

Do Bookmakers Predict Outcomes Better than Betters?

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Abstract

Recent research has suggested that bookmakers display superior skills to bettors in predicting the outcome of sporting events. In this paper we use matched data from traditional bookmaking and person-to-person exchanges to test this hypothesis. Employing a conditional logistic regression model, we find that betting exchange nominal odds have more predictive value than the corresponding bookmaker odds for 693 horse races run in the UK. We attribute this to the favourite-longshot bias. Secondly, we repeat the regressions for probabilities adjusted for bias, and find that the betting exchanges continue to predict outcomes more accurately. Finally, to control for potential spillovers between the two markets, we repeat the analysis for cases where prices diverge significantly. In this case the predictive advantage is reversed, with bookmaker odds apparently yielding more valuable information concerning race outcomes than the exchange equivalents.

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1. Introduction

There have been many studies of the efficiency of horse race betting markets, based either on bettor determined prices (as in pari-mutuel markets), or bookmaker determined prices, the latter mainly based on UK data (see, for example, Smith, Paton and Vaughan Williams 2005; Sung, Johnson and Bruce, 2005). Most studies of bookmaker markets indirectly infer the superiority of bookmaker skills over bettor skills from the existence of persistent negative returns to bettors in aggregate. However, if bettors receive consumption utility from placing wagers, in addition to utility from monetary returns, bettor superiority may be consistent with aggregate negative returns. Furthermore these studies tell us nothing about the abilities of bettors who choose to refrain from entering the market when they judge that bookmaker prices overstate the true chances of race entrants.

A study by Levitt (2004) purports to measure the relative assessments of market makers and bettors, with reference to data from a handicapping competition which is based on US National Football League matches. Levitt characterises the difference between conventional financial asset markets and betting markets as follows: in the former the complexity of information affecting the value of assets is such that market makers cannot gain an advantage through superior processing of information to the market as a whole. In contrast, market makers in betting markets (bookmakers) possess skills in assessing the true chance of various outcomes superior to most bettors, and at least as good as the sub-set of most skilful bettors. Levitt infers this conclusion from his analysis of handicapping data. He suggests that the structural consequences of this differential degree of sophistication are that spot markets equalising supply and demand prevail in conventional financial assets markets, with market makers earning the bid-ask spread, whereas profit maximising bookmakers set prices to exploit bettor biases, constrained only by the presence of the smaller number of

unbiased bettors. Bookmakers therefore earn the equivalent of a bid-ask spread (known as over-round), and an additional return accruing from their exploitation of bettor biases. One consequence of this tendency of bookmakers to act as price makers is that individual books will expose them to positive risk, as bookmakers assume long and short positions exploiting bettor biases.

A disadvantage of the Levitt approach is that, for his data, bookmakers set the terms of the transaction, and bettors respond with a simple decision whether to bet or not. The most skilful players in this situation may be exercising their talents most effectively in cases where they leave specific games alone, but these decisions are not measured in the Levitt study. A more comprehensive test of the relative sophistication of bookmakers and bettors in assessing the true chances of a range of outcomes would permit bettors to express alternative prices to bookmakers, and such prices would constitute the distribution of revealed preferences of bettors.

If one could construct an alternative competition where bookmakers and bettors pitted wits, with each setting their own price for every possible outcome to an event, and repeat this comparison over many events, it would be possible to measure the relative degree of predictive accuracy of the two parties.

We are fortunate that this experiment can now be observed to occur spontaneously in a set of parallel betting markets that has developed in the UK in recent years. The first of these markets is the competitive array of bookmaker fixed odds for specific races available to bettors on the internet. The second is to be found in the person to person markets, or betting exchanges, which have revolutionised the betting industry in the UK in recent years (Jones et al 2006).

In this study we use matched odds data from bookmaker and betting exchange markets for 693 UK horse races in order to measure empirically the accuracy of probability

assessments implicit in the prices. The bookmaker data are traditional fixed prices (odds), whereas betting exchanges, whose clients are generally non-bookmakers¹, offer a parallel set of fixed odds, enabling an assessment of which set of prices has the greatest predictive value. In this way it is possible to directly compare the relative evaluative skills of bookmakers and bettors in assessing the outcome of races. It is unlikely given the nature of these markets that price manipulation by those seeking to distort price accuracy will be a problem but in any case Hanson, Oprea and Porter (2006) argue convincingly that there is little scope for manipulators to distort price accuracy.

