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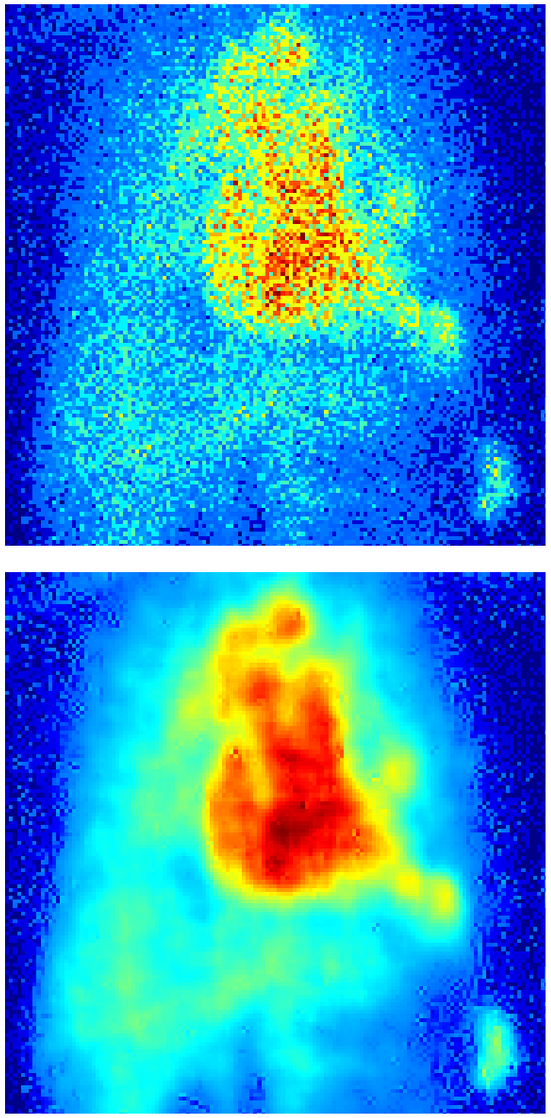
Iterative Convolutional Neural Networks for Image Restoration

