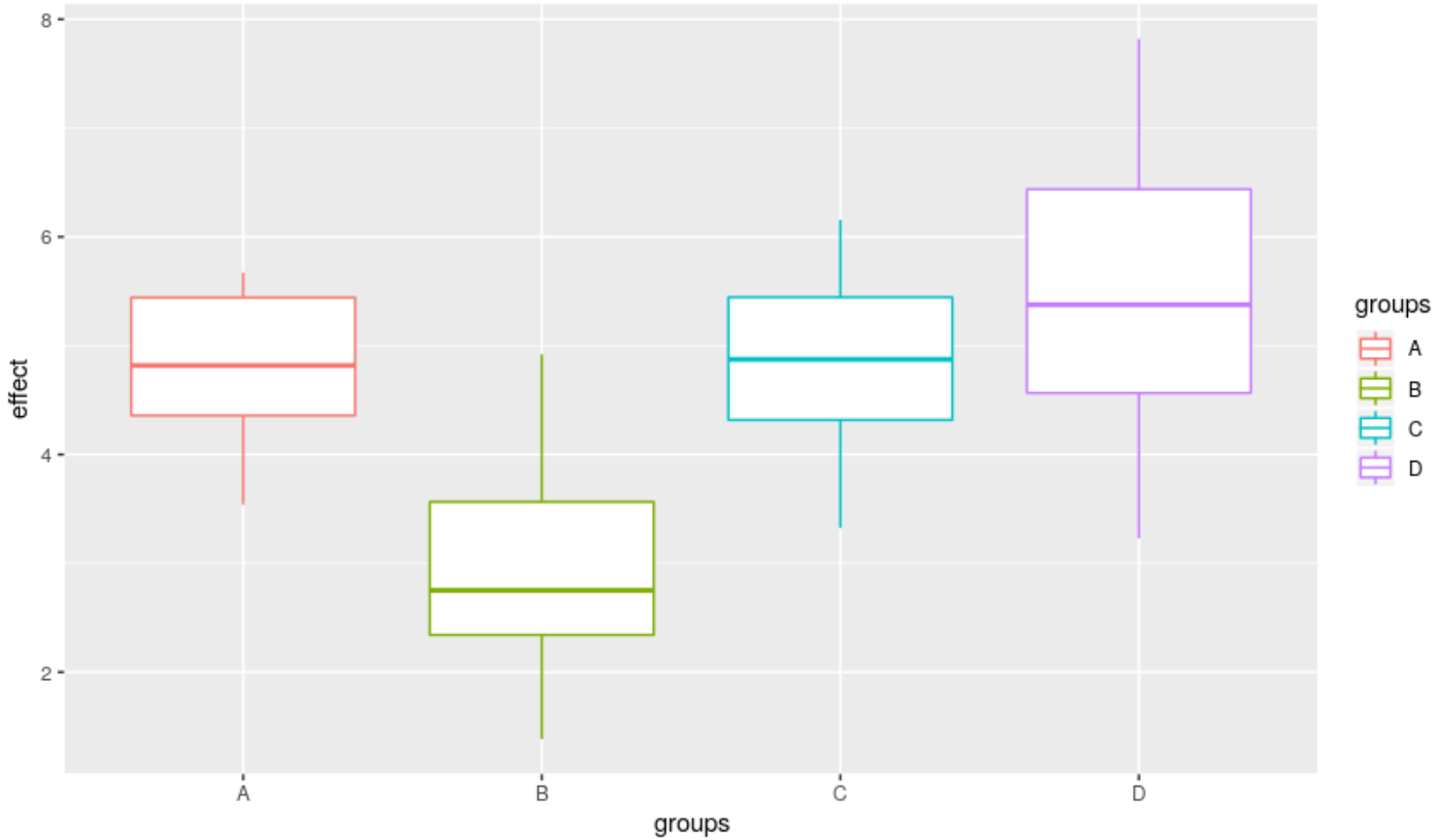
Why?

OK. But maybe we can run several t-tests?

