

Majorana-Kondo coexistence in a double quantum dot.

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(To be written)

I. INTRODUCTION

In the last few decades the interest in the so called Majorana fermions has been increasing. The particle proposed by the physicist Ettore Majorana as the real field solution of the Dirac equation describes a fermion which is its own antiparticle, hence it has no charge nor mass. To the date no fundamental particle with these characteristics has been found. However, theoretical research predicts that Majorana Fermions emerge as quasi-particles at the boundary of certain topological superconductors Kitaev¹. Recently, the new technological innovations allowed the observation of Majorana signatures in different topological materials²⁻⁵.

Despite the positive experimental results, certain skepticism about the existence of Majorana Fermions still remains in the field. One of the reasons is that some properties of Majorana quasiparticles like the expected non-abelian statistics have not been measure. This property is of especial interest due to its promising applications in topological quantum computing. The braiding protocol based on Majorana's non-abelian statistics is the key to create fault-tolerant quantum devices⁶.

A promising method to detect majorana modes consists in attaching a Quantum Dot (QD) to the edges of a majorana chain in the topological phase and executing transport measurements through the QD⁷. The majorana mode at the end of the chain then leaks inside the QD⁸ which produces a zero-bias conductance peak of half a quanta $\frac{e^2}{2h}$ through the dot. This is a majorana signature which produces half of the expected peak by a regular fermion. Recently, experiments including hybrid Majorana-QD systems have been performed⁹. In addition, the similarity of this phenomenon with the Kondo effect, where the zero-bias conductance peak takes $\frac{e^2}{h}$, motivated the study of combined Kondo-Majorana physics in this system^{10,11}. This project revealed the existence of a region of parameters where both, Kondo and Majorana physics, coexist.

The idea of using hybrid quantum dot-majorana heterostructures to implement quantum gates has acquired wide interest in the last years. other advantage is the possibility of moving around Majoranas in multidot systems by shifting the QD gate voltages and Hence the approach is suitable for the implementation of braiding procedures. The simplest system where Majorana manipulation is possible is a Double Quantum Dot (DQD) coupled to a majorana chain. By tuning the QD gate voltages and the

majorana coupling we will be able to probe the mobility of the majorana modes through the dots.

One of the insights of this method is that the information of the qubit is not completely destroyed as in other methods like tunneling spectroscopy. When majoranas are connected to multidot systems it is possible to manipulate the majorana mode inside the QDs by tuning the gate voltage and tunnel couplings. This fa

When both dots are coupled to the leads the Double Quantum Dot exhibits an antiferromagnetic interaction known as Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction¹²⁻¹⁴. On the other hand, when only one QD is coupled to the leads and the other Dot is attached to the QD, the Kondo effect is annihilated due to the destructive interference generated by extra dot¹⁵. Our study includes how the Majorana mode interacts with these two effects.

II. MODEL AND METHODS

We consider the setup shown in Figure 1 in which a Majorana Bound State (MBS) at the edge of Topological Superconductor(TS) is coupled to a double quantum dot (DQD), which is attached to a single metallic lead. The Hamiltonian of this system can be partitioned in four terms: the DQD Hamiltonian H_{DQD} , the Lead Hamiltonian H_{Lead} , the DQD-lead interaction $H_{DQD-Lead}$ and the coupling between the DQD and the Majorana mode H_{M-DQDs} and

$$H = H_{DQD} + H_{Lead} + H_{DQD-Lead} + H_{M-DQD} \quad (1)$$

The interacting Anderson Model describes the DQD-lead system

$$H_{DQD} = \sum_{i \in \{1,2\}} \sum_{\sigma \in \{\downarrow, \uparrow\}} \left(\epsilon_{di} + \frac{U_i}{2} \right) \hat{n}_{i\sigma} + \frac{U_i}{2} (\sum_{\sigma} \hat{n}_{i\sigma} - 1)^2 \\ + \sum_{\sigma \in \{\downarrow, \uparrow\}} t_{dots} (d_{1\sigma}^\dagger d_{2\sigma} + d_{2\sigma}^\dagger d_{1\sigma}), \quad (2)$$

and

$$H_{Lead} = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} \quad (3)$$

$$H_{DQD-Lead} = \sum_{\mathbf{k}\sigma} \sum_{i \in \{1,2\}} V_{i\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger d_{i\sigma} + V_{i\mathbf{k}}^* d_{i\sigma}^\dagger c_{\mathbf{k}\sigma}, \quad (4)$$



