Math 308 Discussion Problems #9
Due December 9, 11:59pm

(1) Find a 3×3 matrix A with eigenvectors $\vec{\mathbf{v}}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ with $\lambda = 1$, $\vec{\mathbf{v}}_2 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$ with $\lambda = 2$ and $\vec{\mathbf{v}}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ with $\lambda = 10$. You may take for granted that $\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3\}$ is a basis for $\mathbb{R}^3$.

(**Hint**: A must be diagonalizable, $A = PDP^{-1}$. Figure out P and D, then compute A directly.)

P =

1	0	1
2	-1	1
3	1	1

A = PDP^1 =

16	3	-3
44/3	-4/3	-10/3
46/3	-11/3	-5/3

D =

1	0	0
0	2	0
0	0	10

P^-1 =

-2/3	1/3	1/3
1/3	-2/3	1/3
5/3	-1/3	-1/3

(2) Let A be a 3×3 matrix whose eigenvalues are

$$\lambda_1 = 1 \quad \lambda_2 = -1 \quad \lambda_3 = \frac{1}{2}$$

and whose corresponding eigenvectors are

$$\vec{u}_1 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \quad \vec{u}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad \vec{u}_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

(a) What is nullity $(A - I)$? What is rank $(A - I)$? Explain.

Nullity(A-I) is 1, since by changing the eigenvalues of the matrix by one by subtracting by I, you've created an eigenvalue of 0, which means there's a non-zero vector that can be multiplied by the matrix that gives you the zero matrix.
With a 3x3 matrix, if you have a nullity of 1, it means you must have a rank of 2, by rank-nullity theorem.

(b) What is nullity (A)? What is rank (A)? Explain.

I assume since A doesn't have zero as an eigenvalue that it has a nullity of zero, since if it had a vector that could be multiplied by it to get the zero vector, it would be an eigenvector. Thus by rank nullity theorem, A has a rank of 3.