Our study is completed in three stages, in which we utilise conditional logistic regression, a maximum likelihood estimation (MLE) technique, as well as measures of returns to a simple unit wagering strategy. At stage 1 these methods are applied to the datasets in aggregate, with odds probabilities normalised to give a rational probability distribution, and non runners removed. At this stage we expect betting exchange prices to be more accurate than bookmaker data as predictors of race results. This is because it has been shown that, whereas both bookmaker and betting exchange markets hold a structural bias known as the *favourite-longshot bias* (whereby low probability runners or longshots are overbet, and high probability runners are underbet), this phenomenon is more extreme in bookmaker markets than in the betting exchanges (Smith, Paton & Vaughan Williams 2006).

Shin (1991, 1992, 1993) argues that the bias evident in bookmaker prices is their response to asymmetric information and adverse selection due to the presence of insiders, rather than a fundamental inability of bookmakers to evaluate true probabilities. Therefore a fairer test of the relative skills of odds makers would be to firstly adjust odds for this known

¹ Witness evidence by *Betfair*, the world's biggest betting exchange, to a UK Parliamentary committee indicates that only 0.71% of its accounts belonged to active customers who made more than £15,000 from trading in the previous year (Joint Committee on the Draft Gambling Bill, 2004). Even so, there is anecdotal evidence that bookmakers set up accounts with the exchanges to help manage their liabilities. We take steps to mitigate against the consequent feedback between the two odds sets by recording bookmaker prices at or close to the time they are first published and before bookmaker operations in the exchanges gather momentum, and by including stage 3 of our analysis (outlined in the narrative).

bias before employing MLE. At stage two of the analysis we therefore derive Shin probabilities, an unbiased estimate of objective probabilities (Cain et al 2000, 2002), from the nominal bookmaker and exchange odds, before applying the MLE model.

Our dataset comprises prices set very early in the market so as to minimise the risks of feedback between the two markets. However, we cannot eliminate entirely the possibility of prices having already converged somewhat through bettors using the two markets as benchmarks in arbitrage processes, and bookmakers operating within the exchanges to hedge their liabilities on specific horses. Given this possibility, at stage 3 the MLE procedure is repeated for the subset of horses for which the divergence between bookmaker and exchange odds probabilities is greatest. These horses are arguably subject to the least feedback between the two markets and therefore may permit further insights into the issue of concern to us, namely differences of opinion between bookmakers and bettors as to the chances of the runners in question.

The structure of the paper is as follows. Section 2 outlines in more detail the recent developments in betting markets referred to above. Section 3 describes the data used drawn from bookmaker and betting exchange markets. Section 4 outlines the methodology employed in more detail. Section 5 shows the results of the present study, with discussion. Section 6 concludes.

2. Recent developments in the UK horse race betting industry

Betting exchanges exist to match people who want to bet on a future outcome at a given price with others who are willing to offer that price. The person who bets on the event happening at a given price is the backer. The person who offers the price to an identified sum of money is known as the layer of the bet.

The advantage of this form of wagering for the bettor is that, by allowing anyone with access to a betting exchange to offer or lay odds, it serves to reduce margins in the odds compared to the best prices on offer with traditional bookmakers. Exchanges allow clients to act as backers or layers at will, and indeed to back and lay the same event at different times during the course of the market.

The way in which this operates is that the major betting exchanges present clients with the three best odds and stakes which other members of the exchange are offering or asking for. For example, for a horse named Take The Stand to win the Grand National, the best odds on offer might be 14 to 1, to a maximum stake of £80, 13.5 to 1 to a further stake of £100 and 12 to 1 to a further stake of £500. These odds, and the staking levels available, may have been offered by one or more other clients who believe that the true odds are longer than they have offered.

An alternative option available to potential backers is to enter the odds at which they would be willing to place a bet, together with the stake they are willing to wager at that odds level. This request (say £50 at 15 to 1) will then be shown on the request side of the exchange, and may be accommodated by a layer at any time until the event begins. Every runner in the race will similarly have prices offered, prices requested, and explicit bet limits.

The margin between the best odds on offer and the best odds sought tends to narrow as more clients offer and lay bets, so that in popular markets the real margin against the backer (or layer) tends towards the commission levied on winning bets by the exchange. This commission normally varies from about 2 per cent to 5 per cent. Clients can monitor price changes, which are frequent, on the Internet website pages of the betting exchange, and execute bets, lay bets, or request a price, instantly and interactively on the website.

Bookmakers have also innovated to take advantage of the Internet; and for many races will offer competitive prices for most or all runners in a race. Bettors can access the array of

prices for runners in matrices displayed on sites such as the *Racing Post* or *Oddschecker*. As with the exchanges, bettors can place bets instantly and interactively. Unlike the betting exchanges, however, bet limits are generally not stated, and clients cannot lay or request prices.

