A Reexamination of the Efficiency of the Betting Market on National Hockey League Games

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A recent paper by Woodland and Woodland examines the efficiency of odds betting on professional hockey games, finding that actual returns on underdog bets consistently exceed expected returns and evidence of a reverse favorite-longshot bias. This article corrects the Woodland and Woodland calculation of bookmaker commissions for unchanged money lines. The authors' revision substantially lowers the commission and thus is potentially important for tests of efficiency. The article also examines the impact of changes in money lines, which further reduce commissions but raise actual returns. The impact of these revisions on tests of efficiency is examined using the Woodland and Woodland sample. In general, the authors show that their revised no line change test statistic is a more stringent test of efficiency than either the Woodland and Woodland test statistic or any reasonable line change test statistic. However, the authors' revised test statistics continue to find the inefficiency documented by Woodland and Woodland.

Keywords: *market efficiency; National Hockey League; odds betting; sports economics*

INTRODUCTION

Sports betting markets have proved to be fruitful arenas for the examination of market efficiency. Although much of the research in this area has focused on the point spread betting markets on football and basketball games, several recent studies have begun to investigate the efficiency of other sports betting markets. For example, the efficiency of the money line betting market on Major League Baseball (MLB) game winners is examined by Woodland and Woodland (1994) and Gandar,

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Zuber, Johnson, and Dare (2002); Brown and Abraham (2002) investigate the efficiency of the over-under betting market on runs scored in MLB games, and Woodland and Woodland (2001) examine the efficiency of the betting market on game winners in the National Hockey League (NHL).

This study focuses on the latter paper. Woodland and Woodland (2001) test the strict version of market efficiency that returns to any betting strategy should produce losses equal to the bookmaker's commission. Using a sample of closing betting lines and game outcomes for six NHL seasons (1990 to 1991 through 1995 to 1996), Woodland and Woodland find that actual returns to betting game underdogs consistently exceed the negative of the commission. Furthermore, they find that returns on underdogs increase with odds to the extent that high odds (low win probabilities) underdogs earn positive returns. In essence, Woodland and Woodland's results both question efficiency in this market and find a reversal of the well-known favorite-longshot bias of racetrack betting.

In his review of the literature on betting markets, Sauer (1998) notes substantial evidence that prices (betting odds or point spread betting lines) in sports betting markets are, almost invariably, unbiased forecasts of actual outcomes and that sightings of profitable betting strategies frequently disappear upon further investigation. However, Sauer also notes that the favorite-longshot bias of racetrack betting (where bettors overbet longshots and underbet favorites so that returns to favorites, although still generally negative, consistently exceed returns to underdogs) remains a longstanding anomaly.

For these reasons, Woodland and Woodland's (2001) results are rather surprising and, we believe, warrant further investigation. This study first reexamines the test methodology used by Woodland and Woodland. We revise their calculation of bookmaker commissions for standard and double-negative money lines. Our revisions have the impact of lowering commissions (raising expected returns) and thereby may alter conclusions about efficiency in this betting market. Next, we examine the impact of changes in money lines from the opening to the closing of betting on the calculation of commissions and actual returns. We show that line changes further reduce commissions. However, in that line changes also tend to raise actual returns, whether or not a consideration of line changes makes a difference for tests of efficiency remains an open question. Consequently, we examine the impact of our (no line change and line change) revisions on tests of efficiency using the Woodland and Woodland sample. In general, we show that our revised no-line-change test statistic is a more stringent test of efficiency than either the Woodland and Woodland test statistic or any reasonable line-change test statistic. However, our revised tests continue to find the inefficiencies documented by Woodland and Woodland.

The article is organized as follows. Section 2 examines and corrects the Woodland and Woodland calculation of bookmaker commissions for both standard and double-negative money lines. The impact of line changes on commissions and actual returns is examined in Section 3. Section 4 compares the results of statistical tests of the difference between expected and actual returns using these revised commissions and returns with those obtained by Woodland and Woodland. Section 5 concludes.

2. RECALCULATING BOOKMAKER COMMISSIONS

Under the null hypothesis of market efficiency, the expected return on any bet is the negative of the bookmaker's commission. This section examines and revises the Woodland and Woodland (1994, 2001) determination of commissions. Before doing this, we provide a brief explanation of money lines. Normally a money line on a game establishes a favorite and an underdog team. For example, a 40-cent money line on a game might be listed as (–150, +110): the bettor wagers \$1.50 to win \$1 on the favorite or \$1 to win \$1.10 on the underdog (the 40-cent line notation stems from the difference between the \$1.50 favorite bet and the \$1.10 underdog return). Following Woodland and Woodland, we denote such money lines as standard lines. Occasionally, with fairly evenly matched teams, more than a dollar has to be wagered on either team in order to win a dollar. For example, at the 40-cent line of (–130, –110), the bettor wagers \$1.30 to win \$1 on the favorite or \$1.10 to win \$1 on the underdog. Again, following Woodland and Woodland, we denote these as double-negative lines. Below, we examine the Woodland and Woodland derivation of the commission and our correction for both types of lines.

The Commission for Standard Lines[1]

Woodland and Woodland's (1994) study of money line betting on MLB games provides the first derivation of bookmaker commissions (and subjective probabilities) for standard lines. Although betting in the MLB market mostly uses a 10-cent line rather than the 30- and 40-cent lines used in NHL betting in the Woodland and Woodland sample period, the derivation of the commission is identical (see Woodland & Woodland, 2001, p. 986).