FIG. 1: DQD-Majorana set-up. Solid lines: standard coupling. Dashed lines: majorana spin- \downarrow effective couplings (6). The atomic energy levels appear inside each QD. Red dashed horizontal lines represent the Fermi level.

where ϵ_{di} is the energy level of Dot i , U_i is the Coulomb repulsion and t_{dots} is the coupling parameter between both QDs. The operator $d_{i\sigma}^\dagger$ creates a particle in Dot i with spin σ and $\hat{n}_{i\sigma} := d_{i\sigma}^\dagger d_{i\sigma}$ is the particle number operator of state i . $c_{\mathbf{k}\sigma}^\dagger$ is the creation operator a particle with momentum \mathbf{k} and spin σ in the lead. $\epsilon_{\mathbf{k}l}$ is the corresponding energy and $V_i(\mathbf{k})$ describes the tunneling coupling between the lead and Dot i .

The majorana modes are modeled as a superposition of the creation and annihilation operators of a spin \downarrow particle f_\downarrow

$$\gamma_1 := \frac{1}{\sqrt{2}} (f_\downarrow^\dagger + f_\downarrow), \gamma_2 := \frac{i}{\sqrt{2}} (f_\downarrow^\dagger - f_\downarrow). \quad (5)$$

This makes possible to define an effective coupling between the Majorana Mode and the DQD by attaching γ_1 with the spin- \downarrow channel in the QDs

$$H_{M-DQD} = \sum_{i=1}^2 t_i (d_{i\downarrow}^\dagger \gamma_1 + \gamma_1 d_{i\downarrow}) + \epsilon_M \gamma_1 \gamma_2. \quad (6)$$

where t_i is the coupling parameter between the majorana mode and QD i . ϵ_m is the coupling energy between both majorana modes. This effective model is able to reproduce the physical effects of more elaborated couplings¹¹ such

Ruiz-Tijerina *et al.* showed that this effective coupling is able to reproduce effectively the transport effects of the Kitaev chain in the topological phase is attached to a single QD.

A. Methods

To study the non-interacting case ($U = 0$), we use Zubarev's ballistic transport approach¹⁶ to compute the Green functions associated to both quantum dot operators ($G_{d_1 d_1^\dagger}(\omega), G_{d_2 d_2^\dagger}(\omega)$). The detailed procedure is included in ???. The transport equations define a linear system where the Hamiltonian parameters ($t_1, t_2, \epsilon_1 \dots$) and the energy ω are taken as fix variables. The flow graph in FIG.2 depicts the linear map associated to the transport in an hybrid Majorana-DQD system (see (??)). The energies of each operator are located at the center of the vertexes while the vertex couplings represent the off-diagonal terms. It is possible to observe the Majorana mode divides this graph in two parts, both of them representing a DQD. The upper DQD is conformed by annihilation operators while the lower one is formed by creation operators. The couplings in the lower part are the negative upper parameters. Meanwhile, the majorana fermion is the vertex acting as a connector between both regions.

To simplify the solution of this system we used a graph linear algorithm¹⁷ to speed-up the process of Gauss-Jordan elimination. The idea consists in "popping-out" the vertexes of the graph one-by-one. Each pop-out process must be understood as the Gaussian elimination of the line and the column in the matrix containing that vertex. Depending on the order of elimination of these vertexes different representations of the final polynomial fraction can be obtained. So the real insight of this method is that it allows us to select the order of vertex elimination to obtain the simplest representation of the final this function.