4. Data

To facilitate the study we required two sets of odds for the same races: one set attributable to bookmakers and the other to bettors. The first set of prices collected were fixed odds offered by bookmakers. Unlike pari-mutuel prices, these odds do not vary with subsequent fluctuations in the market. The only exception to this is when there are withdrawals of runners in the race, in which case a differential reduction is applied, based on the probability of success of the withdrawn runner or runners implied in the odds.

Bookmakers' prices were gathered for 799 horse races run in the UK during 2002. Sample races were drawn from the 2001-02 National Hunt season, the 2002 Flat season and the 2002-03 National Hunt season. In order to minimise liquidity issues, sampling was restricted to Saturdays and other days where overall betting turnover was likely to be most vigorous. One advantage of sampling over the full calendar year 2002 is that our data should not suffer in aggregate from seasonal bias. Prices were taken from the Internet site of the *Racing Post*, the major daily publication dealing with horse racing and betting in the U.K. Taking prices from the Internet site allows for a direct comparison with betting exchange data. After excluding races with poor liquidity and races abandoned due to adverse weather conditions, we were left with 693 races.

The bookmaker data were matched with corresponding betting exchange prices, both collected at the same time each day, 10.30 a.m. This time was chosen as it gave the market

sufficient time to achieve acceptable levels of liquidity, whilst being early enough to avoid the likelihood of sustained bookmaker operations within the exchanges.

To ensure that bookmaker prices were not merely nominal, a trial was conducted whereby bets were placed to establish that the prices stated could be obtained. Actual bets were small (ranging from £5 to £20), but enquiries were also made with individual bookmakers as to whether much larger bets would be accepted. There was evidence of some limits to bet size set by bookmakers on occasions, but not frequently enough to raise concerns about the integrity of prices in general. In contrast low liquidity was a feature of some of the exchange markets, and we have qualified the results accordingly.

Bet limits on the exchanges are explicit and evidenced by the amounts layers state that they are prepared to accept in wagers on individual runners. Where bet limits were small, the prices offered were ignored, and races where overall betting volume was trivially low were excluded from the sample of races, on the grounds that the market did not have sufficient liquidity to warrant treating such observations as representative². A minimum acceptable aggregate turnover threshold (£2000 per race, by 10.30 am) was applied as a filter to the races in the sample in respect of *Betfair* prices; races where this turnover threshold was not met were screened out of the analysis. After exclusion of races on grounds of low turnover exactly 700 races remained for analysis, seven of which were subsequently abandoned due to adverse weather conditions.

Section 4: Empirical models employed

Our principal test of predictive accuracy in relation to the two markets involves a maximum likelihood technique, giving log likelihood calculations which can be judged

² To avoid sample bias, we were careful to exclude only races where turnover was low with *both Betfair and* bookmakers, as evidenced explicitly on the *Betfair* website, and by inference from bet sizes in trade press results sections in the case of bookmakers, and enquiries made with bookmakers as to bet limits.

against the χ^2 distribution. Least squares regression is an unsuitable choice of estimation technique where predictions of the probability of race entrants winning a race are concerned, as it places no limits on extreme estimated values. In contrast conditional logistic regression derives a function that observes the lower and upper bounds of 0 and 1 respectively which characterise probability distributions. It is a technique that is frequently employed in empirical studies of predictive models in horse race wagering (for good examples, see Figlewski 1979, Bolton and Chapman 1986, and more recently Sung, Johnson & Bruce 2005).

For an individual race j , with n runners, the conditional logistic regression model can be expressed as

$$p_i = \frac{e^{Z_i\beta}}{\sum_{j=1}^N e^{Z_j\beta}} \quad (1)$$

where p_i is the conditional probability of horse i winning race j ; Z is the vector of predictors, in this case the subjective probabilities implied by the odds pertaining to horse i (numerator), and all race entrants, $i = 1 \dots N$ (denominator); and β is the vector of coefficients attached to the predictors.

As a maximum likelihood technique, the estimated coefficients β are those which maximise the likelihood function:

$$L = \prod_{k=1}^M P_w^k \quad (2)$$

where P_w^k is the estimated probability associated with the horse winning the k^{th} race in a sample of M races (using the notation employed by Figlewski 1979).

L is initially calculated with restrictions on the variables Z , i.e. the coefficients are all set to zero. In this initial calculation $p_i = 1/N$ for all runners in the race, representing the situation where there is no information about the race entrants. After iterative estimation of

the coefficients, L is re-calculated by dropping the restriction(s) on one or more of the predictor variables Z . The interpretation of the change in L is that the closer its value is to zero, the more closely are the race outcomes explained by the information held in the predictors, in this case the odds.