Woodland and Woodland denote the favorite and underdog prices as β_1 and β_2 (for example, at the above money line of (–150, +110), β_1 and β_2 are 1.5 and 1.1). Denoting the number of unit bets on the favorite and underdog as X and Y, the bookmaker's net receipts are $Y - X$ if the favorite wins and $\beta_1 X - \beta_2 Y$ if the underdog wins. Applying the balanced book assumption (net receipts are balanced regardless of whether the favorite or underdog wins) produces $Y - X = \beta_1 X - \beta_2 Y$, or $X = Y(1 + \beta_2) / (1 + \beta_1)$. Woodland and Woodland define the commission as net receipts divided by the total number of unit bets, or

$$c^W = \frac{Y - X}{X + Y} = \frac{\beta_1 X - \beta_2 Y}{X + Y},$$

which simplifies to

$$c^W = \frac{\beta_1 - \beta_2}{\beta_1 + \beta_2 + 2}.$$

The problem here lies in Woodland and Woodland's definition of the bookmaker's commission. The standard definition of this commission is net receipts ("hold") divided by total dollars bet ("handle"). In Woodland and Woodland's notation, total dollars bet are the sum of dollars bet on the favorite, $\beta_1 X$, plus dollars bet on the underdog, Y. However, instead of total dollars bet, their denominator is the sum of dollars bet on the underdog, Y, plus the (potential) dollar winnings on the favorite, X. Given $\beta_1 > 1$, $(\beta_1 X + Y) > (X + Y)$. That is, their denominator is smaller than total dollars bet and hence, their commission is inflated.

Gandar et al. (2002) show that the problem with the Woodland and Woodland commission stems from their definition of a unit bet on the favorite. With $\beta_1 > 1$, betting β_1 to win \$1 is always something more than a unit bet. Instead, define a unit bet on the favorite as \$1 to win $(1 / \beta_1)$ while leaving the unit bet on the underdog as \$1 to win β_2. With these unit bet definitions, total dollars bet are $(X + Y)$ and net receipts are $Y - (1 / \beta_1)X$ if the favorite wins or $X - \beta_2 Y$ if the underdog wins. With a balanced book, X is $(Y(1 + \beta_2)).(1 + (1 / \beta_1))^{-1}$. The commission, c^R, is net receipts divided by total dollars bet, or

$$c^R = \frac{Y - (1/\beta_1)X}{X + Y} = \frac{X - \beta_2 Y}{X + Y},$$

which simplifies to

$$c^R = \frac{\beta_1 - \beta_2}{2\beta_1 + \beta_1\beta_2 + 1}.$$

The Commission for Double-Negative Lines

In the case of double-negative lines, Woodland and Woodland (2001) define unit bets on the favorite and the underdog as, respectively, β_1 and β_2 to win \$1. Net receipts are $\beta_2 Y - X$ if the favorite wins and $\beta_1 X - Y$ if the underdog wins. Applying the balanced book assumption, X is $(Y(1 + \beta_2)).(1 + \beta_1)^{-1}$. In this case, Woodland and Woodland define the commission as

$$c^W = \frac{\beta_2 Y - X}{X + Y} = \frac{\beta_1 X - Y}{X + Y}$$

which simplifies to

$$c^W = \frac{\beta_1\beta_2 - 1}{\beta_1 + \beta_2 + 2}.$$

The problem with this definition of the commission again lies in the denominator. Instead of equaling the total dollars bet, $\beta_1 X + \beta_2 Y$, their denominator is the combined sum of (potential) winnings on the favorite and the underdog, $X + Y$.

In the Woodland and Woodland setup $\beta_1 > \beta_2 > 1$, so that bets on both the favorite and the underdog are always more than unit bets. Our correction defines a unit bet as \$1 to win $(1 / \beta_1)$ on the favorite and as \$1 to win $(1 / \beta_2)$ on the underdog. Net receipts are $Y - (1 / \beta_1)X$ if the favorite wins or $X - (1 / \beta_2)Y$ if the underdog wins. With a balanced book, X is $[Y(1 + (1 / \beta_2))].[1 + (1 / \beta_1)]^{-1.}$ Defining the commission as net receipts divided by total dollars bet produces

$$c^R = \frac{Y - (1/\beta_1)X}{X + Y} = \frac{X - (1/\beta_2)Y}{X + Y}$$

which simplifies to

$$c^R = \frac{1 - (1/\beta_1)(1/\beta_2)}{(1/\beta_1) + (1/\beta_2) + 2}$$

A Comparison of Commissions

The above commissions for standard lines differ only in their denominators. Given $\beta_1 > 1$, it follows that $c^W > c^R$. That is, at all standard money lines, the Woodland and Woodland commissions are inflated. Table 1 shows the differences between c^W and c^R for both 30- and 40-cent standard money lines. For standard 40-cent money lines, c^W exceeds c^R by almost 20% at the lowest standard line of (–140, +100) and by over 45% at the highest standard line of (–200, +160). Although the absolute differences between c^W and c^R are smaller for standard 30-cent money lines, c^W exceeds c^R by 14% at the lowest standard line of (–130, +100) and by 47% at the highest standard line of (–200, +170).

In that both the numerators and denominators of c^W and c^R differ for double-negative money lines, it is not immediately obvious whether c^W exceeds c^R. However, over the range of observed 40- and 30-cent double-negative money lines, it is always the case that c^W exceeds c^R. As shown in Table 1, the difference is about 20% for 40-cent lines and 15% for 30-cent lines.

