Majorana-Kondo coexistance in a double quantum dot.

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(To be written)

I. INTRODUCTION

Basics of Majorana Bound states: zero-energy edge states in 1D topological superconductors.[1]

Discovery on semiconductor nanowires[2–4] etc. Experimental results[5] Luis *Add others.*

Leaking of Majorana to a QD:[6], Interplay of Kondo and Majorana[7] Important results: Majorana and Kondo co-exist in the quantum dot even if the dot is non-topological.

More recent experimental results: Deng et al.[8] attached a QD to the end of a nanowire.

Here we study the coupling of a MBS to a *double* quantum dot system.

→ *Punchline* Multidot systems offer the possiblity of "moving" Majoranas aroung using gate voltages and couplings. ossibility of Majorana braiding. Here, we study the simplest case, which is a double dot system.

II. MODEL AND METHODS

We consider the setup shown in Figure 1 in which a Majorana Bound State (MBS) at the edge of Topological Superconductor(TS) is coupled to a double quantum dot (DQD), which is attached to a single metallic lead. The Hamiltonian of this system can be expressed as a sum of four terms: the DQD Hamiltonian H_{DQD} , the Lead Hamiltonian H_{Lead} and the interaction that couples the DQD with the Majorana mode H_{M-DQDs} and with the lead $H_{DQD-Lead}$.

$$H = H_{DQD} + H_{Lead} + H_{DQD-Lead} + H_{M-DQDs} \quad (1)$$

For the DQD-lead system, we can implement the Anderson Model so that

$$H_{DQD} = \sum_{\sigma}\sum_{i\in\{1,2\}}\left(\epsilon_{di} + \frac{U_i}{2}\right)\hat{n}_{i\sigma} + \frac{U_i}{2}(\sum_{\sigma}\hat{n}_{i\sigma} - 1)^2 + \sum_{\sigma\in\{\uparrow,\downarrow\}} t_{dots}(d^{\dagger}_{1\sigma}d_{2\sigma} + d^{\dagger}_{2\sigma}d_{1\sigma}),$$

where ϵ_{di} represents the energy level of each QD, U_i is the Coulomb repulsion and t_{dots} is the coupling between both QDs. In addition the operator $d^{\dagger}_{i\sigma}$ creates a particle in dot i with spin σ and $\hat{n}_{i\sigma} := d^{\dagger}_{i\sigma}d_{i\sigma}$ is the particle number operator of this state. On the other hand, the interaction with the Lead is described by

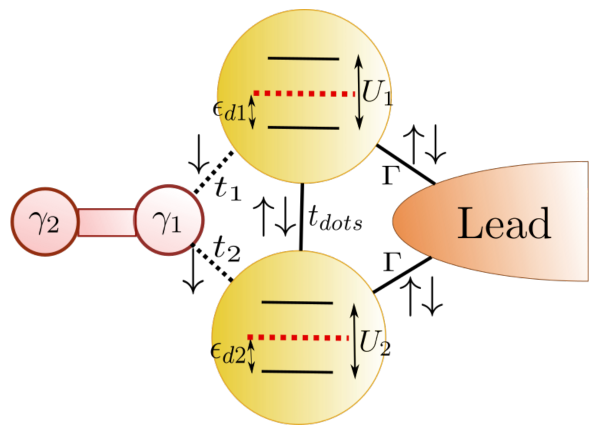

FIG. 1. DQD-Majorana set-up. Solid lines represent standard coupling interactions. Dashed lines represent majorana spin-↓ effective couplings (3). The atomic-limit energy levels are plotted inside each QD. ϵ_{di} is the particle's energy inside QDi and U_i is the corresponding coulomb potential. The red dashed lines represent the Fermi level.

Luis *Pls add a "label" to EVERY equation and EVERY figure.*

$$H_{Lead} = \sum_{\mathbf{k}\sigma}\epsilon_{\mathbf{k}}c^{\dagger}_{\mathbf{k}\sigma}c_{\mathbf{k}\sigma}$$
$$H_{DQD-Lead} = \sum_{\mathbf{k}\sigma}\sum_{i\in\{1,2\}}\Gamma_i c^{\dagger}_{\mathbf{k}\sigma}d_{i\sigma} + \Gamma_i d^{\dagger}_{i\sigma}c_{\mathbf{k}\sigma},$$

where the sum over $\mathbf{k}$ is performed over all possible crystal momentums in the lead. The operator $c^{\dagger}_{\mathbf{k}\sigma}$ creates a particle with momentum $\mathbf{k}$ and spin σ in the lead, $\epsilon_{\mathbf{k}l}$ is the energy of the lead's particles and Γ_i is the hopping exchange term between the lead each QD .