McFadden (1973) shows that the value $2(Lu - Lr)$, where Lu is the likelihood function calculated with unrestricted predictors, and Lr is that with restrictions, closely follows the χ^2 distribution. This test therefore permits us to judge whether a set of odds holds significant information about the outcome of races in our sample.

The vector of predictors, Z , in our sample comprises price probabilities corresponding to the odds in our bookmaker and betting exchange data. Bookmaker prices are expressed as fractional odds values by bookmakers e.g. $2/1 = 0.3333$ odds probability; $4/1 = 0.2$, and so on. The recorded value for each race entrant i is based on the mean of observed bookmakers' odds for that horse, expressed as a probability. The mean of the odds array is chosen rather than the outlier on the basis that the former better represents the consensus of bookmaker opinion. As the mean bookmaker odds for race entrants expressed as a probability typically departs from the probabilities corresponding to discrete fractional odds, each odds value predictor in our record of observations has a range with a continuous probability span. The betting exchange odds adopted are the maximum available to significant bet limits. They are expressed on the website as the return inclusive of one unit stake, in decimal rather than fractional format. Increases in odds at the high probability end of the odds scale are expressed in "ticks" of 0.1 point. For low probability runners, prices increase in 1 point increments.

At stage 1 of the analysis, we simply normalise odds probabilities to sum to unity for each race in the respective markets, removing non runners, so that:

$$p_{ij}^n = \frac{p_{ij}^o}{\sum p_{ij}^o} \quad (3)$$

where p_{ij}^n is an estimate of the race specific true probability of horse i winning race j .

This method of proportional normalisation, commonly adopted in earlier studies (for example, Bird and McCrae, 1987; Tuckwell, 1983), tends to over-inflate the estimated chances of the highest probability runners in fields with many runners, due to the disproportionate impact of the favourite-longshot bias in such races (Shin, 1993), and to similarly deflate the estimate in races with small fields. To minimise the aggregate impact of these biases we here use a sample of races closely matching the variations in field size of the population of races as a whole.

The initial estimation of the likelihood function (2) involves restricting to zero the coefficients of all predictors, from both bookmaker and betting exchange data. The resulting log likelihood, Lr , represents the model's best fit with no prior information. Further estimations of the likelihood function are then carried out as in Table 1.

The statistic Lm , if positive and significant at $p = 0.01$, would provide strong evidence that mean bookmaker prices hold valuable information about the outcome of races. We expect this to be the case, as bookmaker favourites won 26.24% of the races in the sample, representing a large improvement on the expected success rate of a randomly chosen runner in each race, 8.61% (693 winners divided by total number of runners in the sampled races, 8,053). Similarly, we expect the betting exchange data alone to add valuable information, evidenced by Lb .

Lm , $L(b,m)$ shows how the log likelihood for Lm changes by adding the exchange odds to the existing predictor of bookmaker odds. The estimation Lb , $L(b,m)$ reverses the order of predictor additions. These two estimations are key to judging the relative information held by the two odds sets, and hence their predictive value.

At stage 2 a method of adjusting for favourite-longshot bias in the odds is required. Our method of choice is to calculate Shin probabilities from the raw odds data. Shin explains the favourite-longshot bias as a result of bookmaker behaviour in the face of insider trading (Shin 1993, pg.1148). He derives a measure of insider trading, z , which is also interpreted as a proxy measure for the degree of favourite-longshot bias; the higher the value of z , the greater the degree of bias. Detailed outlines of the Shin model can be found in Cain et al (2001 appendix), and Law & Peel (2002 appendix).

Shin is primarily concerned with measuring an aggregate value of z for a sample of races, whereas we wish to calculate values of Shin's z for individual races and derive adjusted probabilities for each runner in a race. We use a method attributable to Jullien and Salanie (1994), later utilised by Cain et al (2000, 2002) in a slightly modified but equivalent format, to compute individual probabilities adjusted for bias.

Jullien and Salanie restated Shin's model to show that, for an individual race, the true probability of winning p_i for horse i can be expressed as:

$$p_i = \frac{\sqrt{\left[z^2 + 4 \frac{\pi_i^2}{\Pi} (1 - z) \right]} - z}{2(1 - z)} \quad (4)$$

where z is Shin's measure of insider trading for that race, π_i is the nominal odds probability associated with horse i , and Π is the sum of π_i in the race. Jullien and Salanie showed that z can be estimated using the equation:

$$\sum p_i \left(\frac{\pi_i}{\sqrt{\Pi}}, z \right) = 1 \quad (5)$$

Through an iterative procedure similar to that employed by Shin, the observed values of π_i are substituted into equations (4) and (5) to derive race specific values of z that will yield probabilities from equation (4), adjusted for insider trading and which sum to unity.

