Homework 04

Multiple variable linear regression by 20175437 신준섭

In [1]:

```
import matplotlib as mpl
import matplotlib.pyplot as plt
import numpy as np
import csv
```

1. Input points

- **Load train datas from `data_train.csv`**

$$\{(x^{(i)}, y^{(i)}, z^{(i)}, h^{(i)})\}$$

- **Load test datas from `data_test.csv`**

$$\{(x^{(i)}, y^{(i)}, z^{(i)}, h^{(i)})\}$$

In [2]:

```
# read data points from .cvs files
def read_datafiles(filename):
    datas, target = [], []
    with open(filename, newline='') as the_file:
        reader  = csv.reader(the_file, delimiter=',')
        for i in reader:
            datas.append([float(1.0), float(i[0]), float(i[1]), float(i[2])])
            target.append([float(i[3])])
    return np.matrix(datas), np.matrix(target)

train_data, train_target = read_datafiles("data_train.csv")
test_data, test_target = read_datafiles("data_test.csv")
```

2. Linear Model

* Linear model of 3 inputs

$$f_\theta(x, y, z) = \theta_0 + \theta_1 x + \theta_2 y + \theta_3 z$$
$$where, \theta_0, \theta_1, \theta_2, \theta_3 \in \mathbb{R}$$

* Vector equation form

$$\Theta = [\theta_0, \theta_1, \theta_2, \theta_3]^T$$
$$\mathbf{x} = [1, x, y, z]^T$$
$$f(\Theta, \mathbf{x}) = \mathbf{x}^T \cdot \Theta$$

* Matrix form

$$X = \begin{bmatrix} \mathbf{x}_1^T \\ \cdots \\ \mathbf{x}_m^T \end{bmatrix}$$

$$F = X \cdot \Theta$$

where for m data points,

$$F \in \mathbb{R}^{m \times 1}, X \in \mathbb{R}^{m \times 4}$$

3. Objective function

* The target value for $\mathbf{x}_i$ is h_i so

$$H = [h_1, \cdots, h_N]^T$$

* Scalar representation

$$J(\theta_0, \theta_1, \theta_2, \theta_3) = \frac{1}{2m} \sum_{i=1}^{m} (\theta_0 + \theta_1 x^{(i)} + \theta_2 y^{(i)} + \theta_3 z^{(i)} - h^{(i)})^2$$

* Matrix representation

$$J(\Theta, \mathbf{x}) = \frac{1}{2m} \sum_{i=1}^{m} [(F - H)^2]_i$$

In [3]:

```
def getLoss(X, T, H):
    m = H.shape[0]
    return (1.0 / (2.0 * m)) * np.sum(np.square(np.matmul(X, T) - H))
```

4. Gradient Descent

- $\theta_0^{(t+1)} := \theta_0^{(t)} - \alpha \frac{1}{m} \sum_{i=1}^{m} (f_\theta(x^{(i)}, y^{(i)}, z^{(i)}) - h^{(i)})$
- $\theta_1^{(t+1)} := \theta_1^{(t)} - \alpha \frac{1}{m} \sum_{i=1}^{m} (f_\theta(x^{(i)}, y^{(i)}, z^{(i)}) - h^{(i)}) x^{(i)}$
- $\theta_2^{(t+1)} := \theta_2^{(t)} - \alpha \frac{1}{m} \sum_{i=1}^{m} (f_\theta(x^{(i)}, y^{(i)}, z^{(i)}) - h^{(i)}) y^{(i)}$
- $\theta_3^{(t+1)} := \theta_3^{(t)} - \alpha \frac{1}{m} \sum_{i=1}^{m} (f_\theta(x^{(i)}, y^{(i)}, z^{(i)}) - h^{(i)}) z^{(i)}$

In [4]:

```
def getGradient(X, T, H):
    m = H.shape[0]
    gradients = np.matrix([[0.0], [0.0], [0.0], [0.0]])
    diff = np.matmul(X, T) - H

    idx = 0
    while idx < X.shape[1]:
        gradients[idx] = (1.0/m) * np.sum(np.multiply(diff, X[:,idx]))
        idx = idx + 1

    return gradients

def gradientDescent(X, T, H, lr = 0.00001):
    grads = getGradient(X, T, H)
    return T - lr * grads

def testConvergence(X, T, H, threshold = 0.8):
    grads = getGradient(X, T, H)
    # testing convergence by the magnitude of the gradients
    return np.sqrt(np.sum(np.square(grads))) < threshold
```

