ASKEM Queries as risk-based problems December demo

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1 Risk-based formulation

- SEIHRD ODE system:
$$\frac{\mathrm{d}\boldsymbol{X}}{\mathrm{d}t} = f(\boldsymbol{X}, t; \boldsymbol{u}, \theta), \tag{1}$$
where $\boldsymbol{X}(t; \boldsymbol{u}, \theta) = [S, E, I, R, H, D]^\top$.

- **Control parameters:** Let $\boldsymbol{u} \in \mathcal{U} \subseteq \mathbb{R}^{n_u}$ be a vector of controllable parameters, e.g. vaccination rate, percentage of masking.

- **Uncertain parameters:** n_θ random variables $\theta \in \Omega \subseteq \mathbb{R}^{n_\theta}$ with the probability density function π_θ (through inverse UQ)

- **Decision metric:** $M : \mathcal{U} \times \Omega \mapsto \mathbb{R}$, e.g. hospitalization (decision metric not dependent on random variables: $M : \mathcal{U} \mapsto \mathbb{R}$)

- **Risk metrics:** $\mathcal{R} : \mathcal{U} \times \Omega \mapsto \mathbb{R}$
robust (linear combination of mean and standard deviation), probability of failure, buffered probability of failure, α-quantile, α-superquantile.

- **Risk-based optimization under uncertainty problem formulation** for a given risk metric $\mathcal{R}$:

  - *Risk metric as objective:*
  $$\boldsymbol{u}^* = \underset{\boldsymbol{u} \in \mathcal{U}}{\arg\min}\ \mathcal{R}(M(\boldsymbol{u}, \theta)) \tag{2}$$

  - *Risk metric as constraint:* decision metric M_1 does not depend on random variables; decision metric M_2 depends on random variables
  $$\begin{aligned} \boldsymbol{u}^* = \underset{\boldsymbol{u} \in \mathcal{U}}{\arg\min}\ & M_1(\boldsymbol{u}) \\ \text{s.t. } & \mathcal{R}(M_2(\boldsymbol{u}, \theta)) \leq \mathcal{R}_{\text{threshold}} \end{aligned} \tag{3}$$

2 December demo

2.1 Scenario 1

- Looking back to this time, which interventions could we have implemented to keep below a hospitalization threshold of 3k covid patients, over the winter 2020 season (Dec. 1st 2020 to March 1st 2021)? (Can this be stated probabilistically? How likely would an intervention have enabled us to reach our goal?)
  (1) Very limited social distancing and masking policies (say this would only apply to healthcare settings, assume 5% decrease from normal contact/transmission levels) beginning right at the start of the period on Dec 1, 2021, through March 1, 2021.
  (2) Stronger social distancing and masking policies, wait until after holidays and begin on Jan 1st, 2021 through end of 3 months (until March 1, 2021). What is the severity of intervention required for this option to have been successful? (for CHIME model this intervention maps to % decrease from baseline transmission levels).
  Modeling constraint/goal: keep covid hospitalizations less than threshold over next $0 \leq t < 3$ months
  - Control mask mandate through changing β
  - Decision metrics could be
    * maximum hospitalization in the given time-frame (leads to a conservative decision):
      $$M(\boldsymbol{u}, \theta) = \max_{0 \leq t < 90 \text{ days}} H(t; \boldsymbol{u}, \theta),$$
    * vector of hospitalizations each day from $0 \leq t < 90$ days
      $$M_i(\boldsymbol{u}, \theta) = H(t_i; \boldsymbol{u}, \theta), \forall i = 0, \ldots, 90$$
  - Then risk can be defined as:
    * probability of exceeding a threshold hospitalization of 3000 as $\mathcal{R}(M(\boldsymbol{u}, \theta)) = \mathbb{P}(M(\boldsymbol{u}, \theta) > 3000)$ and define success by choosing the risk threshold $\mathcal{R}_{\text{threshold}}$ of say 5%