Our redefinition of unit bets on underdogs at double-negative lines also alters actual returns to bets on underdogs. Woodland and Woodland (2001) calculate these returns, R_l^W, as $(\beta_{2l} + 1)\hat{\pi}_l - 1$ for standard money lines and as $(\beta_{2l} + 1)\hat{\pi}_l - \beta_{2l}$ for double-negative lines, where $\hat{\pi}_l$ is the observed frequency of underdog wins at the l^{th} line. Although our revisions do not change actual returns for standard lines

TABLE 1: Commissions and Actual Returns

$(-\beta_{1l}.100, \pm \beta_{2l}.100)$	c_l^W	c_l^R	c_l^Δ	R_l^W	R_l^R	R_l^Δ
40-cent money line						
(–120, –120)	0.1000	0.0833	0.0722	—	—	—
(–125, –115)	0.0994	0.0829	0.0717	–0.0545	–0.0474	–0.0355
(–130, –110)	0.0977	0.0818	0.0705	–0.1484	–0.1349	–0.1263
(–135, –105)	0.0949	0.0798	0.0685	–0.0673	–0.0641	–0.0619
(–140, +100)	0.0909	0.0769	0.0660	–0.0565	–0.0565	–0.0424
(–145, +105)	0.0889	0.0738	0.0633	–0.0762	–0.0762	–0.0626
(–150, +110)	0.0870	0.0708	0.0607	–0.0110	–0.0110	0.0316
(–155, +115)	0.0851	0.0680	0.0583	–0.0524	–0.0524	–0.0392
(–160, +120)	0.0833	0.0654	0.0560	–0.0522	–0.0522	–0.0393
(–165, +125)	0.0816	0.0629	0.0538	0.0217	0.0217	0.0354
(–170, +130)	0.0800	0.0605	0.0518	0.1500	0.1500	0.1600
(–175, +135)	0.0784	0.0583	0.0499	0.0313	0.0313	0.0445
(–180, +140)	0.0769	0.0562	0.0481	0.0875	0.0875	0.1011
(–185, +145)	0.0755	0.0542	0.0464	–0.0540	–0.0540	–0.0424
(–190, +150)	0.0741	0.0523	0.0447	0.0417	0.0417	0.0542
(–195, +155)	0.0727	0.0505	0.0432	0.0200	0.0200	0.0320
(–200, +160)	0.0714	0.0488	0.0417	0.0138	0.0138	0.1506
30-cent money line						
(–115, –115)	0.0750	0.0652	0.0530	—	—	—
(–120, –110)	0.0744	0.0648	0.0526	–0.0655	–0.0596	–0.0502
(–125, –105)	0.0727	0.0635	0.0514	0.0086	0.0082	0.0106
(–130, +100)	0.0698	0.0612	0.0495	0.0055	0.0055	0.0206
(–135, +105)	0.0682	0.0586	0.0473	–0.0596	–0.0596	–0.0459
(–140, +110)	0.0667	0.0562	0.0453	–0.0623	–0.0623	–0.0489
(–145, +115)	0.0652	0.0539	0.0435	0.1359	0.1359	0.1517
(–150, +120)	0.0638	0.0517	0.0417	–0.2235	–0.2235	–0.2129
(–155, +125)	0.0625	0.0497	0.0401	–0.0574	–0.0574	–0.0449
(–160, +130)	0.0612	0.0478	0.0385	0.1265	0.1265	0.1412
(–165, +135)	0.0600	0.0460	0.0370	0.0071	0.0071	0.0200
(–170, +140)	0.0588	0.0443	0.0356	–0.2563	–0.2563	–0.2470
(–175, +145)	0.0577	0.0426	0.0343	0.1049	0.1049	0.1184
(–180, +150)	0.0566	0.0411	0.0331	0.1628	0.1628	0.1767
(–185, +155)	0.0556	0.0396	0.0319	–0.1774	–0.1774	–0.1677
(–190, +160)	0.0546	0.0383	0.0308	0.5220	0.5220	0.5395
(–200, +170)	0.0526	0.0357	0.0287	0.0059	0.0059	0.0171

NOTE: Three estimates of the bookmaker's commission are shown: the Woodland and Woodland (2001) no line change commission, c_l^W; the revised estimate of this commission, c_l^R; and the line change commission, c_l^Δ. Similarly, three estimates of actual returns to unit bets on underdogs are shown: the Woodland and Woodland (2001) return, R_l^W; the revised return, R_l^R (which differs from R_l^W only for double negative lines); and the line change return, R_l^Δ. All returns use the number of games, n_l, and underdog wins, w_l, reported by Woodland and Woodland (2001).

(that is, $R_l^R = R_l^W$), they do for double-negative lines where returns, R_l^R, are $((1/\beta_2 l) + 1)\hat{\pi}_l - 1$.

To calculate the actual returns shown in Table 1, we estimate $\hat{\pi}_l$ using the number of games (n_l) and number of underdog wins (w_l) reported by Woodland and Woodland (2001). For double-negative lines, our revision of returns, R^R, relative to the Woodland and Woodland returns, R^W, raises negative returns and lowers positive returns.

3. RECALCULATING THE COMMISSION WHEN MONEY LINES CHANGE

Both the Woodland and Woodland estimate of the commission and our correction assume that money lines do not change from the open to the close of betting. In this section, we examine the impact of line changes on the estimated commission. As with point spread betting, the bookmaker's goal is to post an opening line that evenly divides the dollars bet on both sides of the proposition. If this occurs, the bookmaker earns a riskless profit from the commission. When betting is not balanced at the opening line, the bookmaker will change the line in an effort to attract bets on the less heavily bet side of the proposition and thereby even the betting action.

The impact of any line change is to lower the commission earned by the bookmaker. Why this occurs is most easily seen with a numerical example. Suppose the 40-cent money line on an NHL game opens at (–140, +100). At this line, there are only bets on the favorite. To attract bets on the underdog the bookmaker moves the line up. To simplify, suppose that the bookmaker moves the money line 10 units and at the new line of (–150, +110), bets on the underdog balance the book. In this example, the relevant favorite and underdog prices, β_1 and β_2, are 1.4 and 1.1. In effect, this line change causes the bookmaker to move from a 40-cent money line to a 30-cent money line. As a result, instead of a (revised) commission of 0.0708 for the closing 40-cent money line of (–150, +110), the effective commission is 0.0562 calculated for the 30-cent line of (–140, +110).