5. Training & Testing

In [5]:

```
thetas = np.matrix([[0.0], [0.0], [0.0], [0.0]])

train_losses = [] # recording the train process
test_losses = [] # recording the testset performence
theta0_records = [] # recording parameters
theta1_records = [] # recording parameters
theta2_records = [] # recording parameters
theta3_records = [] # recording parameters

iterations = 0

while testConvergence(train_data, thetas, train_target) == False:
    thetas = gradientDescent(train_data, thetas, train_target)
    theta0_records.append(thetas[0].tolist()[0][0])
    theta1_records.append(thetas[1].tolist()[0][0])
    theta2_records.append(thetas[2].tolist()[0][0])
    theta3_records.append(thetas[3].tolist()[0][0])
    train_losses.append(getLoss(train_data, thetas, train_target))
    test_losses.append(getLoss(test_data, thetas, test_target))
    iterations = iterations + 1
```

```
[-0.05684602981820183, -0.05684890544084223, -0.05685178104592281, -0.05685465663344
567, -0.056857532203412886, -0.05686040775826556, -0.05686328329068876, -0.05686615
880800158, -0.05686903430776711]
```

6. Plotting the results

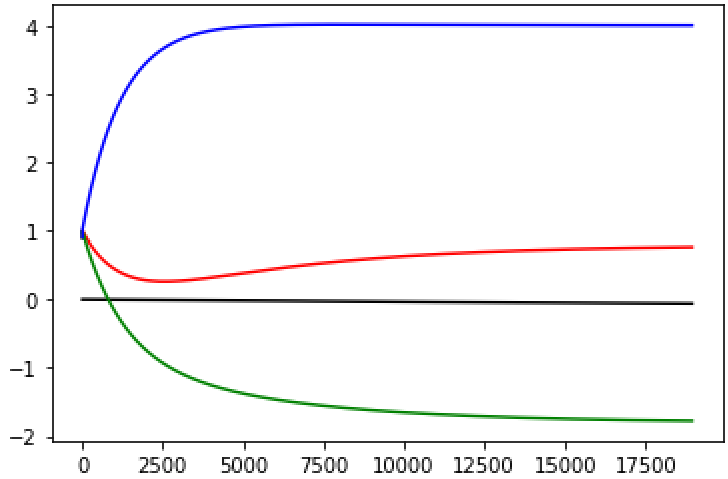
1. Plot the estimated parameters using the training dataset

In [6]:

```
iter_x = list(range(0, iterations))

plt.plot(iter_x, theta0_records, color = 'black')
plt.plot(iter_x, theta1_records, color = 'red')
plt.plot(iter_x, theta2_records, color = 'green')
plt.plot(iter_x, theta3_records, color = 'blue')

plt.show()
```

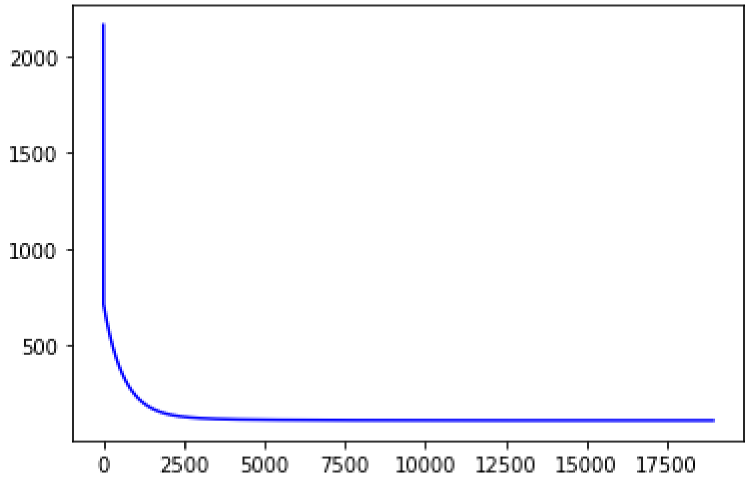


2. Plot the training error using the training dataset

In [7]:

```
plt.plot(iter_x, train_losses, color='blue')

plt.show()
```



3. Plot the testing error using the test dataset at every iterationof gradient descent until convergence

In [8]:

```
plt.plot(iter_x, test_losses, color='red')

plt.show()
```