All this process can be observed graphically in figure 2. In the first step we pop-out the extreme vertexes $c_{k,\downarrow}, c_{k,\downarrow}^\dagger, d_{2,\downarrow}, d_{2,\downarrow}^\dagger$ in that order. This condenses the transport information in the vertexes $d_{1,\downarrow}, d_{1,\downarrow}^\dagger$ and f_\downarrow , such that the energies ϵ_{DQD}^\pm accumulate the information of both double quantum dots while the couplings are modified to T_\pm . The last step is to pop-out vertexes $d_{1,\downarrow}^\dagger$ and f_\downarrow . At the end of the whole process the energy of the remaining vertex $d_{1,\downarrow}$ contains the green function $G_{d_{1\downarrow} d_{1\downarrow}^\dagger}(\omega)$ giving this very compact expression

$$G_{d_{1\downarrow} d_{1\downarrow}^\dagger}(\omega) = \frac{1}{\omega - \epsilon_{DQD}^+ - \frac{\|T_+\|^2}{\omega - \epsilon_{M2} - \frac{\|T_-\|^2}{\epsilon_{DQD}}}}. \quad (7)$$

This graph approach turns out to be especially good for Majorana systems since the Majorana fermion is a natural cutting point that divides the graph in two sections, allowing us to exploit the graph structure to simplify the solution of the system. Note also that the spin- \uparrow green functions can be obtained by replacing the majorana couplings $t_1, t_2 = 0$. The final result will depend on the broadening Γ_i defined by

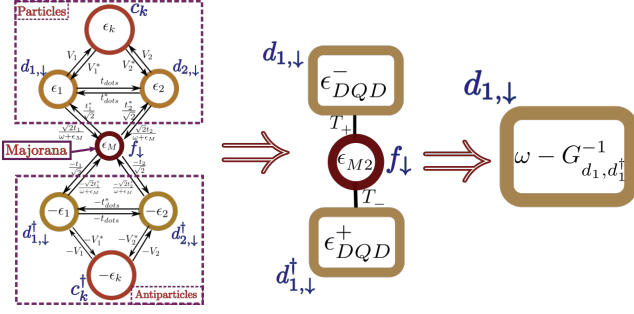


FIG. 2: Transport flow in a DQD majorana system.

$$-i\Gamma_i = \sum_k \frac{V_i^* V_i}{\omega - \epsilon_k}. \quad (8)$$

By convention we will take Γ_1 as the energy unit for the rest of the project. Finally we compute the DOS

$$\rho_{1\sigma}(\omega) = -\frac{1}{\pi} \text{Im} \left[G_{d_{1\sigma}, d_{1\sigma}^\dagger}(\omega) \right]. \quad (9)$$

Similar results can be obtain for the DOS of the second $\rho_{2\sigma}$. Comparing these results for both dots at the Fermi

energy we will be able to determine which dot exhibits a majorana signature.

B. Non-interacting Case (NRG)

For the non-interacting case we used the Numerical Renormalization Group (NRG) approach^{18,19}. To improve the efficiency of the code we took advantage of the preserved symmetries, which are spin- \uparrow particle number \hat{N}_\uparrow and the spin- \downarrow parity. Spin- \downarrow particle number is not preserved due to the term $\hat{P}_\downarrow = \pm$ (+ even, - odd). To implement the NRG code it is necessary to define a fix coulomb repulsion cut-off energy $D^?$. To unify the units of the interacting and non-interacting case we pick $U = 8.69\Gamma_1$ and we let $D = 2U_1 = 17.3\Gamma_1$. The final spectral functions are then computed with the Density Matrix Renormalization Group (DM-NRG) approach²⁰.

III. CONCLUDING REMARKS

Conclusion goes here.

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- ¹ A. Y. Kitaev, *Physics-Uspekhi* **44**, 131 (2001).
- ² V. Mourik, K. Zuo, S. M. Frolov, S. R. Plissard, E. P. a. M. Bakkers, and L. P. Kouwenhoven, *Science* **336**, 1003 (2012).
- ³ A. Das, Y. Ronen, Y. Most, Y. Oreg, M. Heiblum, and H. Shtrikman, *Nature Physics* **8**, 887 (2012).
- ⁴ M. T. Deng, C. L. Yu, G. Y. Huang, M. Larsson, P. Caroff, and H. Q. Xu, *Nano Letters* **12**, 6414 (2012).
- ⁵ H. Zhang, C.-X. Liu, S. Gazibegovic, D. Xu, J. A. Logan, G. Wang, N. van Loo, J. D. S. Bommer, M. W. A. de Moor, D. Car, R. L. M. Op het Veld, P. J. van Veldhoven, S. Koelling, M. A. Verheijen, M. Pendharkar, D. J. Pennachio, B. Shojaei, J. S. Lee, C. J. Palmstrm, E. P. A. M. Bakkers, S. D. Sarma, and L. P. Kouwenhoven, *Nature* **556**, 74 (2018).
- ⁶ S. D. Sarma, M. Freedman, and C. Nayak, *npj Quantum Information* **1**, 15001 (2015).
- ⁷ D. E. Liu and H. U. Baranger, *Physical Review B* **84** (2011), 10.1103/PhysRevB.84.201308, arXiv: 1107.4338.
- ⁸ E. Vernek, P. H. Penteado, A. C. Seridonio, and J. C. Egues, *Physical Review B* **89**, 165314 (2014).
- ⁹ M. T. Deng, S. Vaitiekenas, E. B. Hansen, J. Danon, M. Leijnse, K. Flensberg, J. Nygard, P. Krogstrup, and C. M. Marcus, *Science* **354**, 1557 (2016).