Example: Comparing four groups



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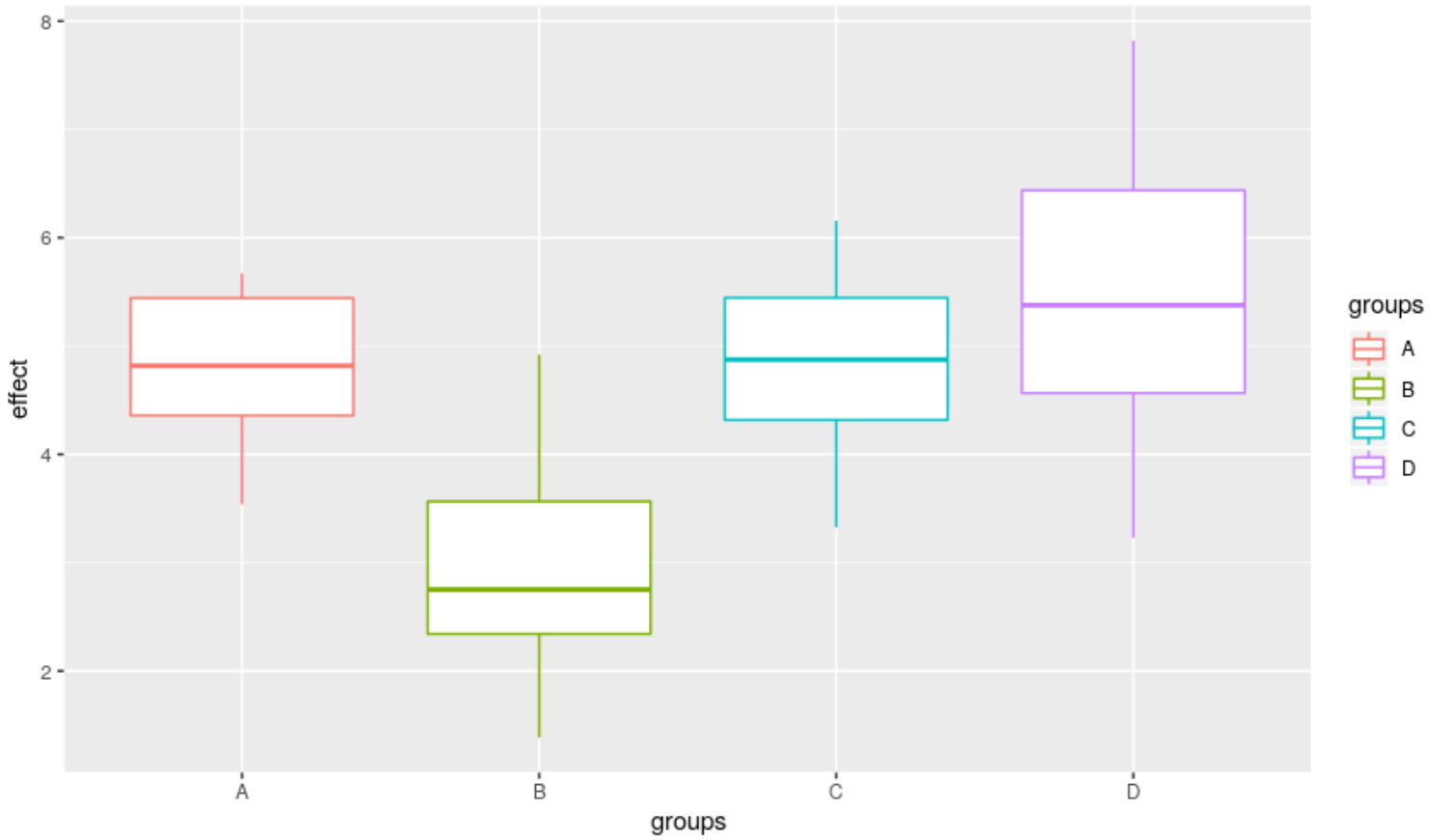

How many t-tests would you need to run?

Example: Comparing four groups

Comparing:

A to B
A to C
A to D
B to C
B to D
C to D

t-test review

What is the probability of getting a false-positive result if there really is no difference?

If you are not sure, think about what happens when we do a t-test. What does your p-value mean? How do you use it to decide?

t-test review

- When computing the p-value we ask:
  If H0 is true, what is the probability of seeing a result as or more extreme as the one we saw in our experiment?