The argument works in reverse. Suppose the bookmaker takes only bets on the underdog at the opening line of (–150, +110), moves the line down to (–140, +100), and takes bets on the favorite to balance the book. Here the relevant favorite and underdog prices, β_1 and β_2, are again 1.4 and 1.1. In this case, instead of a (revised) commission of 0.0769 for the closing 40-cent money line of (–140, +100), the effective commission is again 0.0562.

The argument is general. If the opening money line is a 40-cent line and the bookmaker changes the line by increments of 5, 10, 15, and so on, the bookmaker's line is effectively a 35-cent line, a 30-cent line, a 25-cent line, and so on. Likewise, with a 30-cent opening line and line changes of 5, 10, and 15, the effective line is a 25-cent line, a 20-cent line, and a 15-cent line.

What are the frequency and magnitude of line changes in the NHL betting market? Unfortunately, we have not been able to locate information on both opening and closing money lines for the Woodland and Woodland sample period of the

1990s. However, we have obtained this information for the 2001-2002 NHL season.[2] In this single season there are 514 games with both an opening and closing *Stardust* money line (as opposed to an opening and/or closing composite money line and puck line). No line changes occur in 28% of games, 34% have 5-cent line changes, 16% have 10-cent line changes, 9% have 15-cent line changes, and 13% have line changes of 20 cents or greater (with two games having line changes of 40 cents). These line changes are approximately symmetric: although slightly more than half of line changes move lines down, for each of the above groups of line changes the null hypothesis that the proportion of line changes moving lines down is one half cannot be rejected at conventional levels of significance.

To calculate a more realistic commission for the Woodland and Woodland sample period, we simplify the above line change frequencies and magnitudes. For each individual closing money line, we assume 30% of games have no line changes, 35% have 5-cent line changes, 20% have 10-cent line changes, and 15% have 15-cent line changes. Furthermore, we assume that line changes are symmetric within each of these line change groups. For example, consider the closing 40-cent money line (–150, +110). We assume that there are no changes in this line from open to closing for 30% of observations; that 17.5% of observations opened at both (–145, +105) and (–155, +115); that 10% opened at both (–140, +100) and (–160, +120); and last, that 7.5% opened at both (–135, –105) and (–165, +125).

We then calculate c^R for each of these line change possibilities using the relevant β_1 and β_2 from each opening and closing money line. For the no line change games, $(\beta_1,\beta_2) = (1.5, 1.1)$ and the relevant commission is 0.0708. For the two groups of 5-cent line change games, the relevant (β_1,β_2) are (1.45, 1.1) and (1.50, 1.15) and the commissions are 0.0637 and 0.0611. For the two groups of 10-cent line change games, the relevant (β_1,β_2) are (1.40, 1.1) and (1.50, 1.20) and the commissions are 0.0562 and 0.0517. Finally, for the two groups of 15-cent line change games, (β_1,β_2) are (1.35, 1.1) and (1.50, 1.25) and the commissions are 0.0482 and 0.0426. The weighted average of these various commissions, 0.0607, represents an estimate of the effective commission at the closing line of (–150, +110).

These composite line change commissions for individual lines (labeled c_l^{Δ}) are shown in Table 1. Over the range of 40-cent lines, c_l^{Δ} is about 14% smaller than the no line change commission, c_l^{R} and between 28% and 42% smaller than the Woodland and Woodland no line change commission, c_l^{W}. Likewise, over the range of 30-cent lines, c_l^{Δ} is about 19% smaller than c_l^{R} and between 29% and 45% smaller than c_l^{W}.

Although line changes reduce the bookmaker's commission, they also have the effect of raising actual returns to underdog bets. With line changes, actual returns to underdog bets, R_l^{Δ}, are $(\beta_{2l}^{\Delta} + 1)\hat{\pi}_l - 1$ at standard lines and $((1/\beta_{2l}^{\Delta}) + 1)\hat{\pi}_l - 1$ at double-negative lines, where β_{2l}^{Δ} is the weighted average of the relevant β_2 determining the payout to winning underdog unit bets. Under the above line change assumptions β_{2l}^{Δ} is always larger than β_2, so that $R_l^{\Delta} > R_l^{R}$. For example, again consider the money line of (–150, +110). Woodland and Woodland report 310 games and 146 winning underdog bets at this line. Under the no line change assumption, each winning underdog unit bet nets \$1.10, so that the actual return per underdog bet (R_l^W or R_l^R) is –0.0110. Under the above line change assumptions, whereas 65% of underdog bets are made at a β_2 of 1.1, 35% of these bets are made at β_2 above 1.1 (17.5% at 1.15, 10% at 1.20, and 7.5% at 1.25), so the weighted average net payout for winning underdog unit bets, β_{2l}^{Δ}, is \$1.13 and R_l^{Δ} is +0.0032. Table 1 reports R_l^{Δ} for all individual money lines. R_l^{Δ} is always higher than either R_l^W and R_l^R, generally by 1 to 1.5 cents per dollar bet.

4. REDOING THE STATISTICAL TESTS OF EFFICIENCY

This section first examines the Woodland and Woodland individual money line and aggregate tests of market efficiency and provides our modification of these tests for both unchanged money lines and line changes. Next, the impact of these modifications on test results for the Woodland and Woodland sample of NHL games and betting lines is examined. Finally, sensitivity analysis using a number of different possible line change scenarios is conducted.