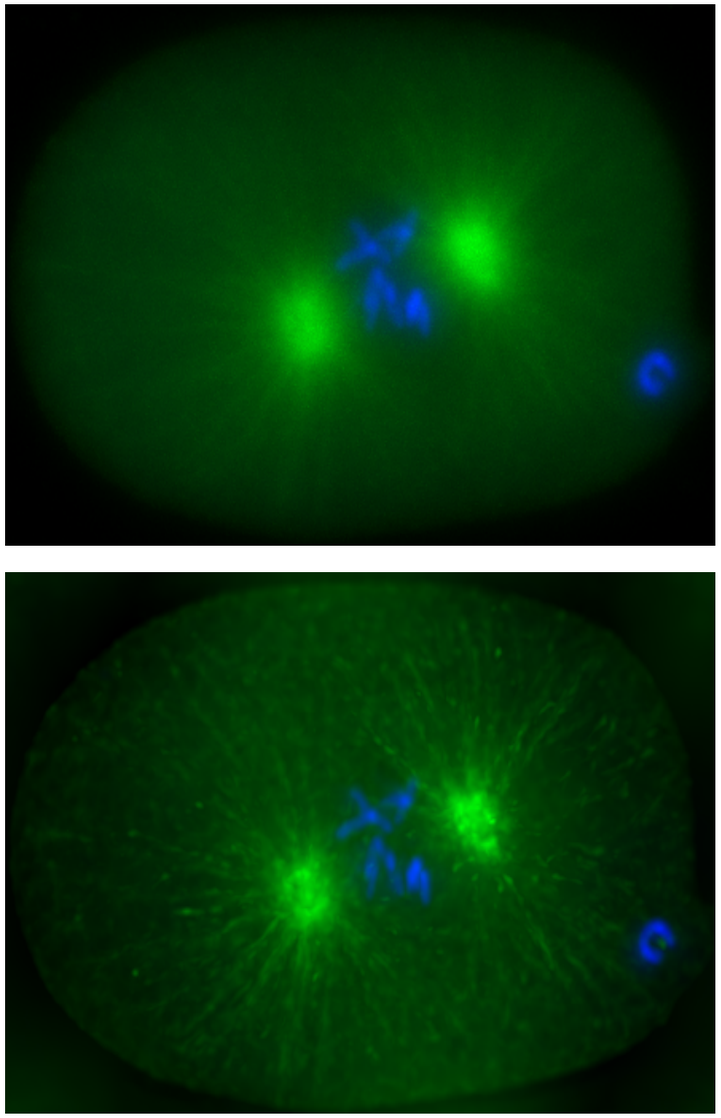
Stamatis Lefkimmiatis

Skolkovo Institute of Science and Technology

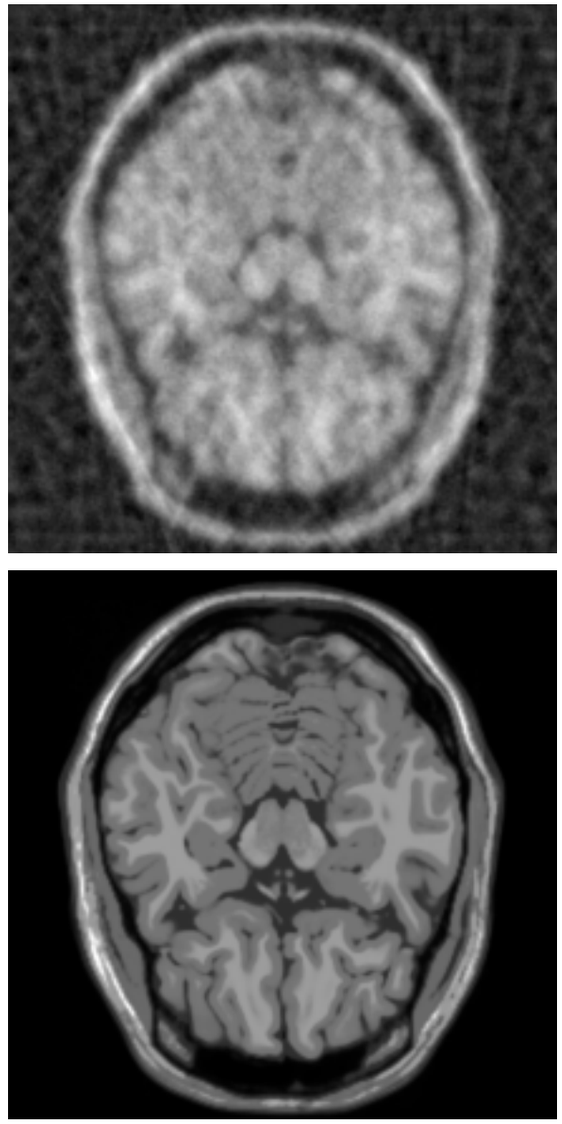
Inverse Problems in Image Processing



denoising



deblurring



MRI reconstruction

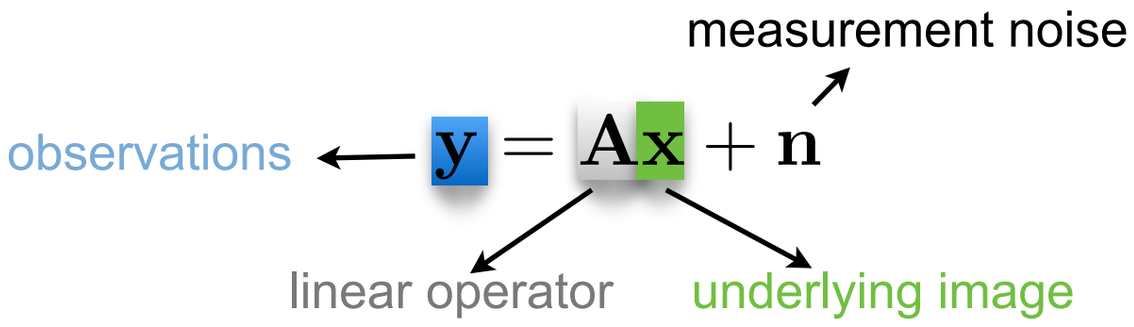
$\mathbf{y}$

$\mathbf{x}$

- Goal : recover $\mathbf{x}$ from a low-quality observed image $\mathbf{y}$

Variational / Bayesian Approach : Overview

- Forward model : Image acquisition can often be modeled as a linear process



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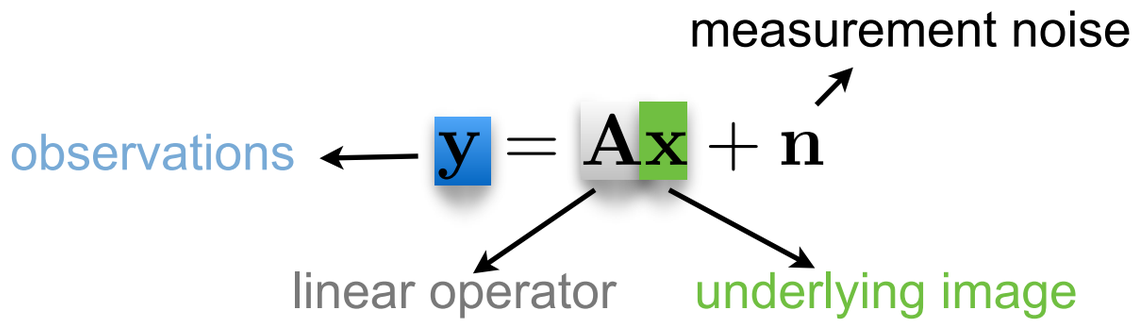

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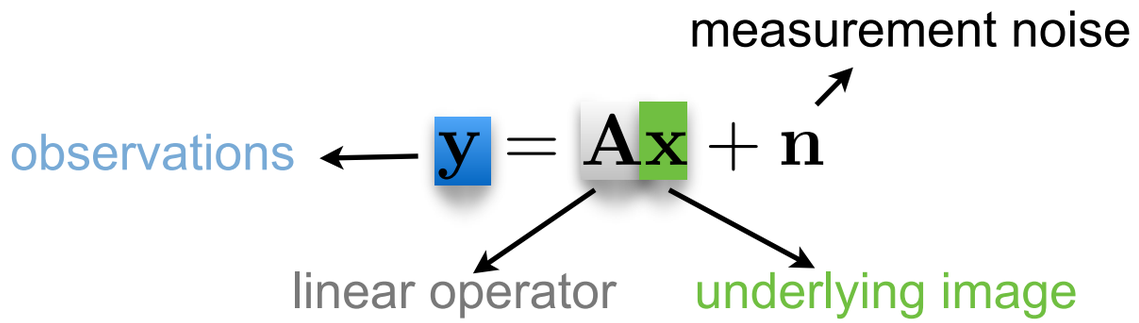

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  - **A** : Identity operator $\Rightarrow$ denoising

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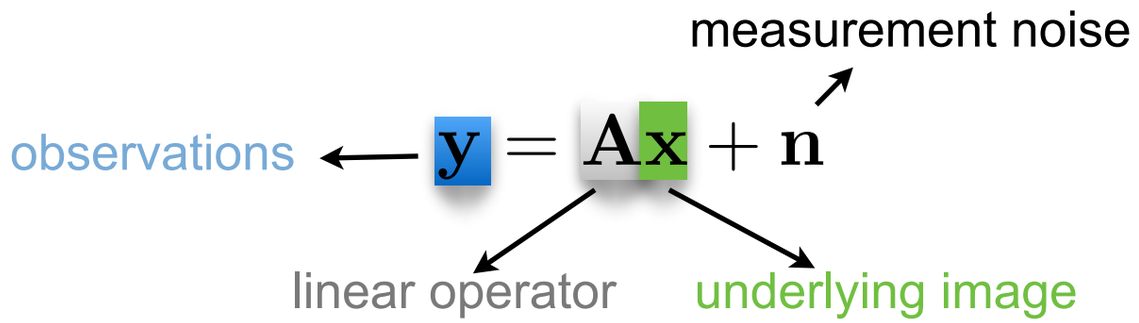

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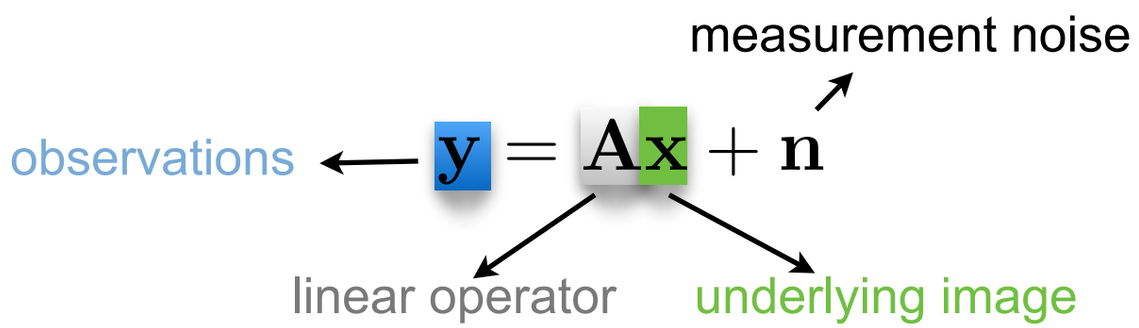

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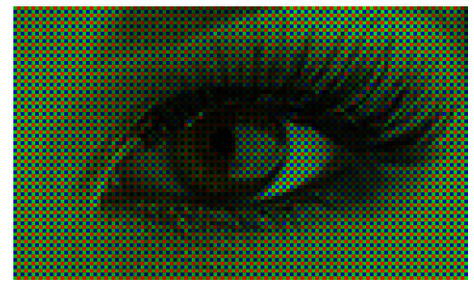