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- But every so often, even if H0 is true, we will see a result as or more extreme as the one we saw. *How often exactly?*

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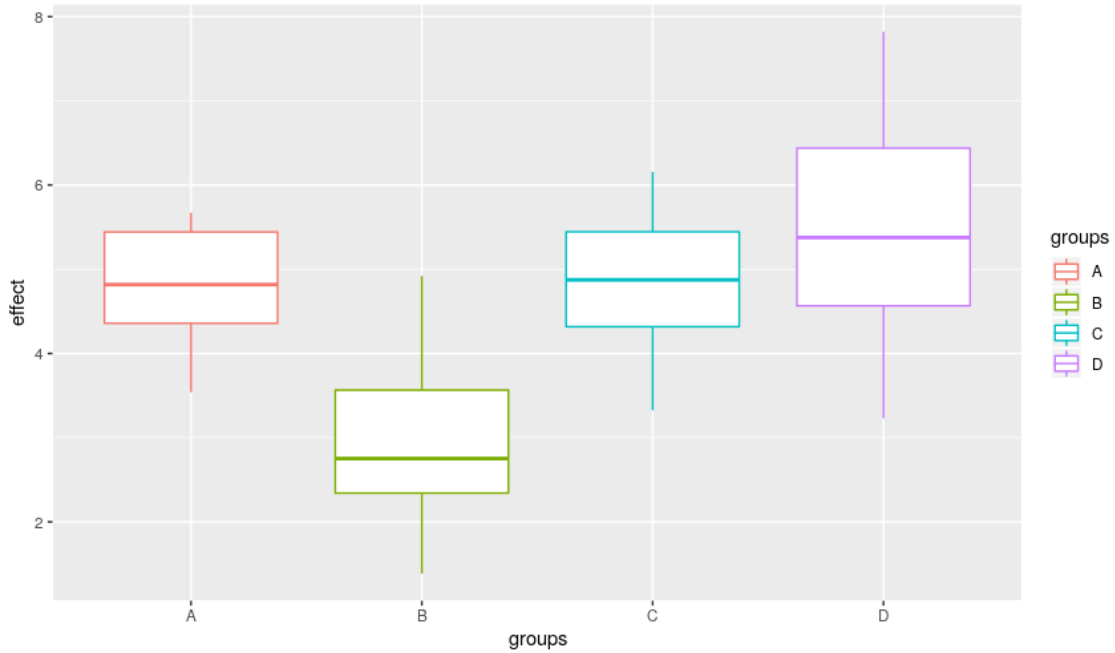
- If we set α to 0.05, this means that we are happy with p-values lower than 0.05
- But every so often, even if H0 is true, we will see a result as or more extreme as the one we saw. *How often exactly?*
- By setting α, we accept a certain risk of seeing a false positive (given H0 is true). This is exactly what α is.