The Individual Line Efficiency Test

Woodland and Woodland (1994, 2001) conduct efficiency tests at individual money lines and for lines in the aggregate. In both cases, their null hypothesis is that actual and expected returns to underdog bets are equal, where expected returns are the negative of the bookmaker's commission. At each individual line, the above null hypothesis is most easily tested by the equivalent test that the objective underdog win probability at the l^{th} line, π_l, equals the subjective underdog win probability. By setting expected returns to favorites and underdogs at each line equal to the negative of the commission, c_l^W, Woodland and Woodland derive this subjective probability, ρ_l^W, as $(2 / (\beta_{1l} + \beta_{2l} + 2)$ for standard lines and $(\beta_{2l} + 1) / (\beta_{1l} + \beta_{2l} + 2)$ for double-negative lines. Substituting the actual underdog win frequency, $\hat{\pi}_l$, for π_l in the null hypothesis, their test statistic[3] is

$$z_l^W = \left(\hat{\pi}_l - \rho_l^W\right) \Big/ \left[\left(\left(\rho_l^W\left(1-\rho_l^W\right)\right)\big/ n_l\right)\right]^{1/2}.$$

Our modification is straightforward (details are provided in Gandar et al., 2002). Again, setting expected returns to unit bets on favorites and underdogs at each money line equal to the negative of the commission, c_l^R, the subjective probability, ρ_l^R, is $(\beta_{1l} + 1) / (2\beta_{1l} + \beta_{1l}\beta_{2l} + 1)$ for standard lines and $((1 / \beta_{1l}) + 1) / ((1 / \beta_{1l}) + (1 / \beta_{2l}) + 2)$ for double-negative lines. Using these subjective probabilities the test statistic is

$$z_l^R = \left(\hat{\pi}_l - \rho_l^R\right) \Big/ \left[\left(\left(\rho_l^R\left(1-\rho_l^R\right)\right)\Big/ n_l\right)\right]^{1/2}.$$

In the case of line changes, the subjective probability at a given line, ρ_l^Δ, is the weighted average of the relevant ρ_l^R and the test statistic is

$$z_l^\Delta = \left(\hat{\pi}_l - \rho_l^\Delta\right) \Big/ \left[\left(\left(\rho_l^\Delta\left(1-\rho_l^\Delta\right)\right)\Big/ n_l\right)\right]^{1/2}.$$

Aggregate Efficiency Test

The Woodland and Woodland aggregate test compares actual returns with expected returns across groupings of lines. For any grouping of lines, $l = 1, \ldots L$, actual returns, $\overline{R}$, are $\Sigma_{l=1}^{L} \gamma_l R_1$, where R_1 is the actual return at each money line, $\gamma_l = n_1 / N$, and $N = \Sigma_{l=1}^{L} n_1$. Substituting R_l^W, R_l^R, or R_l^Δ into $\overline{R}$ produces $\overline{R}^W$, $\overline{R}^R$, or $\overline{R}^\Delta$. The variance of returns, σ_l^2, is $(\beta_{2l} + 1)^2\pi_l(1 - \pi_l)$ at each standard line and $((1 / \beta_{2l}) + 1)^2\pi_l(1 - \pi_l)$ at each double-negative line. Substituting ρ_l^W, ρ_l^R, and ρ_l^Δ for π_l produces the estimated variances σ_l^{2W}, σ_l^{2R}, and $\sigma_l^{2\Delta}$. For any grouping of lines, $l = 1, \ldots L$, the variance for $\overline{R}$ (that is, either $\overline{R}^W$, $\overline{R}^R$, or $\overline{R}^\Delta$), σ_R^2, is $(1 / N)\ \Sigma_{l=1}^{L} \gamma_l \sigma_l^2$ when the appropriate substitutions are made for σ_l^2. Finally, for any grouping of lines, $l = 1, \ldots L$, the commission, $\overline{C}$ (that is, either $\overline{C}^W$, $\overline{C}^R$, or $\overline{C}^\Delta$) is $\Sigma_{l=1}^{L} \gamma_l c_l$. Thus, the test statistic for the efficiency null hypothesis that actual returns equal expected returns is

$$Z = (\overline{R} - \overline{C}) / \sigma_R^2.$$

Appropriate substitutions produce Z^W, Z^R, and Z^Δ.

Individual Line Test Results

Individual money line test statistics (z_l^W, z_l^R, and z_l^Δ) are reported in Table 2. Woodland and Woodland (2001) find a close correspondence between underdog subjective win probabilities and observed underdog win frequencies. Their estimated test statistic, z_l^W, is statistically significant (at ρ values close to 0.000) at only 2 of 32 individual money lines. The revised no line change test statistic, z_l^R, and the line change test statistic, z_l^Δ, although consistently smaller than z_l^W, do not change Woodland and Woodland's results (both statistics reject the efficiency null hypothesis at the same two individual money lines as z_l^W). There is also an interesting pattern in these statistics: although z_l^Δ always lies between z_l^R and z_l^W, this statistic is consistently closer to the former than to the latter. It appears that of the three statistics, the revised no line change test statistic provides the most stringent test of the efficiency null hypothesis.