- ¹⁰ M. Lee, J. S. Lim, and R. Lopez, *Physical Review B* **87**, 241402 (2013).
- ¹¹ D. A. Ruiz-Tijerina, E. Vernek, L. G. G. V. Dias da Silva, and J. C. Egues, *Physical Review B* **91**, 115435 (2015).
- ¹² M. A. Ruderman and C. Kittel, *Physical Review* **96**, 99 (1954).
- ¹³ T. Kasuya, *Progress of Theoretical Physics* **16**, 45 (1956).
- ¹⁴ K. Yosida, *Physical Review* **106**, 893 (1957).
- ¹⁵ L. G. G. V. Dias da Silva, N. Sandler, K. Ingersent, and S. E. Ulloa, *Physica E: Low-dimensional Systems and Nanostructures* **40**, 1002 (2008).
- ¹⁶ D. N. Zubarev, *Soviet Physics Uspekhi* **3**, 320 (1960).
- ¹⁷ D. A. Spielman, *Algorithms, Graph Theory, and Linear Equations in Laplacian Matrices*, Proceedings of the International Congress of Mathematicians (2010).
- ¹⁸ K. G. Wilson, *Reviews of Modern Physics* **47**, 773 (1975).
- ¹⁹ M. Sindel, *Numerical Renormalization Group studies of Quantum Impurity Models in the Strong Coupling Limit*, Text.PhDThesis, Ludwig-Maximilians-Universitt Mnchen (2005).
- ²⁰ W. Hofstetter, *Physical Review Letters* **85**, 1508 (2000).

1. Appendix

In Zubarev's fermionic ballistic transport approach¹⁶ the green functions associated to two operators $A(t)$, $B(t)$ is defined as that Fourier transform of the time-ordered anti-commutator of A and B

$$G_{A,B}(\omega) = \mathcal{F}\{\mathcal{T}[\{A(t), B(t')\}]\}(\omega). \quad (10)$$

The Fourier transform of Schrodinger evolution determines the transport equations

$$\omega G_{A,B}(\omega) = \delta_{A^\dagger, B} + G_{[A,H],B}(\omega). \quad (11)$$

We can apply this to Hamiltonian (1) by replacing A and B by the creation and annihilation operators. To simplify the complexity of the system we fix $B = d_{1\downarrow}^\dagger$. In addition note that the transport equations for f_\downarrow and f_\downarrow^\dagger are

$$(\omega - \epsilon_M) G_{f_\downarrow, d_{1\downarrow}^\dagger}(\omega) = \frac{t}{\sqrt{2}} \left(G_{d_{1\downarrow}, d_{1\downarrow}^\dagger}(\omega) - G_{d_{1\downarrow}^\dagger, d_{1\downarrow}}(\omega) \right) \quad (12)$$

$$(\omega + \epsilon_M) G_{f_\downarrow^\dagger, d_{1\downarrow}^\dagger}(\omega) = \frac{t}{\sqrt{2}} \left(G_{d_{1\downarrow}, d_{1\downarrow}^\dagger}(\omega) - G_{d_{1\downarrow}^\dagger, d_{1\downarrow}}(\omega) \right), \quad (13)$$

which allows us to take $G_{f_\downarrow^\dagger, d_{1\downarrow}^\dagger}(\omega) = \frac{\omega + \epsilon}{\omega - \epsilon} G_{f_\downarrow, d_{1\downarrow}^\dagger}(\omega)$. Therefore, we can eliminate $G_{f_\downarrow^\dagger, d_{1\downarrow}^\dagger}(\omega)$ from the equations even before we start Gauss-Jordan process.