Assume there is really no difference

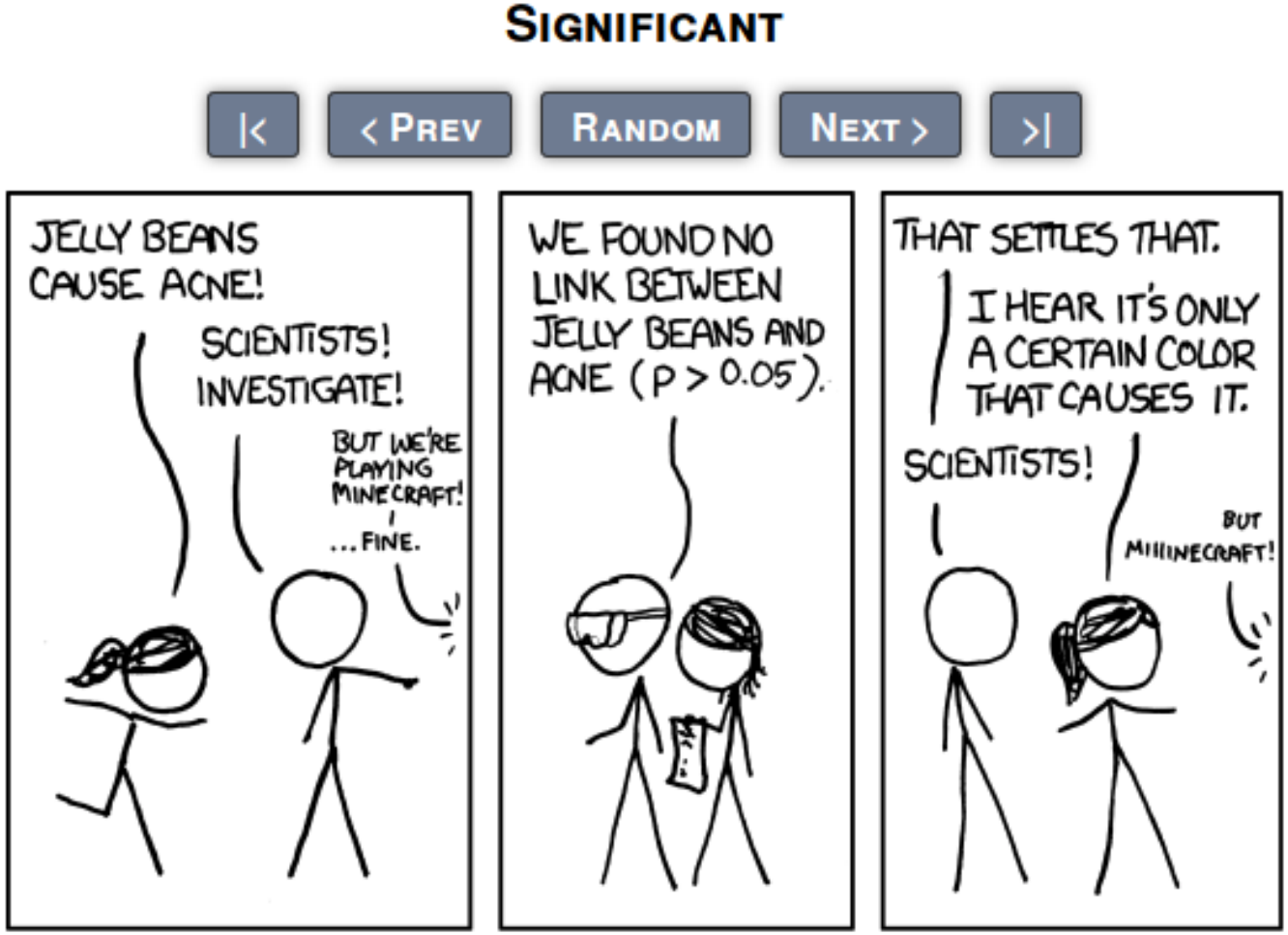


In 6 t-tests with $\alpha = 0.05$ for each, what is the probability of getting at least one false positive result?

Let's do the math

$$
\begin{aligned}
& P(\text{at least one false positive}) = \\
= \quad & 1 - P(\text{no false positives}) = \\
= \quad & 1 - (0.95)^6 = \\
= \quad & 1 - 0.735 = \\
= \quad & 26.5\,\%
\end{aligned}
$$

Very relevant xkcd comic



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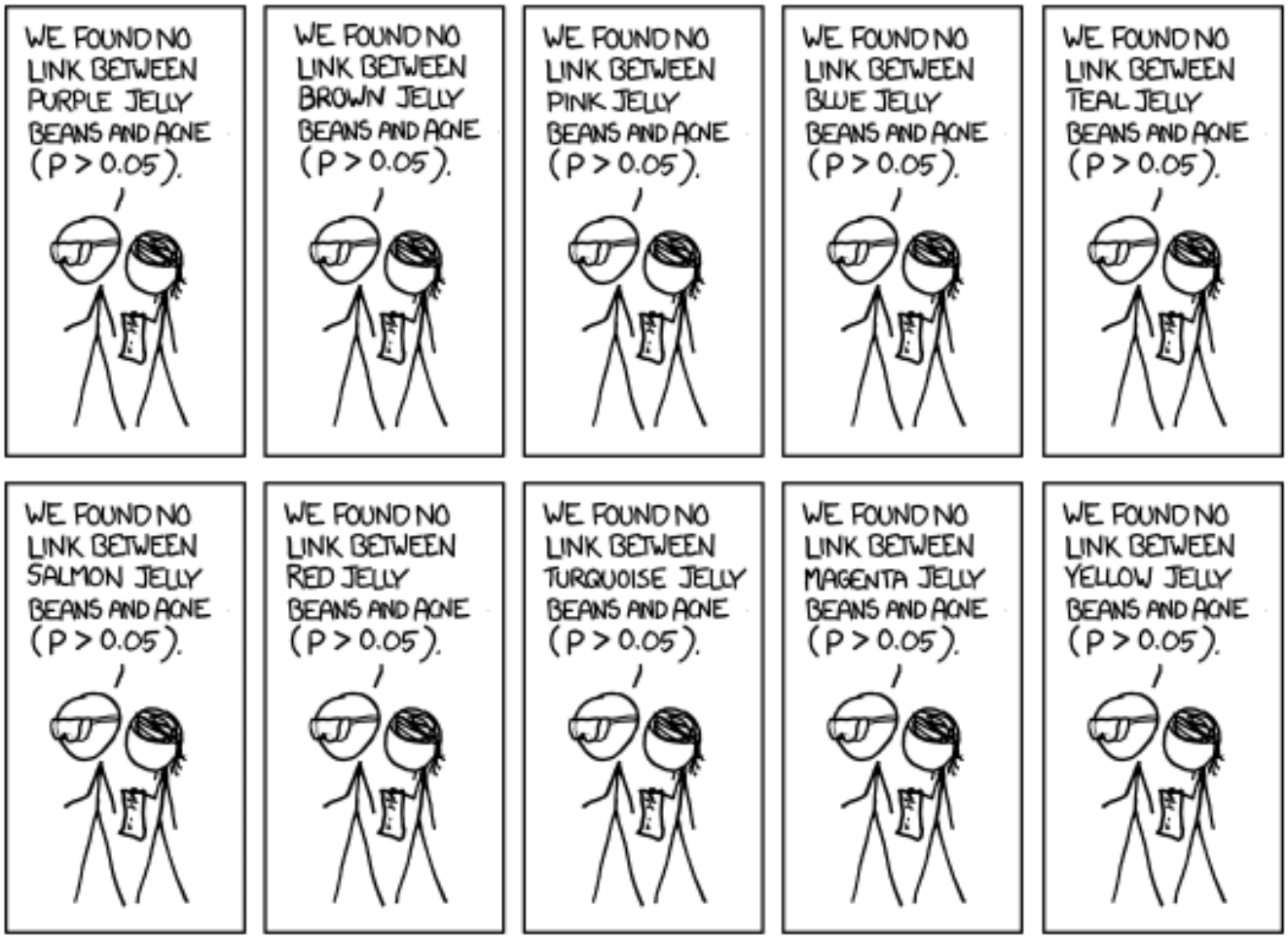