TABLE 2: Underdog Bettor Test of Efficiency at Individual Lines

$(-\beta_{1l}.100, \pm \beta_{2l}.100)$	n_l	$\hat{\pi}_l$	z_l^W	z_l^R	z_l^Δ
40-cent money line					
(–120, –120)	216	—	—	—	—
(–125, –115)	420	0.5095	0.8564	0.7790	0.7957
(–130, –110)	395	0.4532	–0.9593	–1.1068	–1.0753
(–135, –105)	315	0.4794	0.4789	0.2851	0.3246
(–140, +100)	407	0.4717	0.6968	0.4130	0.4643
(–145, +105)	304	0.4507	0.2180	–0.0408	0.0049
(–150, +110)	310	0.4710	1.2852	1.0099	1.0576
(–155, +115)	270	0.4407	0.5055	0.2405	0.2857
(–160, +120)	253	0.4308	0.4570	0.1929	0.2373
(–165, +125)	207	0.4541	1.3449	1.0975	1.1387
(–170, +130)	172	0.5000	2.6771	2.4420	2.4808
(–175, +135)	139	0.4389	1.1275	0.9171	0.9516
(–180, +140)	128	0.4532	1.5932	1.3866	1.4202
(–185, +145)	101	0.3861	0.1820	0.0019	0.0310
(–190, +150)	84	0.4167	0.8787	0.7100	0.7371
(–195, +155)	25	0.4000	0.3780	0.2859	0.3006
(–200, +160)	16	0.4375	0.6708	0.5950	0.6071
30-cent money line					
(–115, –115)	96	—	—	—	—
(–120, –110)	203	0.4926	0.1208	0.0779	0.0901
(–125, –105)	122	0.5164	0.8768	0.8114	0.8286
(–130, +100)	181	0.5028	1.0154	0.8997	0.9258
(–135, +105)	109	0.4587	0.0874	–0.0103	0.0113
(–140, +110)	159	0.4465	0.0532	–0.0734	–0.0459
(–145, +115)	106	0.5283	1.9423	1.8303	1.8544
(–150, +120)	119	0.3529	–1.6016	–1.7202	–1.6949
(–155, +125)	74	0.4189	0.0393	–0.0599	–0.0390
(–160, +130)	98	0.4898	1.6442	1.5231	1.5486
(–165, +135)	49	0.4286	0.4083	0.3221	0.3401
(–170, +140)	71	0.3099	–1.4203	–1.5211	–1.5001
(–175, +145)	51	0.4510	0.9742	0.8814	0.9006
(–180, +150)	43	0.4651	1.1872	1.0998	1.1179
(–185, +155)	31	0.3226	–0.5510	–0.6209	–0.6065
(–190, +160)	41	0.5854	2.9514	2.8578	2.8770
(–200, +170)	51	0.3726	0.3243	0.2296	0.2490

NOTE: The number of games, n_l, and the actual frequency of underdog wins, $\hat{\pi}_l$, are the sample observations reported in Woodland and Woodland (2001). The reported Z statistics (z_l^W, z_l^R, and z_l^Δ) test the null hypothesis that subjective probabilities (ρ_l^W, ρ_l^R, and ρ_l^Δ) equal the objective probability, π_l.

Aggregate Test Results

Aggregate commissions, actual returns to underdogs, and test statistics for all 40-cent lines, all 30-cent lines, and all lines combined are shown in Table 3. The

TABLE 3: Underdog Bettor Tests of Efficiency Across Money Lines

	All Odds	*Low Odds*	*Medium Odds*	*High Odds*
40-cent money lines (1990 to 1991 through 1993 to 1994 National Hockey League [NHL] Seasons)				
N	3,546	1,130	1,291	1,125
$\overline{C}^W$	0.0884	0.0976	0.0883	0.0794
$\overline{R}^W$	–0.0375	–0.0909	–0.0493	0.0297
Z^W	2.8319	0.2139	1.3628	3.2177
$\overline{C}^R$	0.0715	0.0816	0.0728	0.0597
$\overline{R}^R$	–0.0349	–0.0826	–0.0493	0.0297
Z^R	2.0902	–0.0353	0.8213	2.6279
$\overline{C}^\Delta$	0.0614	0.0704	0.0625	0.0511
$\overline{R}^\Delta$	–0.0231	–0.0746	–0.0355	0.0430
Z^Δ	2.1625	–0.1467	0.9272	2.7328
30-cent money lines (1994 to 1995 through 1995 to 1996 NHL Seasons)				
N	1,508	506	493	509
$\overline{C}^W$	0.0656	0.0723	0.0660	0.0585
$\overline{R}^W$	–0.0139	–0.0222	–0.0580	0.0370
Z^W	1.8414	1.0994	0.1684	1.8241
$\overline{C}^R$	0.0540	0.0632	0.0552	0.0438
$\overline{R}^R$	–0.0132	–0.0199	–0.0580	0.0370
Z^R	1.4756	0.9974	–0.0602	1.5403
$\overline{C}^\Delta$	0.0436	0.0512	0.0445	0.0353
$\overline{R}^\Delta$	–0.0012	–0.0102	–0.0447	0.0499
Z^Δ	1.5139	0.9350	–0.0038	1.6038
Combined money lines (1990 to 1991 through 1995 to 1996 NHL Seasons)				
N	5,054	1,636	1,784	1,634
$\overline{C}^W$	0.0816	0.0898	0.0821	0.0729
$\overline{R}^W$	–0.0305	–0.0697	–0.0517	0.0320
Z^W	3.3778	0.7798	1.2409	3.6824
$\overline{C}^R$	0.0663	0.0759	0.0680	0.0548
$\overline{R}^R$	–0.0284	–0.0632	–0.0517	0.0320
Z^R	2.5579	0.5346	0.6613	3.0364
$\overline{C}^\Delta$	0.0561	0.0645	0.0575	0.0462
$\overline{R}^\Delta$	–0.0166	–0.0547	–0.0381	0.0452
Z^Δ	2.6393	0.4080	0.7810	3.1588

NOTE: The number of observations is *N*, weighted commissions and returns are, respectively, $\overline{C}$ and $\overline{R}$ with the superscripts *W*, *R*, and Δ indicating, respectively, the Woodland and Woodland (2001) calculation of commissions and returns at the closing line, the revised calculation of commissions and returns at closing lines, and the calculation of commissions and returns under the line change assumptions given in the text. The *Z* statistics test the null hypothesis that actual returns, $\overline{R}$, equal expected returns, $-\overline{C}$.

table also reports statistics for the odds groupings (low, medium, and high) used by Woodland and Woodland (2001).

Woodland and Woodland's aggregate test results indicate inefficiencies in this market. For all lines combined, actual returns of –0.0305 significantly exceed expected returns of –0.0816 (Z^W of 3.3778, p value of 0.0000). Similar, although somewhat weaker, results hold for all 40-cent lines and all 30-cent lines considered separately (Z^W of 2.8319, p value of 0.0046 for the former and Z^W of 1.8414, p value of 0.0659 for the latter).