Writing the other equations we obtain the linear system

$$T \vec{G}_{d_1^\dagger} = \hat{e}_1 \quad (14)$$

where T is the transport matrix

$$\begin{bmatrix} \omega - \epsilon_1 & -V_1^* & -t_{dots} & \frac{-t_1}{\sqrt{2}} & 0 & 0 & 0 \\ -V_1 & \omega - \epsilon_k & -V_2 & 0 & 0 & 0 & 0 \\ -t_{dots}^* & -V_2^* & \omega - \epsilon_2 & \frac{-t_2}{\sqrt{2}} & 0 & 0 & 0 \\ \frac{-\sqrt{2}t_1^*}{\omega + \epsilon_M} & 0 & \frac{-\sqrt{2}t_2^*}{\omega + \epsilon_M} & \omega - \epsilon_M & \frac{\sqrt{2}t_1}{\omega + \epsilon_M} & 0 & \frac{\sqrt{2}t_1^*}{\omega + \epsilon_M} \\ 0 & 0 & 0 & \frac{t_2}{\sqrt{2}} & \omega + \epsilon_2 & V_2^* & t_{dots}^* \\ 0 & 0 & 0 & 0 & V_2 & \omega + \epsilon_k & V_1 \\ 0 & 0 & 0 & \frac{t_1}{\sqrt{2}} & t_{dots} & V_1^* & \omega + \epsilon_1 \end{bmatrix}, \quad (15)$$

$\vec{G}_{d_1^\dagger}$ is the column vector

$$[G_{d_{1\downarrow}, d_{1\downarrow}^\dagger}(\omega), G_{c_{k\downarrow}, d_{1\downarrow}^\dagger}(\omega), G_{d_{2\downarrow}, d_{1\downarrow}^\dagger}(\omega), G_{f_\downarrow}, d_{1\downarrow}^\dagger(\omega), G_{d_{2\downarrow}^\dagger, d_{1\downarrow}^\dagger}(\omega), G_{c_{k\downarrow}^\dagger, d_{1\downarrow}^\dagger}(\omega), G_{d_{1\downarrow}^\dagger, d_{1\downarrow}^\dagger}(\omega)]^T$$

and \hat{e}_1 is the vector with entries $\hat{e}_{1n} = \delta_{1n}$.

The graph associated to this matrix is the one in FIG.2. The energies inside each vertex are given by subtracting the corresponding diagonal term from ω . The couplings are just the negative of the off-diagonal terms.

2. The Double Quantum Dot

To explain the process of Gaussian elimination we will obtain the green function for the case without Majorana fermion ($t_1 = t_2 = 0$). The transport matrix for this system is

$$\begin{bmatrix} \omega - \epsilon_1 & -V_1 & -t_{dots} \\ -V_1^* & \omega - \epsilon_k & -V_2 \\ -t_{dots}^* & -V_2^* & \omega - \epsilon_2 \end{bmatrix}. \quad (16)$$

The graph associated to this matrix can be observed in FIG3.a). To eliminate the vertex c_k we just need to subtract from (16) the rank-1 matrix that cancels the row and the column corresponding to c_k . This matrix is

$$\begin{bmatrix} \frac{V_1^* V_1}{\omega - \epsilon_k} & -V_1^* & \frac{V_2 V_1^*}{\omega - \epsilon_k} \\ -V_1 & \omega - \epsilon_k & -V_2 \\ \frac{V_2^* V_1}{\omega - \epsilon_k} & -V_2^* & \frac{V_2^* V_2}{\omega - \epsilon_k} \end{bmatrix}. \quad (17)$$