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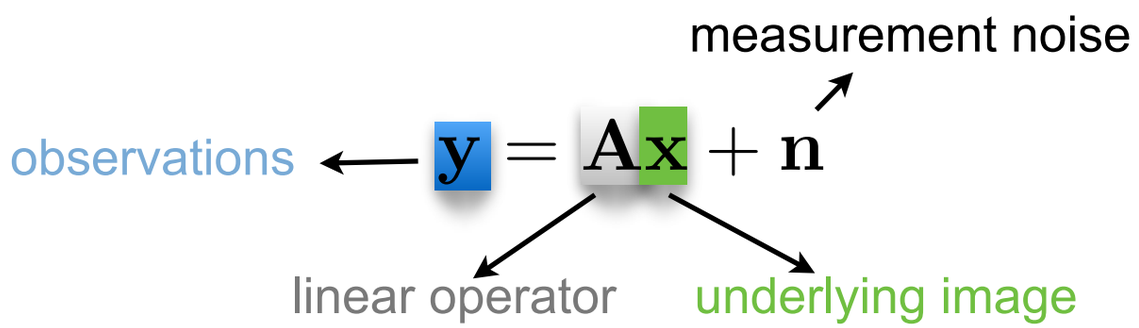

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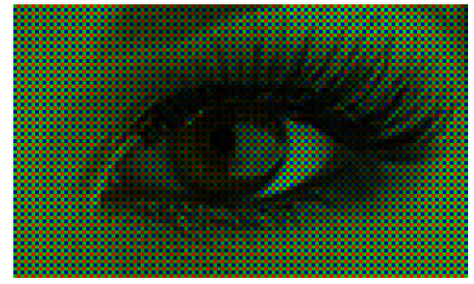




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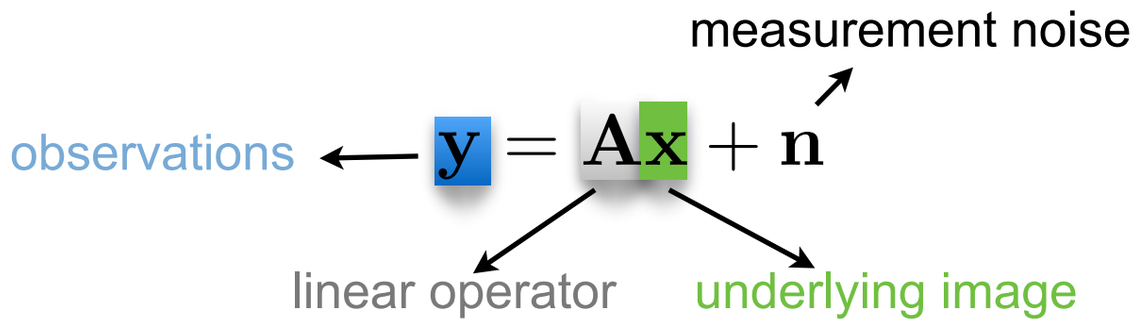

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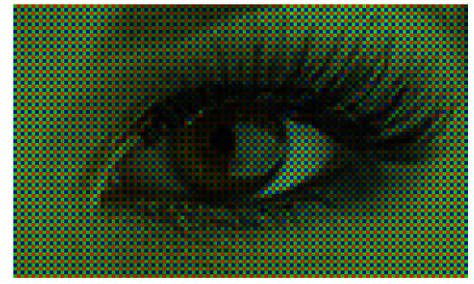
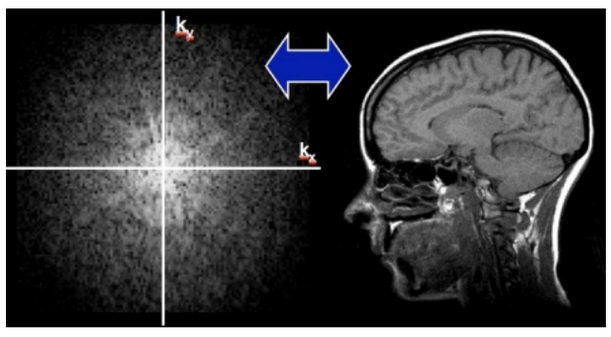
  - **A** : Fourier transform + binary mask $\Rightarrow$ MRI reconstruction



$\vdots$







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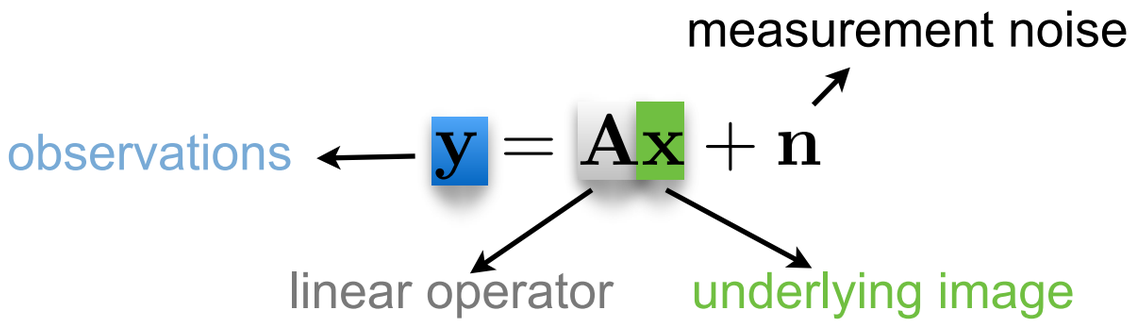

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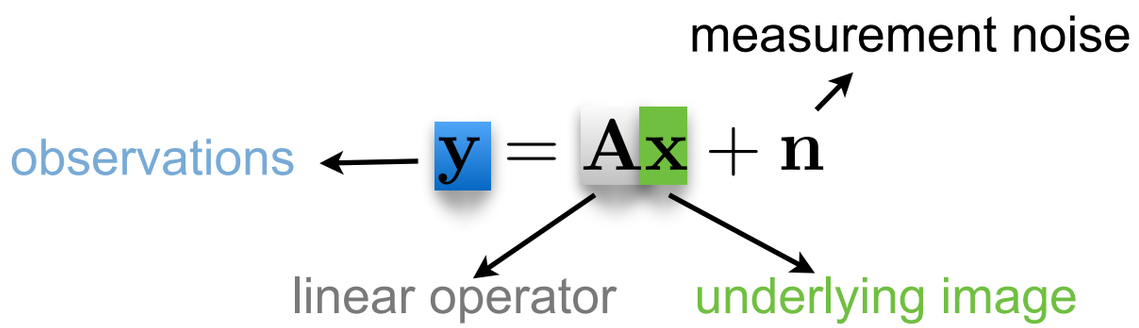

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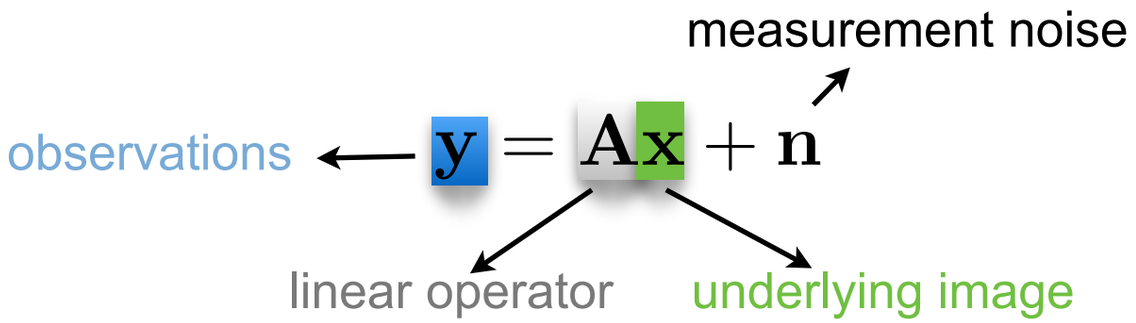

