Implementing quantum Galton boards

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1 Overview

This paper aims to present the implementation of a Quantum Galton Board (QGB) using quantum circuits which mimic the behaviour of probabilistic classical Galton boards. This summary is heavily based on the *Universal Statistical Simulator* [1], which uses Monte Carlo methods on the QGB to produce normal distributions.

2 Classical vs Quantum Galton Boards

A classical Galton board consists of balls and pegs; when the ball hits a peg, it has an equal chance to go to the left or right. Due to the random nature of these events, after a sufficient number of layers, the distributions converge to a normal distribution. Similarly, our QGB uses different gates to achieve the same distribution. A control qubit is used to determine whether the ball went left or right at each peg. By measuring the output, it yields the position of the ball with the correct probabilities, allowing us to estimate the distribution using a Monte Carlo simulation.

3 Circuit Architecture

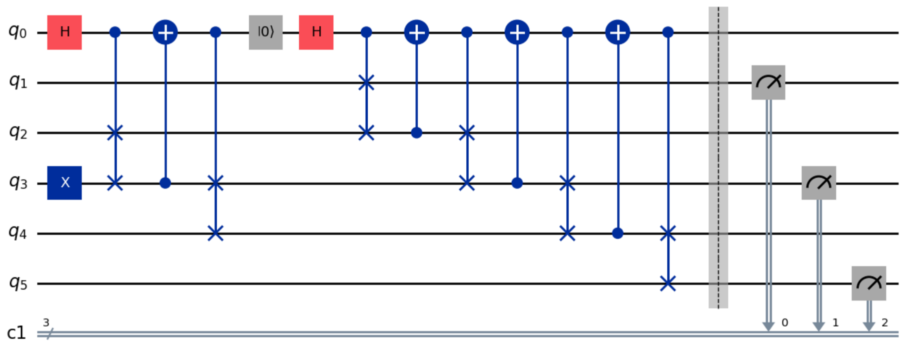

Figure 1: Quantum Galton Board circuit for a board of two layers with the ball starting in the middle using Qiskit. The first qubit q_0 is the control qubit. Looking at the circuit from the right, it resembles a Galton board.

For our circuit in Fig. 1, all the qubits are initialised in the —0¿ position. The ball is placed in the middle of the board by having a X gate act on it. The coin q_0 is used to determine whether the qubit moves up or down. Each layer of the Galton board can be split into sections given by where the coin is reset and flipped. The flipping of the coin in this is applying a Hadamard gate to the coin qubit. This puts the coin in an equal superposition, the ball has a chance to then moved a up a row using a Toffoli gate. If the ball does move up the board the Controlled Not gate straight after swaps the coin to ensure it doesn't also move down. This keeps the classical Galton board of no interference between each peg. At the end of the circuit the qubits which correspond to the bins of the board are then measured. This means it is possible to do Monte-Carlo simulations on the circuit to find the probabilities of each bin.

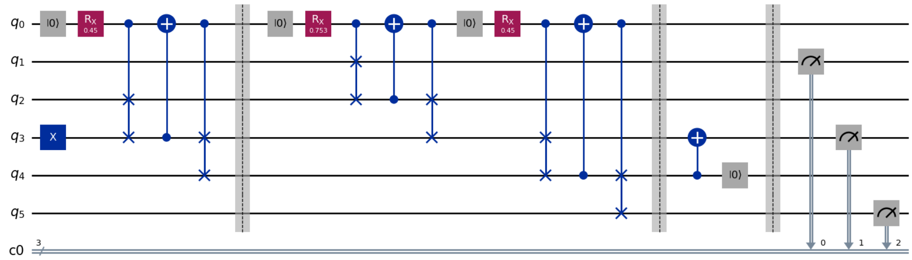

Figure 2: A quantum circuit with different RX gates to adjust the probability of each peg independently.

4 Biasing the Distribution

This bias can be applied uniformly to all pegs or individually to each peg for fine-grained control Fig.2 shows a biased QGB of two layers where each peg is tuned with a different R_x angle, resulting in an exponentially decaying final distribution when measurements are taken. As each peg has a different probability associated to it at each peg the coin qubit is reset and given the correct probability.

At the end of the circuit there is also a need to have a correct $CNOT$ gate at the end of each layer to correct the results and ensure all of the probabilities are correct.

5 Limitations

- **Noise:** There is a large number of multi-qubit gates, this amplifies the affect of noise smearing the output probabilities.
- **Qubit count:** For larger simulations with more layers the qubit number ramps up as 2n + 1 where n is the number of layers. Significant qubit usage
- **Mid-circuit reset:** Control qubit on hardware cannot always be reset. On many superconductor qubits each coin toss must be done on a new qubit to prevent errors. [2]

6 Conclusions

The Quantum Galton Board is easy to implement and generalise for any number of layers as long as you are consistent in the gates. It is a valuable tool for demonstrating quantum walks, probabilistic processes, and Monte Carlo sampling in quantum computing. It is very modular in where you can adjust peg biases to produce various probability distributions, making it useful for simulation tasks.

The QGB illustrates how classical stochastic systems can be mapped to quantum circuits, highlighting both the opportunities and limitations (noise, qubit requirements) of current quantum hardware. With future improvements in hardware, QGBs could become a practical component in larger quantum statistical simulation frameworks.

References

[1] Carney M, Varcoe B. Universal Statistical Simulator. *arXiv preprint* arXiv:2202.01735 [quant-ph]. 2022. https://arxiv.org/abs/2202.01735

[2] DeCross M, Chertkov E, Kohagen M, et al. Qubit-reuse compilation with mid-circuit measurement and reset. *Physical Review X*. 2023;13(4):041057. Available from: https://doi.org/10.1103/PhysRevX.13.041057