The result of (16) - (17) is

$$\begin{bmatrix} \omega - \epsilon_1 - \frac{V_1^* V_1}{\omega - \epsilon_k} & 0 & -t_{dots} - \frac{V_2 V_1^*}{\omega - \epsilon_k} \\ 0 & 0 & 0 \\ -t_{dots}^* - \frac{V_2^* V_1}{\omega - \epsilon_k} & 0 & \omega - \epsilon_2 - \frac{V_2^* V_2}{\omega - \epsilon_k} \end{bmatrix} \quad (18)$$

which is depicted by the graphs in FIG.3.b). The next step is to pop-out the vertex d_2 following the same procedure. At the end, the energy inside the vertex d_1 will be

$$\epsilon_{DQD} = \epsilon_1 + \sum_{\mathbf{k}} \frac{V_1 V_1^*}{\omega - \epsilon_{\mathbf{k}}} + \frac{\left\| t_{dots} + \sum_{\mathbf{k}} \frac{V_1 V_2^*}{\omega - \epsilon_{\mathbf{k}}} \right\|^2}{\omega - \epsilon_2 - \sum_{\mathbf{k}} \frac{V_2^* V_2}{\omega - \epsilon_{\mathbf{k}}}} \quad (19)$$

and the green function of $G_{d_1 d_1^\dagger}(\omega)$ in a DQD will be given by $\frac{1}{\omega - \epsilon_{DQD}}$ (see FIG.3.c)).

3. Solution of the transport equations

The previous procedure can be generalized into the following algorithm:

1. To compute the transport equations with the second term fixed in the creation operator of the dot.
2. To set up the graph associated to the transport system.
3. To pop out the vertexes of the graph. Each pop-out process carries the following steps.
 - (a) To compute the extra-terms in the energies and couplings based on the walks passing through the vertex that will be popped out.
 - (b) To eliminate this vertex from the graph.

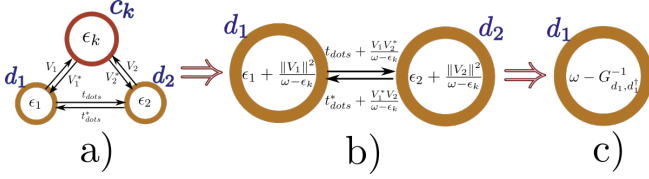


FIG. 3

(c) To iterate till only just one vertex.

4. Invert the last energy to obtain the final green function of the dot.

To solve the general case we start with the graph in FIG.2 and we pop out the vertexes $c_k, c_k^\dagger, d_{2,\downarrow}$ and $d_{2,\downarrow}^\dagger$ in that order. The energies associated to $d_{1,\downarrow}$ and $d_{1,\downarrow}^\dagger$ will be similar to (19) giving

$$\epsilon_{DD}^\pm = \pm\epsilon_1 + \sum_{\mathbf{k}} \frac{V_1 V_1^*}{\omega - \epsilon_{\mathbf{k}}} + \frac{\left\| \pm t_{dots} + \sum_{\mathbf{k}} \frac{V_1 V_2^*}{\omega - \epsilon_{\mathbf{k}}} \right\|^2}{\omega \pm \epsilon_2 - \sum_{\mathbf{k}} \frac{V_2 V_2^*}{\omega - \epsilon_{\mathbf{k}}}}. \quad (20)$$

There is also a correction in the couplings between the majorana mode and $d_{1,\downarrow}, d_{1,\downarrow}^\dagger$ given by

$$T_\pm = \pm t_1 \pm t_2 \frac{\left(\pm t_{dots} + \sum_{\mathbf{k}} \frac{V_1 V_2^*}{\omega - \epsilon_{\mathbf{k}}} \right)}{\omega \pm \epsilon_2 \pm \sum_{\mathbf{k}} \frac{V_2 V_2^*}{\omega - \epsilon_{\mathbf{k}}}}. \quad (21)$$

Finally since the Majorana is in contact with Dot 2, there is an extra-term appearing in the majorana energy given by

$$\epsilon_{M2} = \omega - \epsilon_M - \frac{\frac{\omega}{\omega + \epsilon_M} \|t_2\|^2}{\omega - \epsilon_2 - \sum_{\mathbf{k}} \frac{V_2 V_2^*}{\omega - \epsilon_{\mathbf{k}}}} - \frac{\frac{\omega}{5a + \epsilon_M} \|t_2\|^2}{\omega + \epsilon_2 - \sum_{\mathbf{k}} \frac{V_2 V_2^*}{\omega + \epsilon_{\mathbf{k}}}}. \quad (22)$$

With all the terms of the graph in FIG.2.b) computed, it only remains to pop out vertexes $d_{1,\downarrow}^\dagger$ and f_{\downarrow} in that order to obtain the green function in equation (7).