      Constraint: $\mathbb{P}(M(\boldsymbol{u}, \theta) > 3000) \leq 0.05$
      Multiple constraints case: $\mathbb{P}(M_i(\boldsymbol{u}, \theta) > 3000) \leq 0.05, \forall i = 0, \ldots, 90$
    * α-superquantiles of hospitalization that can help with working around the hard threshold hospitalization of 3000 as $\mathcal{R}(M(\boldsymbol{u}, \theta)) = \overline{Q}_\alpha(M(\boldsymbol{u}, \theta))$ and using α of say $1 - 0.05 = 0.95$; risk-preference can be adjusted by changing the value of α;

      Constraint: $\overline{Q}_{0.95}(M(\boldsymbol{u}, \theta)) \leq 3000$
      Multiple constraints case: $\overline{Q}_{0.95}(M_i(\boldsymbol{u}, \theta)) \leq 3000, \forall i = 0, \ldots, 90$

2.2 Scenario 2

- What vaccination rate(s) would these two groups need to have over the next 3 months, in order to lower the observed case rate for those age groups below 10 cases per 100k population? Modeling constraint/goal: get age-specific case rate less than threshold ***by*** t = 3 months (so by April 1, 2021): Note that in reality, on January 1st these groups were at 81.5 and 78.9 observed cases/100k, respectively. On April 1st, the groups were at 30.5 and 24.1 observed cases/100k respectively.
  - ODE model: SVIIvR
  - Decision metrics
    * Vaccination rate (minimize): $M_1(\boldsymbol{u}) = \boldsymbol{u} = \nu$
    * 7-day average of total infections after 90 days (3 months):

$$M_2(\boldsymbol{u}, \theta) = \frac{1}{7} \sum_{i=0}^{6} I(t = 90 - i; \boldsymbol{u}, \theta) + I_{\mathrm{V}}(t = 90 - i; \boldsymbol{u}, \theta)$$

  - Then risk can be defined as:
    * probability of exceeding a threshold infections of 10 in 100000 as $\mathcal{R}(M_2(\boldsymbol{u}, \theta)) = \mathbb{P}(M_2(\boldsymbol{u}, \theta) > 10)$ and define success by choosing the risk threshold $\mathcal{R}_{\text{threshold}}$ of say 5%:

$$\textit{Constraint: } \mathbb{P}(M_2(\boldsymbol{u}, \theta) > 10) \leq 0.05$$

      The OUU problem can also be equivalently formulated using α-quantile measure as constraint using α of $1 - 0.05 = 0.95$:

$$\textit{Constraint: } Q_{0.95}(M_2(\boldsymbol{u}, \theta)) \leq 10$$

    * α-superquantiles of infections that can help with working around the hard threshold infections of 10 in 100000 as $\mathcal{R}(M_2(\boldsymbol{u}, \theta)) = \overline{Q}_\alpha(M_2(\boldsymbol{u}, \theta))$ and using α of say $1 - 0.05 = 0.95$ (risk-preference can be adjusted by changing the value of α):

$$\textit{Constraint: } \overline{Q}_{0.95}(M_2(\boldsymbol{u}, \theta)) \leq 10$$

  - Risk-based OUU problem formulation minimizes vaccination rate such that the risk of infections exceeding the threshold is below acceptable risk threshold:

$$\begin{aligned} \boldsymbol{u}^* = \underset{\boldsymbol{u} \in \mathcal{U}}{\arg\min}\ & M_1(\boldsymbol{u}) = \nu \\ \text{s.t. } & \mathcal{R}(M_2(\boldsymbol{u}, \theta)) \leq \mathcal{R}_{\text{threshold}} \end{aligned} \tag{4}$$

  - Distribution of random variables for SVIIvR model are shown in Table 1 and Figure 1:
  - Comparing the effect of two vaccination rates of $\nu = \{0.005, 0.05\}$ on the decision metric of 7-day average infections in Figures 2 and 3:
  - Results comparing Q_α-based OUU vs $\overline{Q}_\alpha$-based OUU are provided in Table 2 and Figure 4.

Table 1: Truncated normal distribution parameters of the random variables.