To model the interaction between the DQD and the Majorana Mode we define the majorana operators as the superposition of the creation and annihilation operators of a spin ↓ particle $f_{\downarrow}$:

$$\gamma_1 := \frac{1}{\sqrt{2}}\left(f^{\dagger}_{\downarrow} + f_{\downarrow}\right), \gamma_2 := \frac{1}{\sqrt{2}}\left(f^{\dagger}_{\downarrow} - f_{\downarrow}\right).$$

This makes possible to define an effective coupling between the Majorana Mode and the DQD by attaching γ_1 with the spin-↓ channel in the QDs

$$H_{M-DQD} = \sum_i t_i \left(d^\dagger_{i\downarrow}\gamma_1 + \gamma_1 d_{i\downarrow} \right) \tag{2}$$
$$= \sum_i t_i \left(d^\dagger_{i\downarrow} f^\dagger_\downarrow + f_\downarrow d_{i\downarrow} + d^\dagger_{i\downarrow} f_\downarrow + + f^\dagger_\downarrow d_{i\downarrow} \right) \tag{3}$$

Jesus *Should I put the other terms?*

This effective coupling represents the single majorana mode at the edge of the TS attached to the QD. Ruiz-Tijerina *et al.* proved that the interaction (3) is able to reproduce effectively the results obtained when the Kitaev chain in the topological phase is attached to a single QD.

A. Methods

In order to calculate the properties of the model given by Eq. (3), we used the Numerical Renormalization Group (NRG) approach.[9–11] To to improve the efficiency of the code we used the symmetries of the system. Now, one of the insights of this model is that spin-↓ particle number $\hat{N}_\downarrow$ is not preserved due to the term $(d^\dagger_{i\downarrow} f^\dagger_\downarrow + f_\downarrow d_{i\downarrow})$ in H_{M-DQD} (3). However the spin-↑ particle number $\hat{N}_\uparrow$ and the parity of spin-↓ particles $\hat{P}_\downarrow = \pm$ (+ even, − odd) are still conserved.

To implement the NRG code it is necessary to define a cut-off energy D[11]. By convention, we define this energy to be double the coulomb repulsion of the first dot $D = 2U_1$. For convenience we use D as energy unit.

The local density of states (LDOS) on each dot is obtained by computing the spectral function

$$\rho_{i\sigma}(\omega) = \frac{-i}{\pi}\mathcal{I}\left[\mathcal{G}^R_{i\sigma}(\omega)\right], \tag{4}$$

where $\mathcal{G}^R(\omega)$ is the Fourier transform of the Green function $\mathcal{G}^R(t,T) = -i\theta(t)\langle\{d_{i,\sigma}(t), d^\dagger_{i,\sigma}(0)\}\rangle$. and the spectral functions were calculated with the DM-NRG method.[12]

Jesus *Luis... should I put details about the methods that we used?. Till now, I just mentioned them with references.* Luis *It is fine for now*

B. Atomic limit

In the atomic limit ($\Gamma = 0$) the lead interaction is neglected. Hence the Hamiltonian (1) reduces to

$$H = H_{DQD} + H_{M-DQDs}. \tag{5}$$

The dimension of this Hamiltonian is $2^2 \times 2^2 \times 2 = 32$ (2^2 per QD ↑, ↓ and 2 due to the majorana spin-↓).

A simple analytical solution can be given for the case where $\epsilon_{di} = \frac{-U_i}{2} = \frac{-U}{2}$ and $t_{dots} = 0$. With

$$H = \tfrac{U}{2}\sum_i(\sum_\sigma \hat{n}_{i\sigma} - 1)^2 + \sum_i t_i \left(d^\dagger_{i\downarrow}\gamma_1 + \gamma_1 d_{i\downarrow} \right). \tag{6}$$