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- Variational framework - Energy Minimization

data fidelity

$$E(\mathbf{x}) = d(\mathbf{x}; \mathbf{A}, \mathbf{y}) + \lambda \cdot r(\mathbf{x})$$ regularizer

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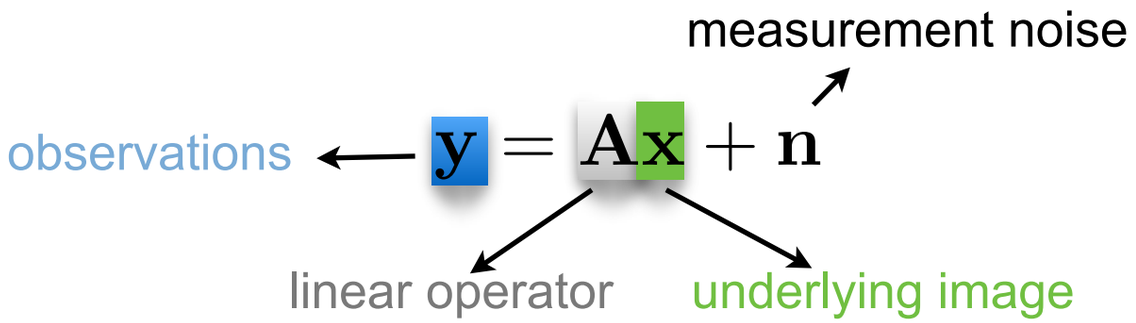

- Image reconstruction
  - Ill-posed problem
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- Variational framework - Energy Minimization

$$E(\mathbf{x}) = \underbrace{d(\mathbf{x}; \mathbf{A}, \mathbf{y})}_{\text{data fidelity}} + \lambda \cdot \underbrace{r(\mathbf{x})}_{\text{regularizer}}$$

- Direct relation with Bayesian methods :
  - data fidelity $\Rightarrow$ observation log-likelihood
  - regularizer $\Rightarrow$ prior distribution
  - minimizer $\Rightarrow$ MAP estimate

Image Regularization

- Regularizer : key component of the reconstruction
  - Constrains the set of plausible solutions
  - Prior knowledge of desirable image properties

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- Generic form of regularization functionals :

$$r(\mathbf{x}) = \sum_{k=1}^{K} \phi(\mathbf{L}_k \mathbf{x})$$

- $\mathbf{L} : \mathbb{R}^N \mapsto \mathbb{R}^{K \times D}$**:** regularization operator (gradient, Hessian, wavelets, …)
- ϕ : potential function - ℓ_2 squared norm (Tikhonov-Miller regularization) , ℓ_1 norm, log function, ℓ_0 pseudo-norm, etc …

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- Quadratic regularizers
  - mathematical simplicity
  - computational tractability
- Non-quadratic regularizers
  - less sensitive to outliers
  - improved reconstruction results

Total Variation

- One of the most popular regularizers for imaging applications [ROF `92]
  - Provides a global measure of the image intensity variations

Continuous-domain definition

$$\mathrm{TV}(u) = \int_{\Omega} \|\nabla u(\boldsymbol{r})\|_2 \, d\boldsymbol{r}$$

Discrete-domain definition

$$\mathrm{TV}(\mathbf{u}) = \sum_{n=1}^{N} \|(\nabla \mathbf{u})_n\|_2$$

- Numerous applications in image processing and computer vision
  - ✔ Invariance properties w.r.t transformations of the coordinate system
  - ✔ L_1 type of behavior that does not over-penalize large intensity variations
  - ✔ Reconstructed images with sharp edges
  - ✔ Convex functional - Efficient minimization techniques
  - ✗ Promotes piecewise constant reconstructions
  - ✗ Presence of staircase artifacts in smooth regions

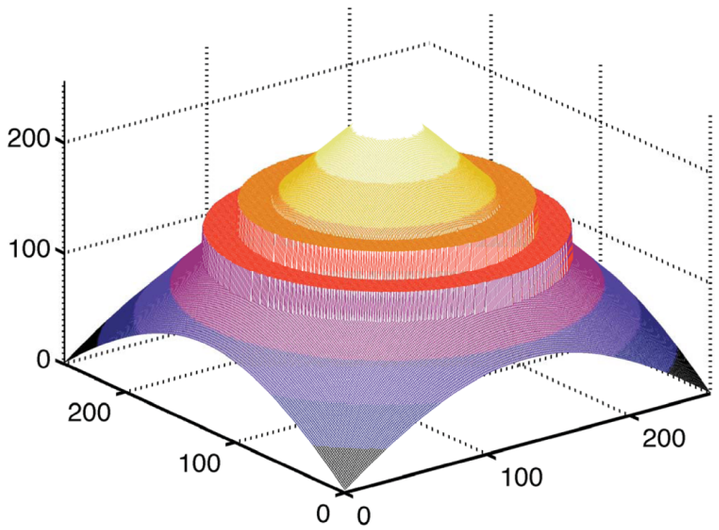
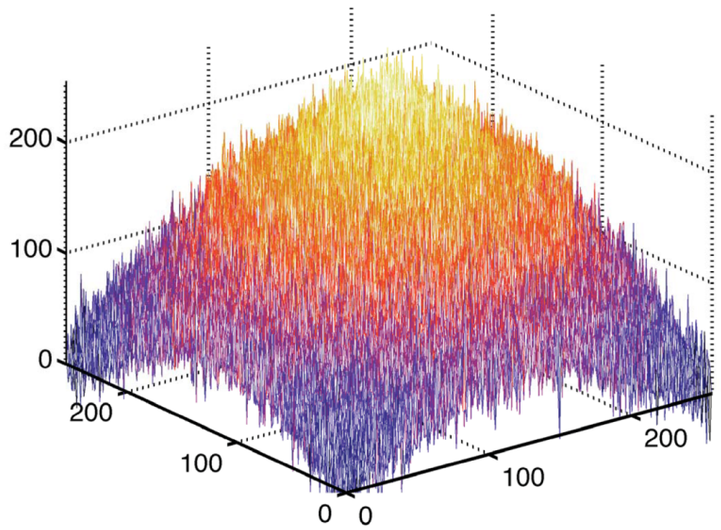
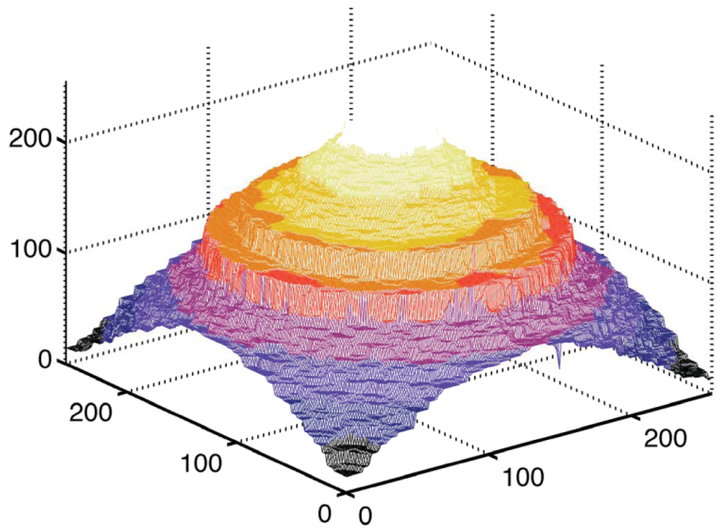