Having derived adjusted probabilities in this way we complete stage 2 by repeating the MLE procedures outlined for stage 1 (above and Table1), substituting the unbiased Shin probabilities for the nominal probabilities.

For stage 3 of our analysis we require a suitable measure of price divergence. Law and Peel (2002) employ a measure of price movement which they claim to be superior to that used by Crafts (1985) in his influential study of insider trading. Crafts employed the ratio of odds probabilities associated with starting odds and forecast odds. Law and Peel adopt the alternative measure, pm , such that:

$$pm = \log\left(\frac{1}{1-p_1}\right) - \log\left(\frac{1}{1-p_2}\right) \quad (6)$$

where for an individual runner in a race p_1 and p_2 are the odds probabilities derived from, for example, starting odds and forecast odds respectively (our notation). Unlike the Crafts ratio, equation (6) weights price movements from initially low odds with greater emphasis than those from initially high odds, reflecting the greater trading volumes required to cause odds to change at low odds. For similar reasons we adopt equation (6) as our measure of divergence, pd , between bookmaker mean and exchange odds for each horse, except that p_1 becomes the highest odds probability (lower odds) and p_2 becomes the lowest odds probability (higher odds). Thus our application differs in that that we use equation (6) to measure differences in odds at a point in time, as opposed to differences over time. For illustrative purposes Table 2 indicates the odds divergence for different levels of odds, required to yield specific values of pd .

To complete stage 3 we then repeat the MLE procedures adopted in the previous stages for the set of horses exhibiting the greatest odds divergence, allowing for possible sensitivity of results to our choice of pd constituting high levels of divergence.

5. Results and discussion

The estimation identifiers used in this section follow the descriptions in Table 1 for the MLE iterations based on nominal odds probabilities (stage 1). The corresponding identifiers for the stages 2 and 3 iterations based on Shin probabilities differ only in the use of a subscript, a . Thus the term “ ma ” indicates Shin adjusted mean bookmaker odds probabilities; the term “ ba ” similarly indicates Shin adjusted exchange odds probabilities.

Table 3 summarises the stage 1 results of the initial log likelihood estimates, for normalised odds probabilities derived from bookmaker mean and exchange odds. The χ^2 test statistics in Table 3 correspond to the various restrictions on predictors outlined in Table 1 above.

The log likelihood values for bookmaker odds alone (Lm) and exchange odds alone (Lb) indicate that each set of odds, individually contribute significant information in predicting the outcomes of the races in our sample; the χ^2 test statistic for each is significant at $p=0.01$.

The further measures $Lm, L(b,m)$ and $Lb, L(b,m)$ permit us to judge whether either of the nominal odds sets holds valuable information in addition to the other set alone. The mean bookmaker odds unadjusted for the favourite-longshot bias yield no significant additional information to that contained in the exchange data, a result indicated by the χ^2 statistic associated with $Lb, L(b,m)$ being insignificant. In contrast, when the order of addition is reversed, the exchange data add significantly to the amount of information concerning race outcomes held in the unadjusted mean odds alone, evidenced by a χ^2 value for $Lm, L(b,m)$ significant at $p = 0.05$.

This result may reflect the greater degree of favourite-longshot bias in nominal bookmaker odds; the Shin’s z value for the bookmaker mean odds associated with the races studied, at 2.17%, is significantly greater than that for the exchange odds, at 0.09% (see

Smith, Paton and Vaughan Williams, 2006, for confirmation of the independence of these Shin's z results for the same dataset).

In order to perform the stage 2 MLE tests, this bias was removed using the Jullien and Salanie method outlined above. As the Shin probabilities are estimated independently of results, a useful test of their efficiency in removing bias is to calculate returns for the dataset at odds corresponding to the Shin probabilities themselves. If the Shin adjustments are successful the distribution of returns arising from these calculations should be equal across different odds values. We regress the notional returns to Shin odds equivalents (dependent variable) against odds probabilities corresponding to actual odds (independent variable), with standard errors adjusted for heteroscedasticity, to see if this is the case. A slope coefficient not significantly different from zero will provide evidence that the Shin probabilities are unbiased. Table 4 summarises the estimated coefficients of this regression for the unadjusted and adjusted bookmaker mean and exchange odds.