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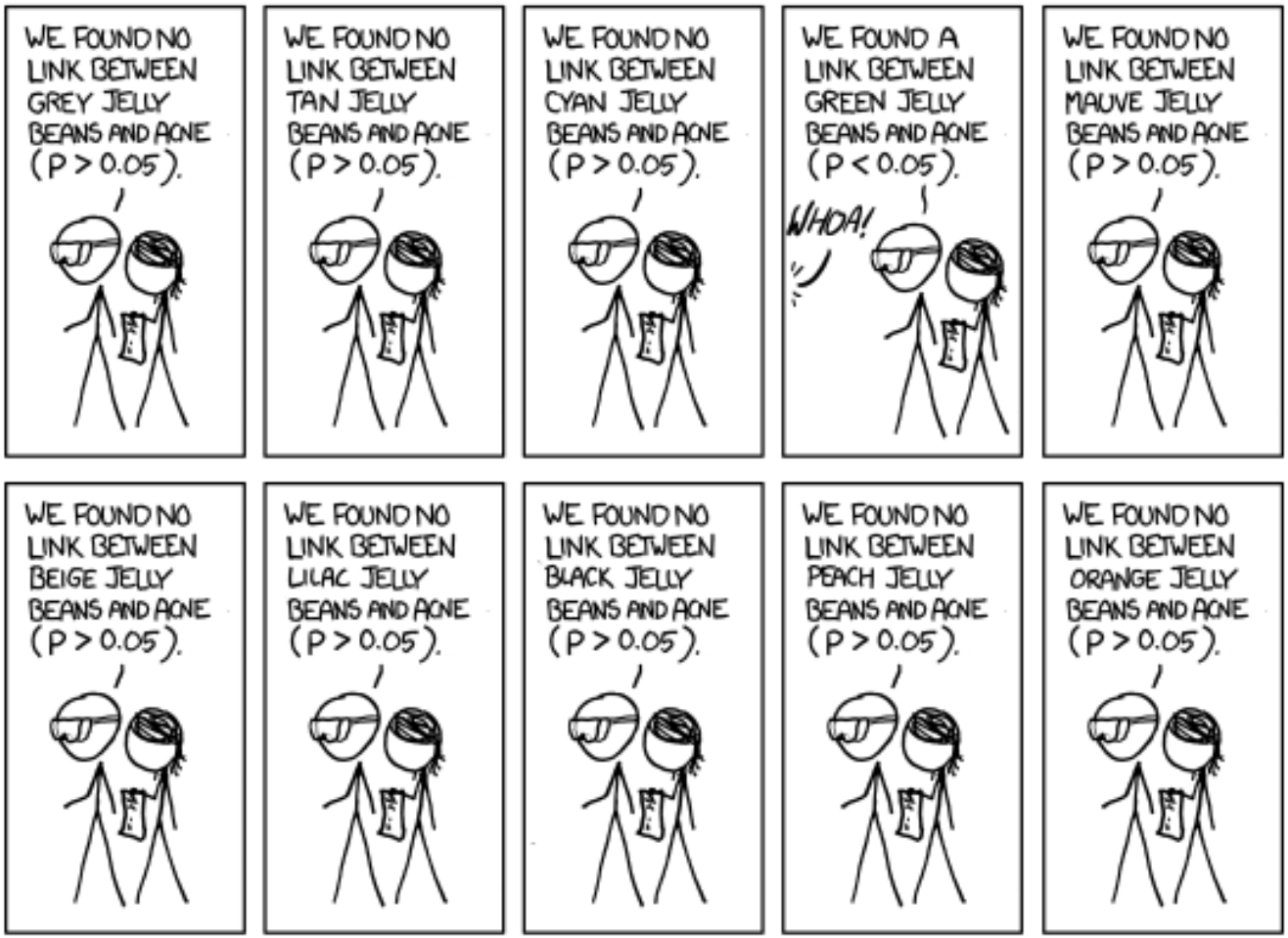