Image intensity profile



Noisy intensity profile



TV reconstruction

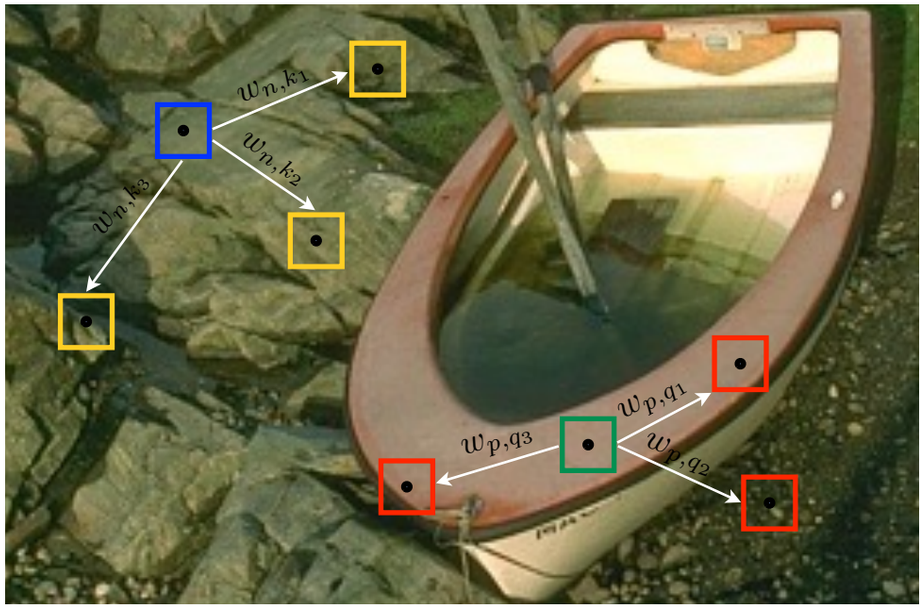
From Local to Non-Local Regularization

- TV-like regularizers only consider local features of the data
- Is there a way to capture more complex image structures?
  - Non-local Means : adaptive filter that considers non-local interactions [Buades et al. `05 & `10]

$$\mathrm{NL}(u)(\boldsymbol{x}) = \frac{1}{Z(\boldsymbol{x})} \int_\Omega e^{-\frac{d_a(u(\boldsymbol{x}),u(\boldsymbol{y}))}{\beta^2}} u(\boldsymbol{y})\, d\boldsymbol{y}$$

patch-distance

$$d_a(u(\boldsymbol{x}), u(\boldsymbol{y})) = \int_\Omega G_a(t) \left|u(\boldsymbol{x}+\boldsymbol{t}) - u(\boldsymbol{y}+\boldsymbol{t})\right|^2 d\boldsymbol{t}$$



- Non-local TV : Variational extension of NL-means [Gilboa & Osher `08, Elmoataz et al. `08]
  - Key component : Non-local gradient on graphs

$$\nabla_w u(\boldsymbol{x}) = (u(\boldsymbol{y}) - u(\boldsymbol{x}))\sqrt{w(\boldsymbol{x},\boldsymbol{y})}, \quad \forall \boldsymbol{y} \in \Omega$$

non-local weight

- NL-TV : similar definition with classical TV
  - Important difference : image is defined on a graph

$$\mathrm{NLTV}(u) = \int_\Omega \|\nabla_w u(\boldsymbol{x})\|_2 \,\mathrm{d}\boldsymbol{x}$$

Machine Learning for Inverse Problems

- Two major challenges under the variational / minimization framework
  - Specify the exact form of the regularization operator and potential function
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- Main challenge for ML
  - How to decide a proper architecture for the graph of the network?
- Principled approach for defining the network graph
  - Unroll an iterative optimization method
  - Use a limited number of updates to construct the network
  - Parametrize the regularization operator and the potential function

Optimization Strategy

- Recover the underlying image as the minimizer of the objective function

$$\arg\min_{\mathbf{x}} d\left(\mathbf{x}; \mathbf{A}, \mathbf{y}\right) + \lambda \cdot r\left(\mathbf{x}\right)$$

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  - FISTA (Majorization-Minimization) [Beck & Teboulle `09]
  - Split-Bregman / ADMM [Osher et al. `05, Goldstein & Osher `09, Esser `09, Afonso et al. `10 & `11, etc]
  - Primal-dual algorithms [Chambolle & Pock `10 & `11]
  - Proximal methods [Parikh & Boyd `13]

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  - The proximal operator is a generalization of the orthogonal projection
  - Can be interpreted as Gaussian denoising of $\mathbf{z}$ using $g\left(\mathbf{x}\right)$ as the regularizer

Image Denoising - Constrained Optimization

- Proximal map - image denoising solution (P1)

$$\mathbf{x}^{\star} = \arg\min_{\mathbf{x}} \frac{1}{2} \|\mathbf{y} - \mathbf{x}\|_2^2 + \lambda \sum_{k=1}^{K} \phi\left(\mathbf{L}_k \mathbf{x}\right)$$

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- Regularization parameter λ : “free” parameter
  - Different values lead to different restoration results of varying image quality
  - **No direct way** to *a priori* relate the value of λ with the restoration quality
  - Networks derived based on this formulation are noise-level specific [Schmidt & Roth `14, Chen et al. `16, Lefkimmiatis `17]
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- Alternative way : Constrained problem formulation (P2)

$$\mathbf{x}^{\star} = \arg\min_{\|\mathbf{y}-\mathbf{x}\|_2 \leq \varepsilon} \left(\sum_{k=1}^{K} \phi\left(\mathbf{L}_k \mathbf{x}\right) \equiv r\left(\mathbf{x}\right) \right)$$

  - $\|\mathbf{y} - \mathbf{x}\|_2 = \|\mathbf{n}\|_2 \propto \sigma$
  - Free parameter ε is easier to tune - directly related to the noise std
  - Available methods for estimating the noise level from observed data

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  - $\|\mathbf{y} - \mathbf{x}\|_2 = \|\mathbf{n}\|_2 \propto \sigma$
  - Free parameter ε is easier to tune - directly related to the noise std
  - Available methods for estimating the noise level from observed data
- The two problems are equivalent in the following sense :
  - For any $\varepsilon > 0$ such that P2 is feasible, the solution of P2 is either the null vector or else it is a solution of P1 for some $\lambda > 0$