Random variable	Description	Mean	Standard deviation	Lower bound	Upper bound
β	contact rate for unvaccinated population	0.2	0.025	0.01	0.3
β_{V}	contact rate for vaccinated population	0.1	0.025	0.01	0.25
γ	recovery rate for unvaccinated population	0.1	0.1	0.05	0.4
γ_{V}	recovery rate for vaccinated population	0.2	0.1	0.1	0.4

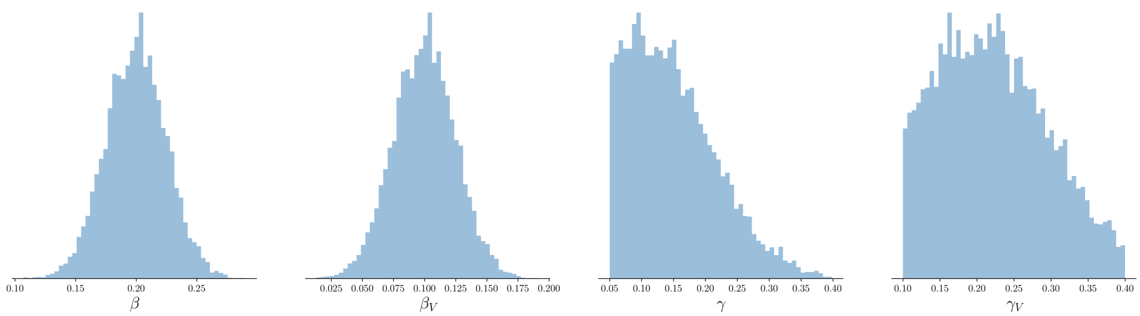
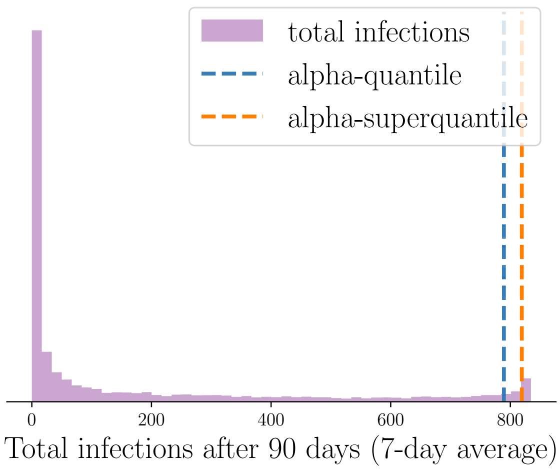


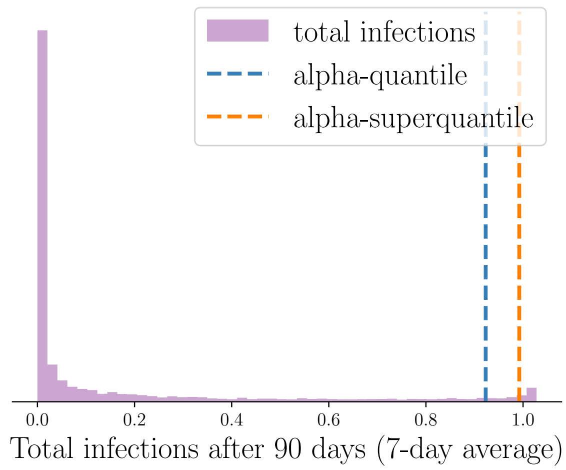
Figure 1: Random variable distribution

Table 2: Risk-based OUU results for optimal vaccination rate.

Method	Optimal vaccination rate	Risk at optimum	
		Q_α	$\overline{Q}_\alpha$
Q_α-based OUU	0.02443	10.14	10.68
$\overline{Q}_\alpha$-based OUU	0.02486	9.53	10.04