We now consider the impact of our revisions on these aggregate results. For all lines combined, the no line change revisions raise actual and expected returns (to –0.0284 and –0.0663), thereby reducing the no line change test statistic but not altering the test's conclusion (Z^R of 2.5579, p value of 0.0105). A similar result holds for 40-cent lines separately (Z^R of 2.0902, p value of 0.0366). However, for 30-cent lines our revisions of actual and expected returns produce a test statistic that does not allow for the rejection of the efficiency null (Z^R of 1.4756, p value of 0.1402). Our line change test statistics are very similar (for all lines combined, for 40-cent lines, and for 30-cent lines respectively, Z^Δ values are 2.6393, 2.1625, and 1.5139, and p values are 0.0082, 0.0302, and 0.1291).

Partitioning their data into three odds groupings, Woodland and Woodland find that actual returns for all lines combined increase (from –0.0697 for the low odds group to +0.0320 for the high odds group) and that the differences between actual and expected returns increase across these groupings. As a result, although their test statistics for the low and medium odds groups cannot reject the null hypothesis, the test statistic for high odds underdogs rejects the efficiency null hypothesis (Z^W of 3.6824, p value of 0.0000).[4]

Our upward revisions of actual and expected returns change neither the above patterns of these returns nor the rejection of the efficiency null hypothesis for the heavy underdog grouping (Z^R of 3.0364 with p value of 0.0024 and Z^Δ of 3.1588 and p value of 0.0000).

With the exception of the low odds grouping of lines,[5] the distribution of the aggregate test statistics shows the same pattern as the individual line test statistics; although Z^Δ falls between Z^W and Z^R, it is consistently closer to Z^R. In general, the revised no line change test statistic is again the most stringent of the three test statistics.

Sensitivity Analysis

This section explores the sensitivity of our line change test results to variations in the assumptions about the frequency, magnitude, and symmetry of line changes. The line changes assumed above are based on those from a sample of the most recent single season of NHL play available, a sample that may or may not be representative of line changes in the earlier Woodland and Woodland sample period. What impact do alternative line change assumptions have on the test results? Furthermore, both the individual line tests and the aggregate tests have the pattern that, although the line change test statistics (z_l^{Δ} or Z^{Δ}) generally lie between the Woodland and Woodland no line change statistics (z_l^{W} or Z^{W}) and our revised no line change statistics (z_l^{R} and Z^{R}), these line change statistics are always much closer to the latter than to the former. Does this pattern continue to hold under different assumptions about line changes?

We initially consider two extreme examples of symmetric line changes. Above, we assumed 30% of games at each line had no change, 35% had 5-cent changes, 20% had 10-cent changes, and 15% had 15-cent changes. First, suppose that the frequency and magnitude of line changes is much smaller. Instead of 70% of lines changing from opening to closing, suppose that 70% are unchanged and that 15%, 10%, and 5% of games have, respectively, 5-cent, 10-cent, and 15-cent line changes. Alternatively, suppose that the frequency and magnitude of line changes is much larger: Lines on all games change; 20% by 5 cents, 30% by 10 cents, and 50% by 15 cents. We label the aggregate Z statistics based on these alternative small and large line change assumptions as $Z_{\text{small}}^{\Delta}$ and $Z_{\text{large}}^{\Delta}$.

Our second set of line change scenarios explores the impact of changing the assumption that all line changes are symmetric. To do this, we revert to the original line change frequencies and magnitudes but drop the assumption that half the line changes at a given increment move β_1 up to the closing line (because of betting on favorites at these earlier lines) and half move β_2 down to the closing line (because of betting on underdogs at these earlier lines). We produce two asymmetric line change scenarios by first assuming that all line changes move only β_1 up to the closing line of (β_{1l},β_{2l}) and then that all line changes move only β_2 down to the closing line of (β_{1l},β_{2l}). We label the aggregate Z statistics based on these two asymmetric line change scenarios as $Z_{\text{favorites}\uparrow}^{\Delta}$ and $Z_{\text{underdogs}\uparrow}^{\Delta}$.

Aggregate test results for these four alternative line change scenarios are shown in Table 4 (for convenience, the table also repeats the results for Z^{W}, Z^{R}, and Z^{Δ}). The alternative symmetric line change scenarios make little difference. The two test statistics, $Z_{\text{small}}^{\Delta}$ and $Z_{\text{large}}^{\Delta}$ are similar in size (the increase in expected returns resulting from the larger symmetric line changes is offset by a similar increase in actual returns) and are very close to the original line change statistic, Z^{Δ}. However, the asymmetric line changes do make a difference: Relative to Z^{Δ}, $Z_{\text{favorites}\uparrow}^{\Delta}$ is markedly smaller and $Z_{\text{underdogs}\uparrow}^{\Delta}$ is markedly larger. In both cases, because expected returns are approximately the same as those for the original symmetric line changes, these differences arise from the different behavior of actual returns. Under the first asymmetric line change scenario, actual returns are much lower than for the original symmetric line changes (because winning underdog bets are all paid off at the given β_{2l}, actual returns are the same as no line change returns). In contrast, under the second asymmetric line change scenario, actual returns are much higher (because winning underdog bets are paid off at β_2 levels at or above the given β_{2l}). The resulting differences in the test statistics could now lead the researcher to either accept the

TABLE 4: Aggregate Efficiency Test Results Under Different Line Change Scenarios

	All Lines	*All 40-Cent Lines*	*All 30-Cent Lines*
No line changes			
Z^{W}	3.3778	2.8319	1.8414
Z^{R}	2.5579	2.0902	1.4756
Symmetric line change results			
Z^{Δ}	2.6393	2.1625	1.5139
Z^{Δ}_{small}	2.5945	2.1223	1.4934
Z^{Δ}_{large}	2.6350	2.1650	1.5025
Asymmetric line change results			
$Z^{\Delta}_{favorites\uparrow}$	1.9386	1.5772	1.1286
$Z^{\Delta}_{underdogs\uparrow}$	3.3316	2.7418	1.8931