Hamiltonian (6) can be written in blocks labeled by the two conserved quantum numbers $\hat{N}_\uparrow$ and $\hat{P}_\downarrow = \pm$. For example the block for 0 spin-↑ and odd spin-↓ particles $\left(\hat{N}_\uparrow = 0, \hat{P}_\downarrow = -\right)$ can be written in terms of the base

$$\{|\downarrow,\downarrow,\downarrow\rangle, |0,0,\downarrow\rangle, |0,\downarrow,0\rangle, |\downarrow,0,0\rangle\}; \tag{7}$$

where the states are labeled by $|QD1, QD2, MZM\rangle$. Hence the representation of block $\left(\hat{N}_\uparrow = 0, \hat{P}_\downarrow = -\right)$ is

$$\begin{array}{c} |\downarrow,\downarrow,\downarrow\rangle \rightarrow \\ |0,0,\downarrow\rangle \rightarrow \\ |0,\downarrow,0\rangle \rightarrow \\ |\downarrow,0,0\rangle \rightarrow \end{array} \begin{bmatrix} 0 & 0 & -t_1 & t_2 \\ 0 & U & t_2 & t_1 \\ -t_1 & t_2 & \frac{U}{2} & 0 \\ t_2 & t_1 & 0 & \frac{U}{2} \end{bmatrix}. \tag{8}$$

The four eigen-energies of this block of the Hamiltonian can be written as Luis *Let's call these energy states something!*

$$\varepsilon^{(1)}_\pm = U/4 \pm \sqrt{\tfrac{U^2}{16} + t_1^2 + t_2^2} \tag{9}$$
$$\varepsilon^{(2)}_\pm = \tfrac{3U}{4} \pm \sqrt{\tfrac{U^2}{16} + t_1^2 + t_2^2}\,.$$

For $t << \frac{U}{4}$, the eigenvalues can be approximated by the Taylor series giving

$$\varepsilon^{(1)}_\pm \approx U/4 \pm U/4\left(1 + 8\tfrac{t_1^2+t_2^2}{U^2}\right) \tag{10}$$
$$\varepsilon^{(2)}_\pm \approx \tfrac{3U}{4} \pm U/4\left(1 + 8\tfrac{t_1^2+t_2^2}{U^2}\right).$$

Comparing this results with the $t = 0$ eigen-values $(0, \frac{U}{2}, U)$, we observe that the displacement of the energy levels generated by the majorana couplings t_1 and t_2 is

$$\delta_{t_1,t_2} = \frac{2t_1^2 + 2t_2^2}{U}. \tag{11}$$

This implies that the energy scale where the majorana effects will be observed will scale as the square root of the energy couplings. A similar result will be obtained for the other quantum numbers.

Jesus *The analysis that I did explains the energy scale where the majorana peaks appear. But I don't know if it is good to do the base change that is the way we used to explain the Kondo satellites.*

Luis *No need to change the basis. But can you add some dashed lines in the plots of Fig. 3 showing these values as a reference?*

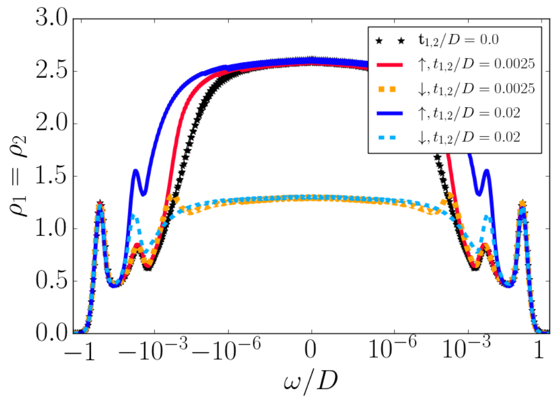

FIG. 2. Density of states at each QD of the horizontal dashed cuts in Figure 3 . The energy is in logarithmic scale. At the Fermi energy ($\omega = 0$) the spin-↑ DOS is $\rho_{\uparrow}(0) \sim \frac{1}{4\pi\Gamma}$. At $t_{1,2} = 0$ $\rho_u p = \rho_{\downarrow}$. However, for $t > 0$ $\rho_{\downarrow}(0) = \frac{\rho_{\uparrow}(0)}{2}$ which is a majorana signature. The spin-↑ and spin-↓ DOS split at an energy scale that depends on the parameter $t_1 = t_2$.