The pre-adjustment slope coefficients in Table 4 are consistent with the Shin values reported above. The table shows that before Shin adjustment the bookmaker odds contain an appreciable bias: the slope coefficient β indicates that for every 1% increase in odds probabilities returns increase by a highly significant 1.34%. After adjustment the coefficient estimate is reduced to 0.44, insignificant at conventional significance levels. The β value for the exchange odds before the Shin adjustment is also 0.44. This is not significantly different from zero, implying little initial bias. The adjustment of exchange odds decreases the estimate of β to a value very close to zero and insignificant at any level. At stage 2 of the analysis, the initial conditional regressions carried out at stage 1 were repeated with the Shin probabilities. The results are summarised in Table 5. From the test $Lma, L(ba,ma)$ it is apparent that following adjustment for bias there is weakly significant evidence that betting exchange odds continue to add further useful information to that contained in the bookmaker

odds. The reverse is not true, with Lba , $L(ba, ma)$ yielding an insignificant χ^2 statistic. For the full sample of races with probabilities adjusted for bias, it appears that the betting exchange odds still have greater predictive accuracy than the bookmaker equivalents, although the margin of advantage is not as significant as for nominal odds.

Stage 3 of our analysis begins with a summary of returns to different magnitudes of price divergence between the two sets of odds. Table 6 shows cumulative returns to minimum filter pd levels.

It is notable that, for both $ma > ba$ and $ba > ma$, losses can be restricted by concentrating on those horses for which $pd \geq 0.01$; the returns achieved are very similar, at -4.64% and -5.24% respectively, suggesting little difference in the predictive value of the two sets of odds. The proximity of these returns is consistent with the existence of feedback between the two markets, but this does not explain why returns to horses for which $pd \geq 0.01$ are superior to those where $pd \leq 0.01$. This may be a feature of the dynamics of the market, with market participants making pricing errors which yield temporary arbitrage opportunities, subject to mean reversion at some time after 10.30am. Our evidence suggests that such errors occur in both sets of odds when $pd \geq 0.01$.

If we consider price divergence filter levels of 0.02 or more, a different outcome is apparent. Returns to $ma > ba$ are increasing with pd at these higher levels, but deteriorating when $ba > ma$. The returns distributions at higher pd levels taken at face value suggest that the mean bookmaker odds assessment of true outcomes is more accurate, yielding positive returns at the higher levels of pd when $ma > ba$. The returns are not statistically significant, however, and the number of horses for which pd is equal to or greater than 0.02 is only 305 when for $ma > ba$. This represents only 8% of the total number of runners in the subset of horses for which exchange prices exceed the corresponding bookmaker odds.

There is also evidence that a disproportionate number of horses exhibiting high levels of price divergence were in races with low liquidity. Table 7 shows that the percentage of horses exhibiting high price divergence ($pd \geq 0.02$) is skewed more to low liquidity races than those with lower levels of price divergence ($pd < 0.02$). This association is confirmed by a highly significant χ^2 statistic. On these grounds we conclude that the instances of high price divergence are atypical and accordingly should not be given undue emphasis.

In order to obtain an additional perspective on the returns distribution reported in Table 6 the conditional logistic regressions were repeated for subsets of horses with varying levels of price divergence. The above discussion of returns suggests that the results may be sensitive to the pd boundary chosen. In order to avoid an arbitrary choice we performed regressions for alternative pd filter levels, beginning with $pd \geq 0$, then $pd \geq 0.01$, $pd \geq 0.02$ and so on.

The outcomes of the stage 3 conditional logistic regressions are summarised in Table 8, organised by pd filter value. For the purpose of comparison the results for the category $pd < 0.01$ are also included. The key values to consider in the current context are those relating to the Lma , $L(ba,ma)$ and Lba , $L(ba,ma)$ regressions. At the filter level $pd \geq 0.01$ neither set of odds is distinguished by holding superior information to the other, evidenced by neither Lma , $L(ba,ma)$ or Lba , $L(ba,ma)$ yielding significant test statistics.

At all filter values of pd equal to or greater than 0.02, the bookmaker odds yield information in addition to that in exchange odds, with the χ^2 statistic for Lb , $L(ba, ma)$ being significant at $p = 0.05$ or better. At only one filter level in this range ($pd \geq 0.04$) do the exchange odds add useful information to that contained in the bookmaker mean odds, and the test statistic is only weakly significant. As noted above, however, this class of observations is associated with low liquidity races, detracting from the significance of this result.

These results, along with the earlier finding that the exchange odds were the better predictors for the sample of races as a whole, imply that we may find, for the subset of horses for which pd is less than 0.01, that the exchange odds will again hold more information concerning race outcomes than do bookmaker odds. Table 8 shows this to be the case, with the Lm , $L(ba, ma)$ regression indicating that exchange data add valuable information to the bookmaker data alone, significant at $p = 0.05$. Conversely, when the order of addition is reversed the bookmaker data add no significant extra information to that contained in the exchange odds. The high liquidity levels of the markets of the races most closely associated with this class of runner lead us to emphasise this result.