NOTE: The listed Z statistics are for the null hypothesis that actual returns to underdogs equal expected returns (the negative of the commission). The no line change statistics Z^{W} and Z^{R} (previously reported in Table 3) are reported here for the reader's convenience. The first symmetric line change scenario (producing the statistic Z^{Δ}) assumes 30% of lines do not change, 35% change by 5 cents, 20% change by 10 cents, and 15% change by 15 cents (this statistic is also repeated from Table 3 for the reader's convenience). The symmetric line change scenario producing the Z^{Δ}_{small} statistic assumes 70% of all games at each money line do not change, that 15% change by 5 cents, 10% change by 10 cents, and 5% change by 15 cents. In contrast, the scenario producing Z^{Δ}_{large} assumes all lines change, 20% by 5 cents, 30% by 10 cents, and 50% by 15 cents. Two asymmetric line change results are also reported here. Both assume the same proportion and magnitude of line changes used for Z^{Δ}. However, for $Z^{\Delta}_{favorites\uparrow}$ we also assume that all line changes stem from bets on the favorites (and hence move β_1 up to the closing β_{1l}) and all payoffs to underdog bets are made at the closing β_2 line and the opposite is assumed for $Z^{\Delta}_{underdogs\uparrow}$—all line changes stem from bets on underdogs (moving β_2 down to the closing β_{2l}) and payoffs to underdogs are made at these higher β_2 as well as at the closing β_{2l}.

null hypothesis of efficiency (in the case of $Z^{\Delta}_{favorites\uparrow}$) or to decisively reject this hypothesis (in the case of $Z^{\Delta}_{underdogs\uparrow}$).

We are uncertain about the plausibility of asymmetric line changes. An implication of Woodland and Woodland's discussion that bettors tend to overbet favorites is that money lines will tend to be bet up. If correct, these asymmetric line changes would tend to move the aggregate test statistic away from Z^{Δ} and closer to $Z^{\Delta}_{favorites\uparrow}$. In contrast, slightly more than half of the line changes moved lines down in our single season of line change data. However, as noted above, the null hypothesis of symmetrical line changes could not be rejected in this small sample. Until future research demonstrates otherwise, we think that the assumption of symmetrical line changes is reasonable.

In summary, alternative symmetric line change assumptions have a minimal impact on the aggregate test's conclusions. Although the statistics Z^{Δ}_{small} and Z^{Δ}_{large} result from radically different assumptions about symmetric line changes, they are similar in size and are very close to our original line change test statistic, Z^{Δ}. In turn, the latter statistic is close to our revised no line change statistic, Z^{R}.

5. CONCLUSION

Woodland and Woodland test a constant returns view of market efficiency in a new betting market, the odds betting market on NHL games. Although their tests of actual returns to underdog bets versus expected returns reveal little inefficiency at individual money lines, their aggregate test decisively rejects the efficiency null hypothesis. Furthermore, Woodland and Woodland find that returns to underdog bets increase across odds groupings to the point that high odds underdogs earn positive returns.

This article corrects the Woodland and Woodland calculation of the bookmaker's commission for both standard and double-negative money lines. This revision has the impact of substantially lowering the estimate of the commission and thus is potentially important for tests of efficiency in any odds betting market. The article also examines the impact of changes in money lines on commissions. We show that line changes further reduce commissions. However, because line changes also raise actual returns, it is an open question whether line changes make a difference for efficiency tests.

We use the Woodland and Woodland sample of betting odds and NHL game outcomes to examine the impact of our (no line change and line change) revisions. Our revisions weaken, but do not alter, the results found by Woodland and Woodland for this sample: We continue to find inefficiency in the all-lines combined case, and, like Woodland and Woodland, we find high odds underdogs significantly underbet. Although our revisions do not change the results obtained by Woodland and Woodland for their sample period, these revisions remain important. First, they correct the calculation of expected and actual returns for all money lines. Second, they allow line changes to be taken into account. Finally, our no line change test statistic is a more stringent test of efficiency than either the Woodland and Woodland test statistic or any likely line change test statistic.

In the light of the efficiency generally documented in other sports betting markets, the finding of a basic inefficiency in the hockey betting market is puzzling. Likewise, the reversal of the longstanding favorite-longshot bias in this market is an anomaly. We believe further research on this market is needed. We suggest that this research focus on retesting efficiency in this market with a data set sufficient to explore the efficiency of money lines at particular puck lines, the impact of recent reductions in commissions, and the distribution of money line changes.

NOTES

1. This section is based on Gandar, Zuber, Johnson, and Dare (2002).

2. These data were collected daily from the Internet site http://www.donbest.com. This site lists the opening and closing lines at a number of Las Vegas and offshore sports books, including the *Stardust*.

3. To permit use of the normal approximation to the binomial distribution, lines are excluded if either $n_l\rho_l < 5$ or $n_l(1 - \rho_l) < 5$.

4. Woodland and Woodland (2001) also conduct a test of the profitability of betting on high odds underdogs. Although actual returns are positive (0.0320) for this group, their test statistic cannot reject the null hypothesis of zero profits for all seasons combined (Z^W of 1.1233, ρ value of 0.3693).

5. The exception occurs at the low odds groupings (generally double-negative lines). For this group, moving from the Woodland and Woodland estimates to the revised no line change estimates causes both average expected returns and average actual returns to increase. However, because the increase in expected returns exceeds that of actual returns, the test statistic becomes negative. The same pattern occurs in moving from the revised no line change estimates to the line change estimates so that the test statistic becomes even more negative.

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