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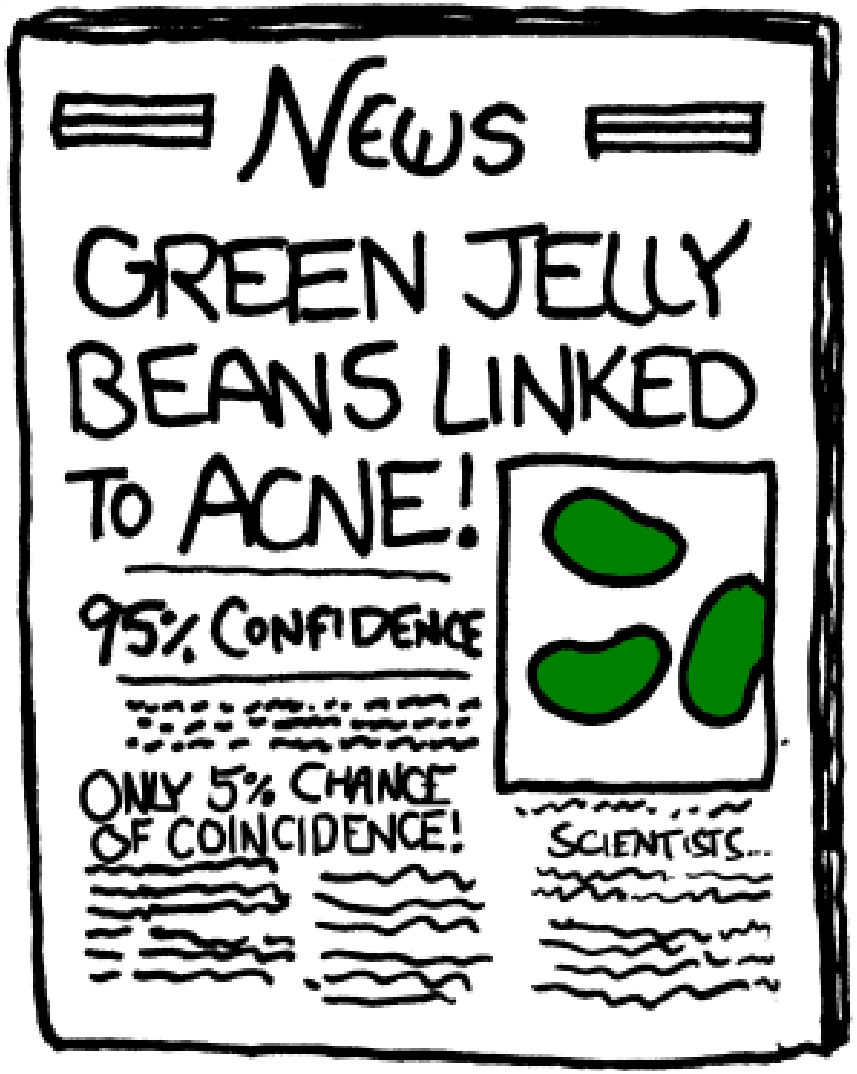

Yes, OK, but . . .

If we can't do a bunch of t-tests, what other option do we have?

Key idea

Looking not at group means, but at variation between individuals.