6. Conclusions

The empirical results confirm our expectation that nominal betting exchange odds have more predictive value than bookmaker odds, due to the lower degree of favourite-longshot bias in the former. After adjustment for bias the exchange odds continue to hold more information concerning race outcomes than bookmaker odds, particularly for horses exhibiting low levels of price divergence. The fact that such horses are disproportionately found in high liquidity races adds weight to this result.

As the odds differential increases there is evidence that bookmaker odds, adjusted for bias, hold more accurate information concerning the true probabilities than do the adjusted exchange odds. The incidence of such cases is low, however, and may be largely attributable to low liquidity exchange markets, simple pricing errors, and possibly the activity of insiders. In the main, therefore, the exchange odds prove to be superior predictors of the results of the sampled races.

Our principal finding contrasts with that of Levitt, who found that bookmakers exhibited superior skills in evaluating objective outcomes in the handicapping contest that

was the medium for his study. The observation was made in the introduction that Levitt's methodology made it likely that the preferences of the most skilled or informed bettors might not be revealed if they decided that the terms of the wagers set by bookmakers were unfavourable, and in consequence chose not to trade. The same would be true of the bookmaker markets studied here. In contrast the betting exchanges offer opportunities for these subsets of bettors to trade which are not available in bookmaker markets. For example, skilled traders, insiders, and bettors seeking hedging opportunities are all able to lay odds on the exchanges which may as a result more accurately reflect the chances of the horses concerned than those offered by bookmakers. In these circumstances we might expect the proportion of turnover attributable to casual bettors to be lower in the exchanges than in bookmaker markets, with a consequent tendency for odds to more closely reflect objective probabilities. This account of the differences between the two markets is consistent with recent transaction cost explanations of the structure of betting markets (see Hurley and McDonough, 1995; Sobel and Raines, 2003; Smith, Paton and Vaughan Williams, 2006).

Differences in the nature of traders and trading activities may therefore explain the greater relative efficiency of the exchanges in reflecting objective outcome probabilities observed in the current study. Similarly, the results presented here may not so much contradict Levitt's findings as reflect a different composition of traders engaged in the respective betting media studied. As the betting exchanges continue to expand in size and liquidity, it will be interesting to monitor how well they continue to predict the outcomes of events for which they offer markets.

Tables

Table 1: Estimations of the likelihood function with various degrees of restriction for race outcome predictors (odds)

Estimation of likelihood function	Restrictions on predictors	Alternative hypothesis tested ¹
L_r	All predictors restricted	
L_m	Only mean bookmaker odds unrestricted	Bookmaker odds hold useful information concerning race outcomes
L_b	Only betting exchange odds unrestricted	Exchange odds hold useful information concerning race outcomes
$L_m, L(b,m)$	Both predictors unrestricted (relative to log likelihood of bookmaker odds alone)	Exchange odds hold useful information <i>additional</i> to that contained in bookmaker odds
$L_b, L(b,m)$	Both predictors unrestricted (relative to log likelihood of exchange odds alone)	Bookmaker odds hold useful information <i>additional</i> to that contained in exchange odds

Note:

1. The test statistic in all cases is χ^2 with 1 degree of freedom

Table 2: Odds differences corresponding to increasing levels of price divergence, $pd^{1,2}$

Higher odds ³	Lower odds ³ , yielding specified pd values				
	$pd = 0.01$	$pd = 0.02$	$pd = 0.03$	$pd = 0.04$	$pd = 0.05$
2	1.94	1.89	1.83	1.78	1.73
5	4.72	4.46	4.23	4.02	3.82
10	9.00	8.18	7.49	6.90	6.39
20	16.51	14.04	12.20	10.77	9.63

Notes:

- pd is a measure of the divergence between the odds probabilities equivalent to the mean of bookmaker array of odds for an individual horse, and the greatest *Betfair* (betting exchange) odds on offer to non trivial stakes for the corresponding horse.
- pd is measured as in equation (6) – see also accompanying narrative
- All odds expressed to a 1 unit stake e.g. “2-1”, “1.94 to 1” and so on.

Table 3: Conditional logistic regression results for whole dataset:
nominal odds probabilities

Model restrictions	- 2 log likelihood
<i>Lr</i>	3271.474
<i>Lm</i>	2972.602*** (298.872)
<i>Lb</i>	2967.236*** (304.237)
<i>Lm, L(b,m)</i>	2967.199** (5.402)
<i>Lb, L(b,m)</i>	2967.199 (0.037)
N	8053

Notes:

1. ***p=0.01, **p=0.05, *p=0.1
2. Figures in parentheses are the relevant χ^2 statistics.