Proximal Gradient Method

- Unconstrained form of P2

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} \sum_{k=1}^{K} \phi\left(\mathbf{L}_k \mathbf{x}\right) + \iota_{\mathcal{C}(\mathbf{y},\varepsilon)}\left(\mathbf{x}\right)$$

indicator function

$$\iota_{\mathcal{C}(\mathbf{y},\varepsilon)}\left(\mathbf{x}\right) = \begin{cases} 0, & \text{if } \|\mathbf{y}-\mathbf{x}\|_2 \le \varepsilon \\ \infty, & \text{otherwise} \end{cases}$$

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$$\text{indicator function} \quad \iota_{\mathcal{C}(\mathbf{y},\varepsilon)}\left(\mathbf{x}\right) = \begin{cases} 0, & \text{if } \|\mathbf{y}-\mathbf{x}\|_2 \leq \varepsilon \\ \infty, & \text{otherwise} \end{cases}$$

- PGM : gradient-descent method that can handle non-smooth functionals

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- Updates of the solution using PGM (assuming a smooth regularizer)

adaptive step size

$$\mathbf{x}^t = \text{prox}_{\gamma^t \iota_C}\left(\mathbf{x}^{t-1} - \gamma^t \nabla_{\mathbf{x}} r\left(\mathbf{x}^{t-1}\right)\right)$$

Proximal Gradient Method Cont'd

- Gradient of the smooth part

$$\nabla_{\mathbf{x}} r(\mathbf{x}) = \sum_{k=1}^{K} \mathbf{L}_k^{\mathsf{T}} \psi(\mathbf{L}_k \mathbf{x}) \equiv h(\mathbf{x})$$

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Normalized residual iterations

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normalized noise realization estimate

Universal Network Architecture

- Cascade of composite layers (“stages”)
- Each layer implements a single PGM update

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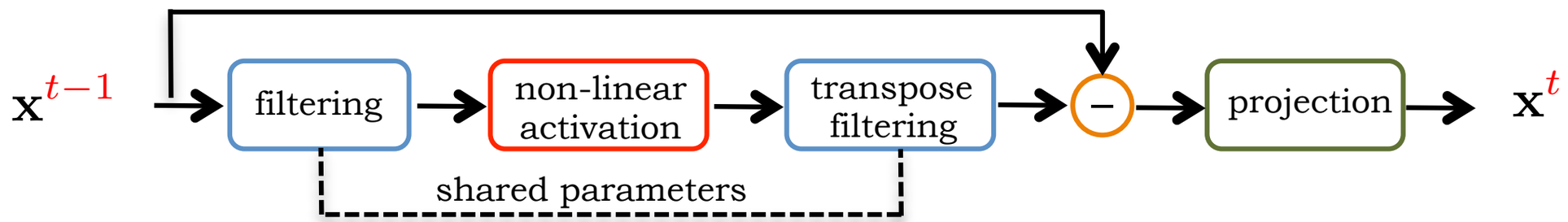

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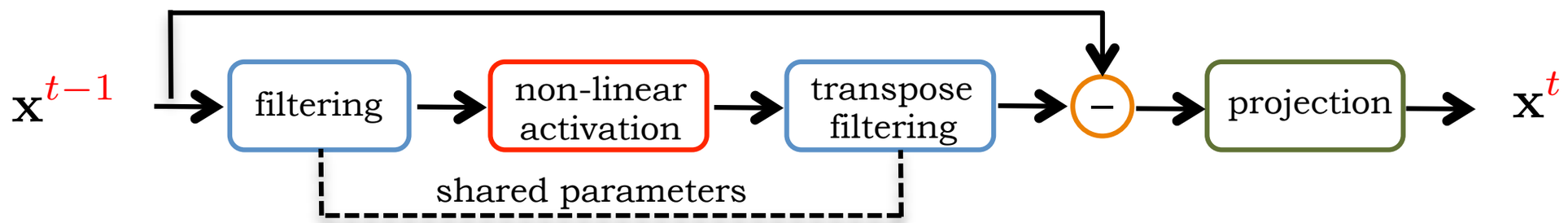
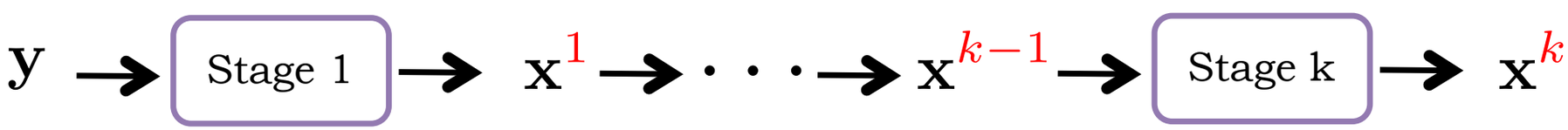


- Feedforward network



Local Deep Network

- Regularization operator $\mathbf{L} : \mathbb{R}^N \mapsto \mathbb{R}^{K \times D}$
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- Similar parametrization has been used in other deep networks for classification tasks to replace batch-normalization [Salimans & Kingma `16]

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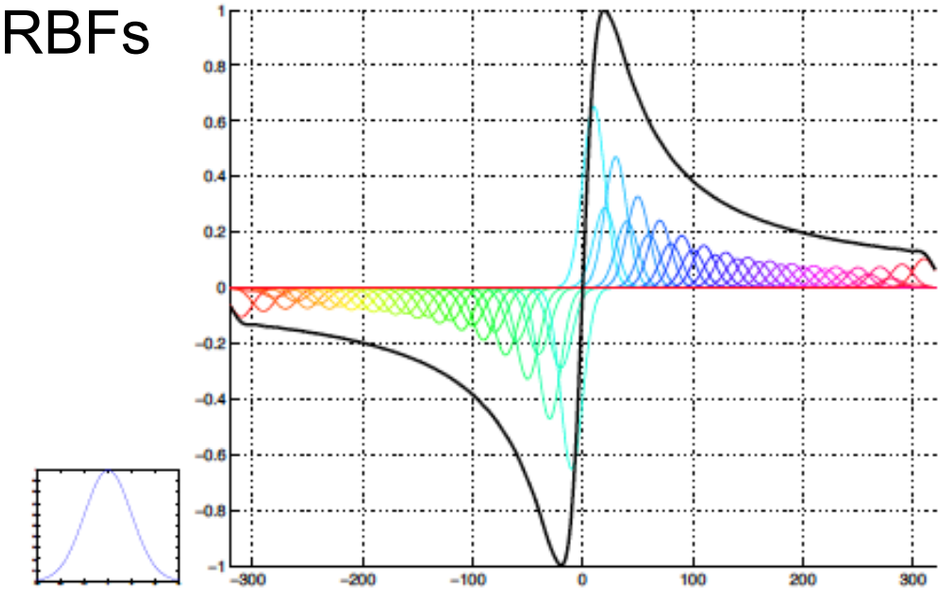
  - Indirect modeling using linear combination of RBFs