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Looking not at group means, but at variation between individuals.

Key question

If I select two individuals from different groups, are they going to be more different than if I select two individuals from the same group?

Can you think of another way of phrasing this question?

Key idea

Looking not at group means, but at variation between individuals.

Key question

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Alternative formulation

How much of the variation between individuals is explained by differences **between** groups (as opposed to differences **within** the same group)?

How does this help?

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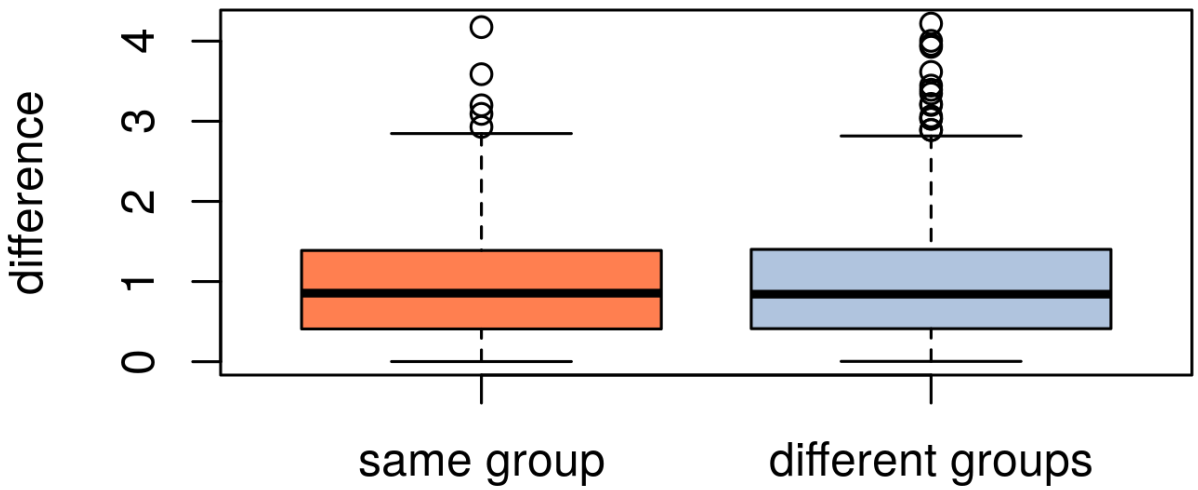
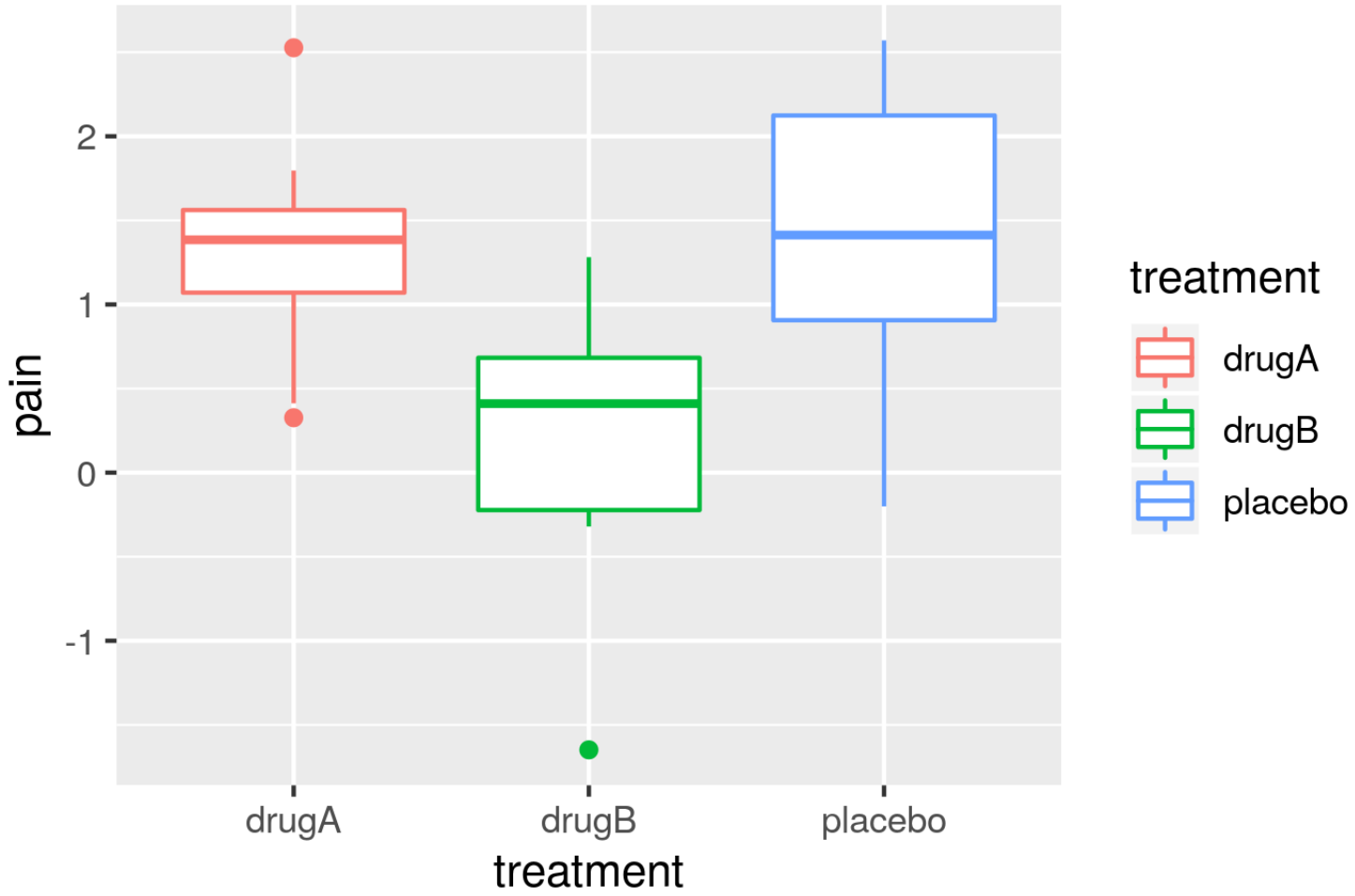