III. NRG RESULTS

A. Connecting the Majorana Mode

Parameters:
$\Gamma \sim 2.83 \times 10^{-2} D,\ t_{dots} = 0,\ U_{1,2} = -2\epsilon_{d1,2} = 0.5D.$
Variable:

$$t_1 = t_2 \in [0\ ,\ 2 \times 10^{-2} D].$$

The first process consists in attaching the Majorana mode to both Quantum Dots symmetrically. For this, we scale up the coupling parameter $t_1 = t_2$ from $0D$ (Decoupled) to $0.02D$ (Completely coupled). The other parameters where chosen with an equilibrium between the dot energy and Coulomb repulsion ($\epsilon_{d1,2} = -\frac{U_{1,2}}{2}$) and without inter-dot coupling $t_{dots} = 0$. These circumstances guarantee that the system preserves Particle Hole Symmetry (PHS). Thus the Density of States (DOS) of particles and holes remains equal at all instances ($\rho(-\omega) = \rho(\omega)$).

For $t_1 = t_2 = 0$ the system consists only of a DQD coupled to a NM lead. Since the model is symmetric for both QDs , the Kondo density of states splits between both dots (See Figure 2). Apart from the coulomb peaks that appear at $\omega = 0.25D = \pm\epsilon_{d1,2}$, two new sided peaks emerge at low-energies ($\omega \sim 10^{-2}D$). These peaks are the result of the of a strong anti-ferromagnetic interaction between both dots caused by the indirect exchange of quantum states through the Lead. This interaction receives the name of Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction[13–15].

Once the MZM is attached $t_1 = t_2 > 0$ the spin-↑ and spin-↓ DOS split at low energies. In both dots the spin-↓ DOS at the Fermi energy ($\omega = 0$) decays to the half of the spin-↑ DOS $\rho_{\downarrow} = \frac{\rho_{\uparrow}}{2}$. It was previously verified by Ruiz-Tijerina *et al.* that the Majorana signature of a QD in the Kondo regime coupled to a TS wire is the half-decay of the spin-↓ DOS. Hence the results from Figure 2 imply that the majorana signature appears in both dots.

Moreover, there is an additional effect caused by the indirect exchange between the QDs through the Majorana mode. The energy scale of this effect was computed for the atomic limit in equation (**??**). Depending on this energy scale the consequences the indirect exchange through the majorana mode will produce different results.

1. If $t_1 = t_2 \ll \Gamma$ two more satellites are formed at very low energies only in the spin-↓ DOS. These peaks correspond to the displacement energy $\frac{4t^2}{U}$ found in the atomic limit (**??**) (See Figure 3 Spin-down $\omega \sim 10^{-3}D$).
2. For $t_1 = t_2 \sim \Gamma$ the Majorana peaks do not appear in the spin-↓ DOS any more. Instead, we observe a combined Kondo-majorana physics in the satellite peaks. This produces an increase of the DOS at the energy scale where the satellite peaks appeared at $t_1 = t_2 = 0$. Since $\rho_{\uparrow}$ must double $\rho_{\downarrow}$ at $\omega = 0$, the spin-↑ satellites are greater (See 2 for $t_{1,2} = 0.02D$). This effect causes the rapid scale-up of satellite peaks for $t_1 = t_2 > 0.01D$ (See 3 Spin-↑, $\omega \sim 10^{-2}D$).

Jesus *Maybe we should include a plot comparing the temperature of Kondo and majorana physics.*

B. Transferring the MZM

Parameters:

$$\Gamma \sim 2.83 * 10^{-2} D, t_{dots} = 0, U_{1,2} = -2\epsilon_{d1} = 0.5$$
$$t_1 = t_2 = 0.02D$$

Variable:

$$\epsilon_{d2} \in [-0.25D\ , -0.05D]$$

This process starts with the DQD coupled symmetrically to the Majorana mode, just as in III A. The idea of this process is to break PHS by increasing the energy of the second QD ϵ_{d2}. This procedure should induce the Majorana to tunnel only into the first dot.

In 4 we observe that both, the Kondo and the MZM peaks are preserved in the first QD as well as the majorana signature (See 5) when ϵ_{d2} is scaled up to -0.1. However, PHS breaking will favor the growth of the spin-↑ hole ($w > 0$) satellite and the spin-↓ particle ($w < 0$) satellite.