Table 4: Coefficients for returns regressed on odds probabilities, bookmaker mean & exchange odds, unadjusted & Shin adjusted

	Bookmaker mean odds		Exchange odds	
	Unadjusted	Shin adjusted	Unadjusted	Shin adjusted
α	-0.3652*** (0.0599)	-0.0223 0.0960	-0.1411* (0.0869)	0.0157 (0.1062)
β	1.343*** (0.3594)	0.4351 (0.5570)	0.4355 (0.4975)	-0.0853 (0.6066)
R^2	0.0009	0.0000	0.0001	0.0000
N	8053			

Notes:

1. *** p = 0.01 ** p = 0.05 * p = 0.1.
2. Figures in parentheses are robust standard errors.

Table 5: Conditional logistic regression results for whole dataset:
Shin probabilities

Model restrictions	- 2 log likelihood
<i>Lr</i>	3271.474
<i>Lma</i>	2970.069*** (301.405)
<i>Lba</i>	2966.504*** (304.970)
<i>Lma, L(ba,ma)</i>	2966.307* (3.762)
<i>Lba, L(ba,ma)</i>	2966.307 (0.197)
N	8053

See notes to Table 3.

Table 6: Returns to subsets of horses showing various degrees of price divergence, based on Shin probabilities.

Divergence of odds probabilities (<i>pd</i>)	N	Profit/loss to unit stake ¹	Return ² %
<i>ma > ba</i>			
All	4080	-420.86	-10.32
$0 \leq pd < 0.01$	2892	-365.70	-12.65
Cumulative, $pd \geq$			
0.01	1188	-55.15	-4.64
0.02	305	30.64	10.04
0.03	109	3.71	3.40
0.04	56	-2.07	-3.69
0.05	35	2.18	6.24
<i>ba > ma</i>			
All	3973	-572.69	-14.41
$0 \leq pd < 0.01$	2679	-504.86	-18.85*** (0.0684)
Cumulative, $pd \geq$			
0.01	1294	-67.82	-5.24
0.02	373	-19.97	-5.35
0.03	134	-16.91	-12.62
0.04	66	-5.13	-7.77
0.05	33	-2.43	-7.35

Notes:

1. *ma > ba* profit/loss based on a unit stake placed at maximum of outlier & exchange odds; *ba > ma* profit/loss based on a unit stake placed at outlier odds.
2. Only one return is significant at $p = 0.1$. Standard errors are not reported for the remainder.
3. Figure in parentheses is robust standard error.

Table 7: Contingency table classifying levels of price divergence by market liquidity of races

	Frequency (% in brackets) by Market Liquidity				
Price divergence	1	2	3	4	Total
$pd < 0.02$	1399 (18.97)	2340 (31.73)	1694 (22.97)	1942 (26.33)	678 (100)
$pd \geq 0.02$	201 (29.65)	270 (39.82)	103 (15.19)	104 (15.34)	7375 (100)
χ^2	94.54				
Critical value of χ^2 at $p = 0.01$	11.34				
Degrees of freedom	3				

Table 8: Conditional logistic regression results for subset¹ of horses with greatest price divergence between mean bookmaker & exchange odds (based on Shin probabilities)

Model restrictions	<i>pd</i> < 0.01	<i>pd</i> ≥ 0.01	<i>pd</i> ≥ 0.02	<i>pd</i> ≥ 0.03	<i>pd</i> ≥ 0.04	<i>pd</i> ≥ 0.05
<i>Lr</i>	1545.455	866.315	158.903	41.249	14.099	8.318
<i>Lma</i>	1425.471*** (122.983)	770.119*** (96.196)	136.091*** (22.811)	36.917** (4.332)	6.086*** (8.013)	3.821*** (4.497)
<i>Lba</i>	1420.679*** (124.776)	770.498*** (95.817)	140.256*** (18.646)	39.405 (1.844)	10.668* (3.431)	6.780 (1.538)
<i>Lma, L(ba,ma)</i>	1420.024** (5.447)	769.362 (0.758)	135.287 (0.805)	34.803 (2.115)	3.187* (2.899)	2.775 (1.046)
<i>Lba, L(ba,ma)</i>	1420.024 (0.665)	769.362 (1.137)	135.287** (4.970)	34.803** (4.603)	3.187*** (7.481)	2.775** (4.005)
N	5571	2482	678	243	122	68

Notes:

1. This subset includes horses for which $ma > ba$ and those where $ba > ma$
2. See also notes to Table 3.

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