It once more gives us two things to compare, reducing the problem to a single test.

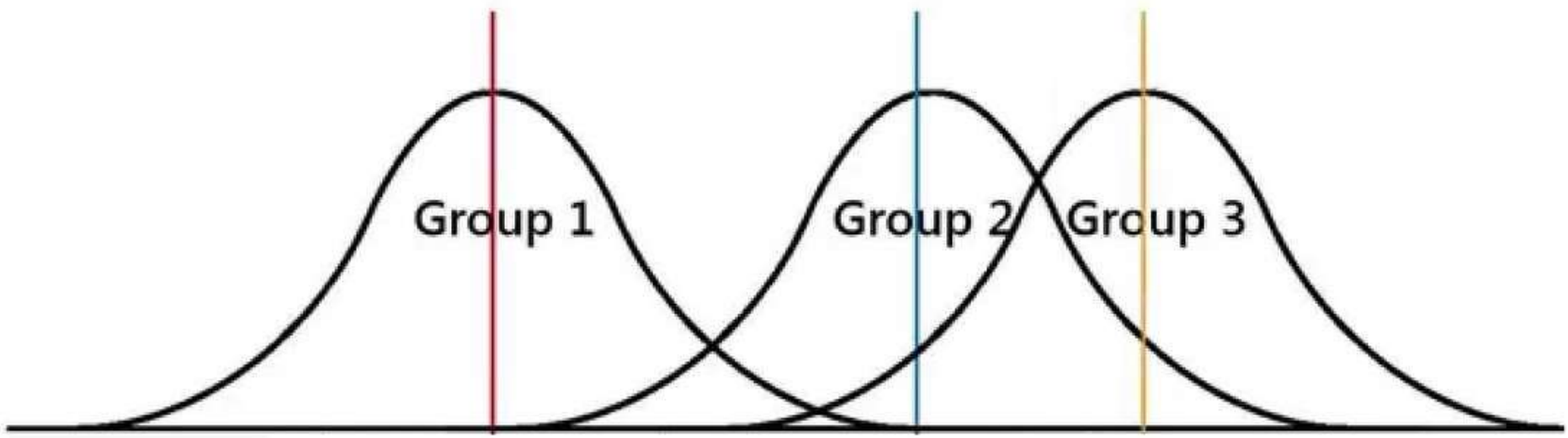
Preview: This afternoon's practical

Using a simulation-based approach to determine whether there are differences between 3 groups.



Preview: Next week's lecture

A more formal look at ANalysis Of VAriance



What questions do you have?

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Image credits

- ANOVA. From a lecture by Nicola Romano for ABMS2, 2018/10.
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