$$\psi_i(x) = \sum_{j=1}^{M} \pi_{ij} \rho_j(|x - \mu_j|)$$

π_{ij} : expansion coefficients

μ_j : centers of basis functions

$\rho_j = \exp\left(-\varepsilon_j r^2\right)$: Gaussian radial functions



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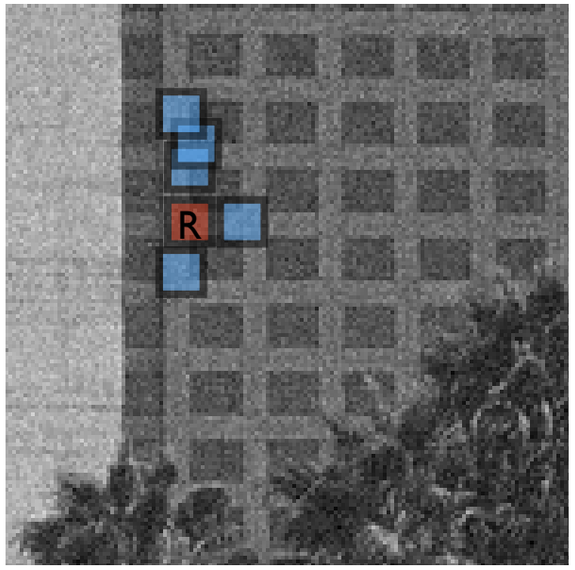

Block-Matching

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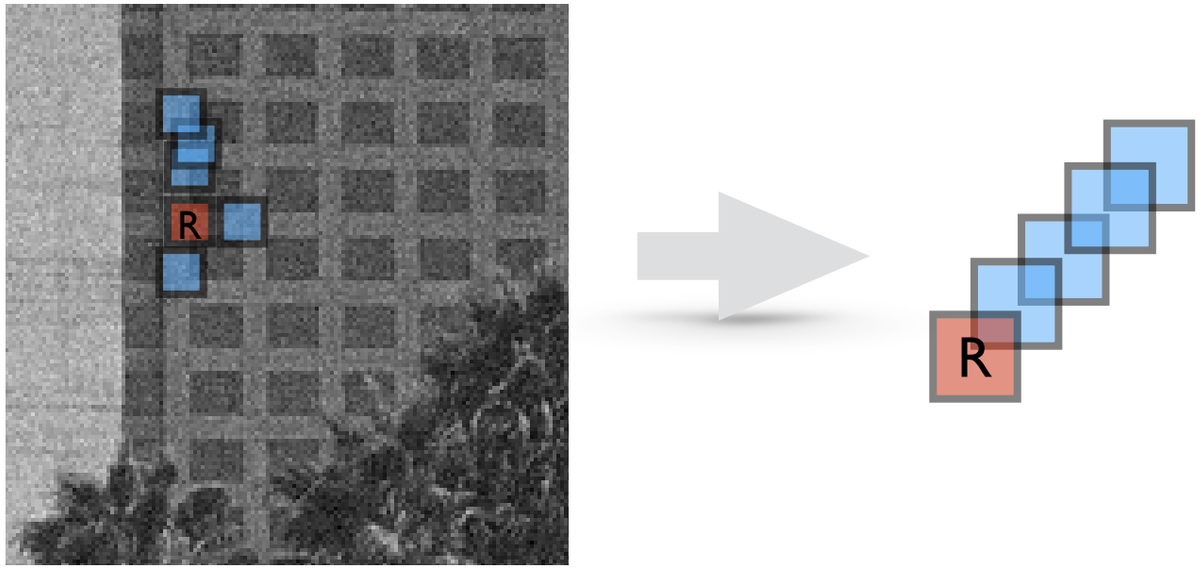

Block-Matching

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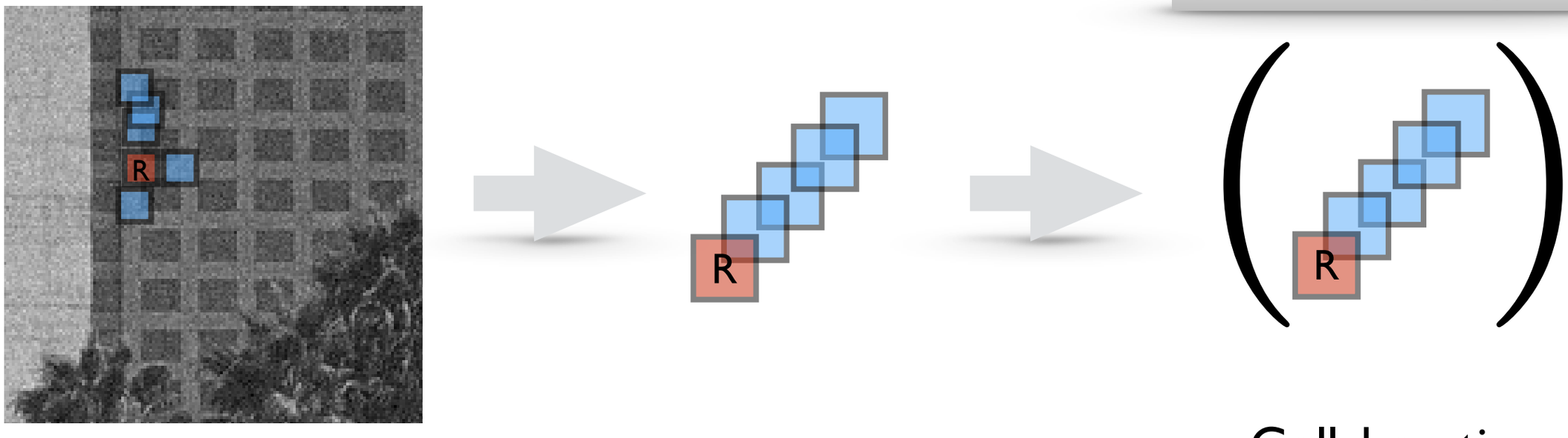

Block-Matching

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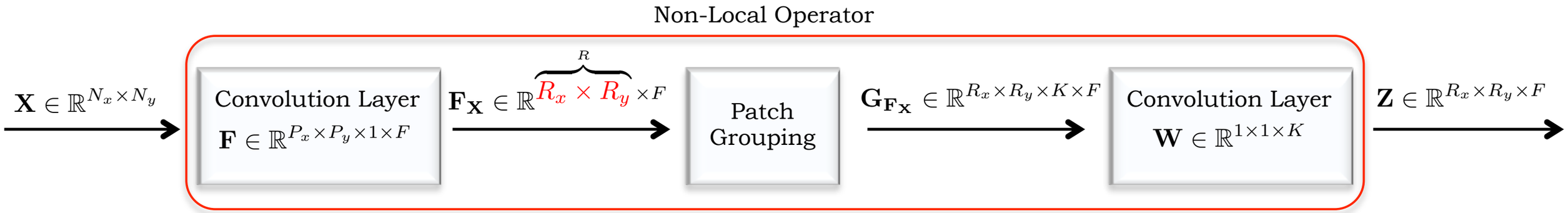
Collaborative Filtering

Non-local Regularization Operator

- Construction of the Non-Local operator
  - Patch extraction & patch-transform can be combined into a single step
    - Pass the input image through a filterbank
    - Filterbank setup : F filters of support $P = P_x \times P_y$ (patch spatial dimensions)
  - Collaborative filtering : Single convolution in the patch dimension after grouping
  - Adjoint NL operator : straightforward implementation as well