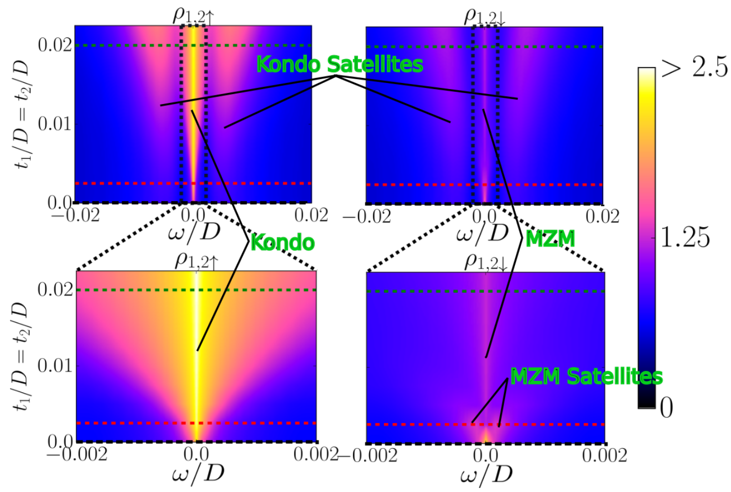

FIG. 3. Evolution of the DOS of both QDs through $t_1 = t_2$ tuning. UP: Energy scale $\omega \sim 10^{-2}D$. DOWN: Energy scale $\omega \sim 10^{-3}D$. LEFT: Spin $\uparrow$. RIGHT: Spin $\downarrow$. Luis *IN the paper plot you need to remove to comments "MZM satellites", etc. We can add small arrows and explain in the main text.*

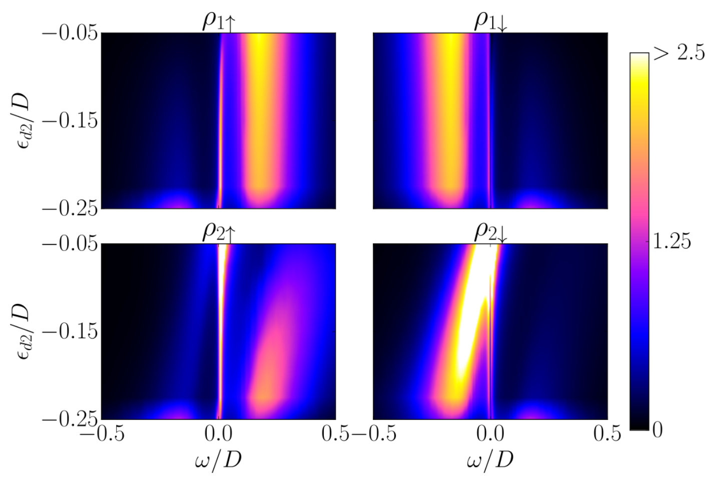

FIG. 4. Evolution of the DOS of both QDs through the ϵ_{d2} tuning. UP: QD1. DOWN: QD2. LEFT: Spin $\uparrow$. RIGHT: Spin $\downarrow$.

IV. CONCLUDING REMARKS

Conclusion goes here.

ACKNOWLEDGMENTS

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Luis *Only ONE bib file*

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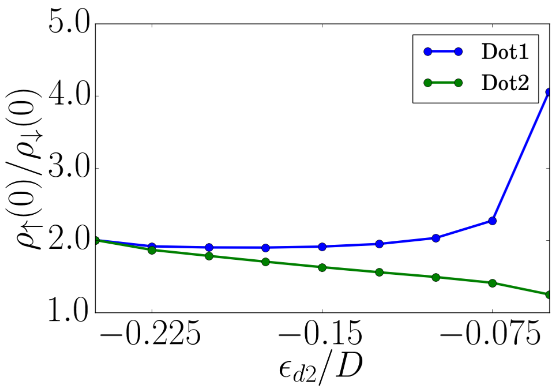

FIG. 5. As described in III A the relation $\frac{\rho_\uparrow(0)}{\rho_\uparrow(0)} = 2$ determines the Majorana Signature . This picture shows the evolution of the relation $\frac{\rho_\uparrow(0)}{\rho_\uparrow(0)}$ for both QDs. While QD2 losses rapidly the Majorana signature, QD1 maintains it till $\epsilon_{d2} \sim -0.1D$. For $\epsilon_{d2} > -0.1D$ the coulomb peaks in QD2 overlap with the fermi energy which destroys the Kondo effect in the first dot (See Figure 4).

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