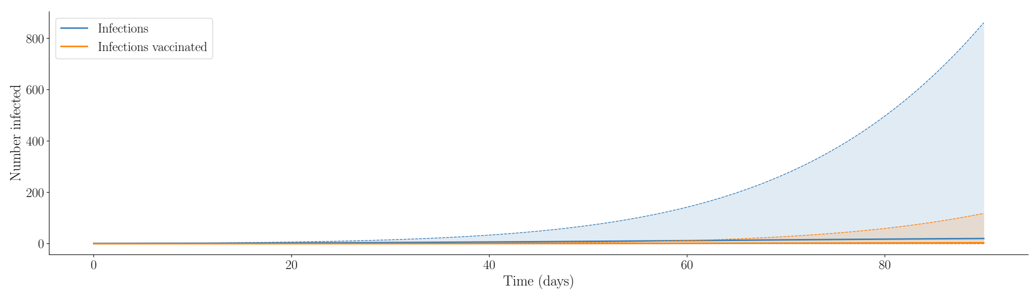


(a) $\nu = 0.005$

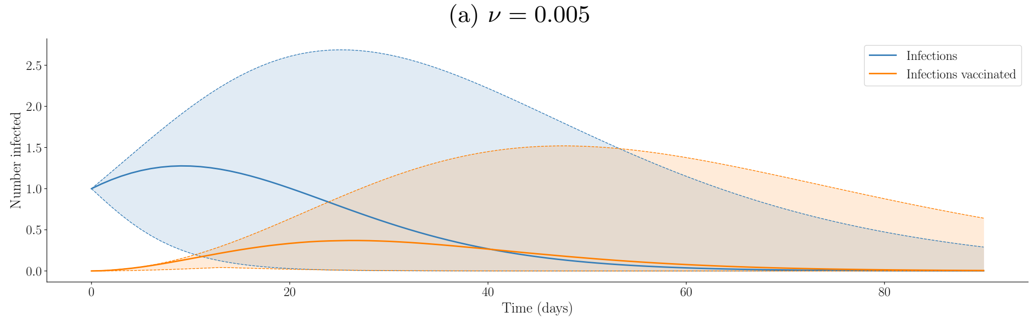


(b) $\nu = 0.05$

Figure 2: Comparing distribution of total infections as a 7-day average under two different vaccination rates.



(a) $\nu = 0.005$



(b) $\nu = 0.05$

Figure 3: Comparing trajectories infections under two different vaccination rates.

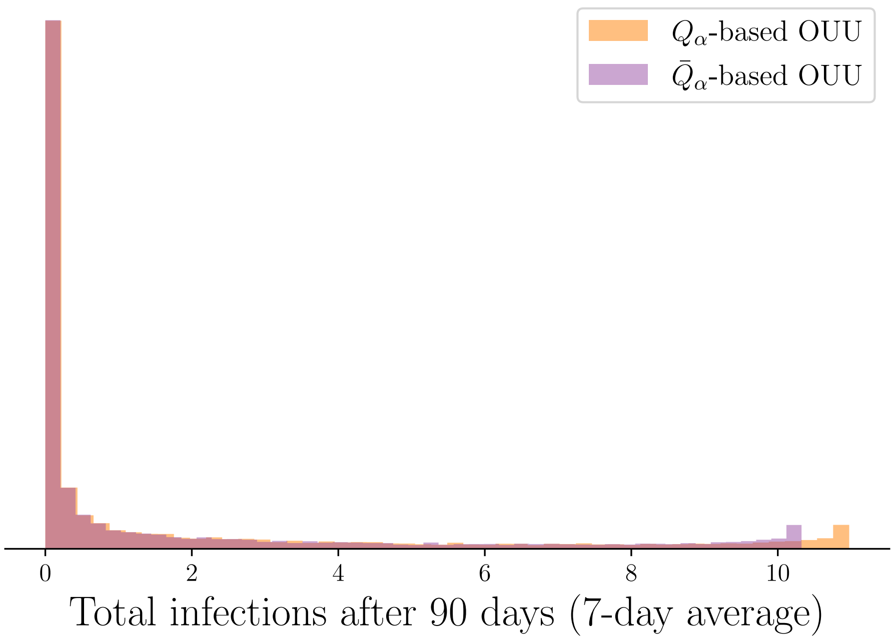


Figure 4: Comparing distributions of 7-day averages of total infections using the optimal vaccination rates obtained from solving the risk-based OUU problems.