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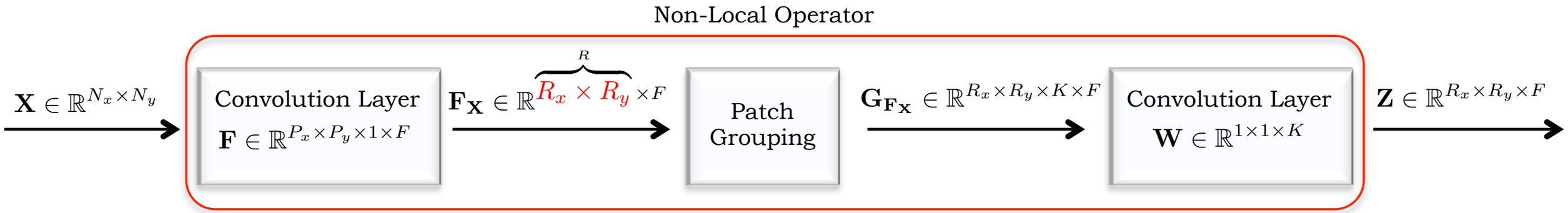
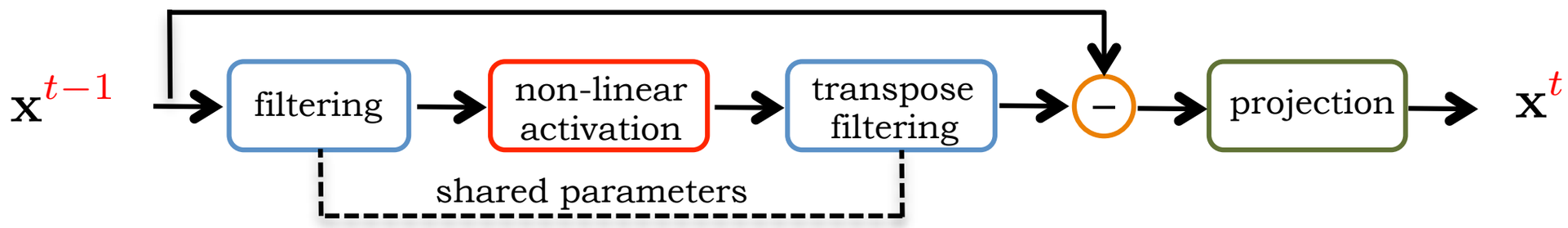


- Efficient NL implementation
  - Highly efficient specialized CPU & GPU libraries for convolutions
  - CUDNN (GPU), OMP (CPU multi-threading)

Non-local Network Architecture

- Cascade of composite layers (“stages”)



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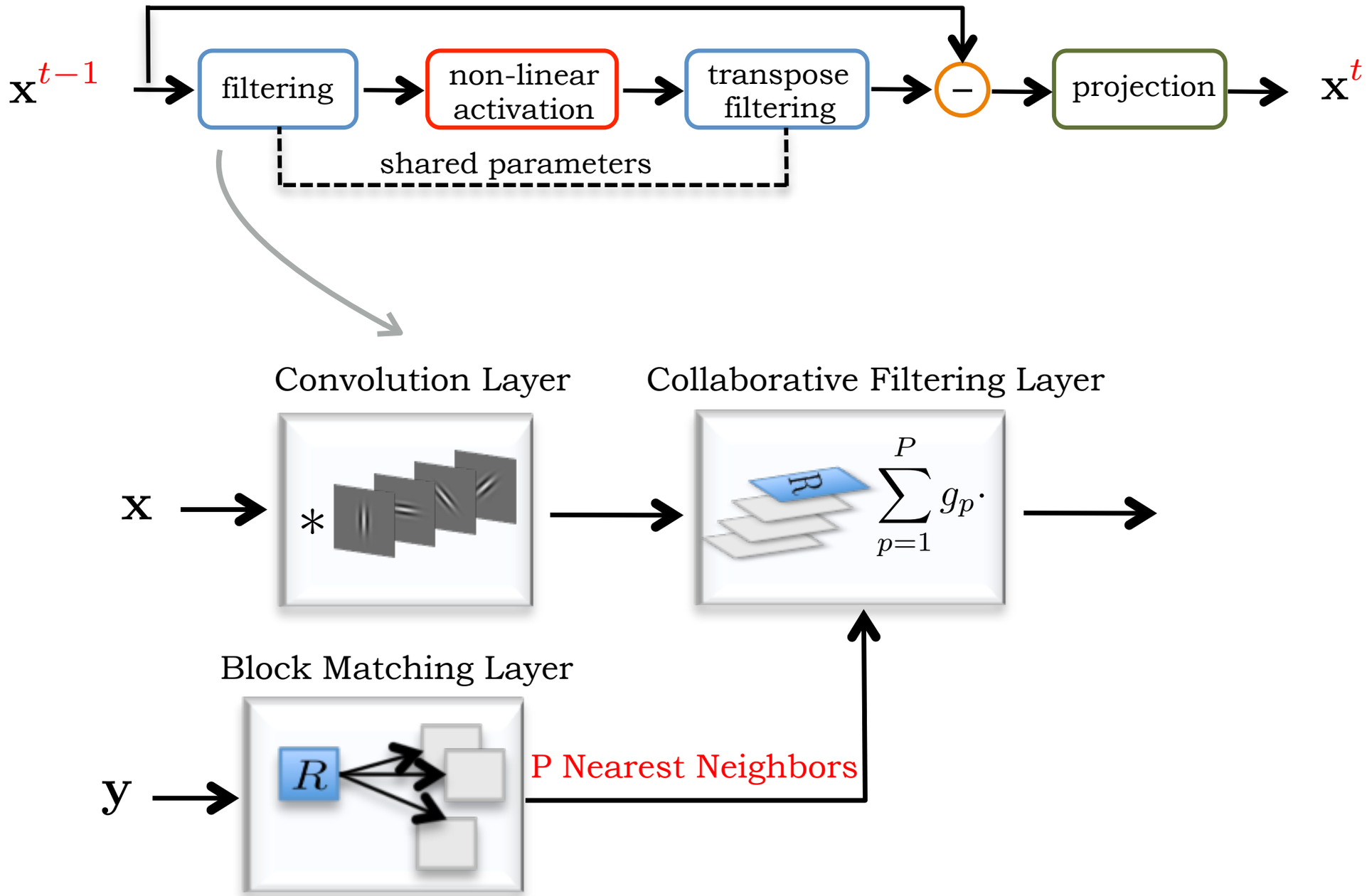


Discriminative Network Training

- Training of a S-stage network
  - Network parameters : $\boldsymbol{\Theta} = \left[\boldsymbol{\Theta}^1, \ldots, \boldsymbol{\Theta}^S\right], \boldsymbol{\Theta}^t = \left\{s^t, \mathbf{v}^t, \mathbf{g}^t, \boldsymbol{\pi}^t, \alpha^t\right\}$
- Learning using a loss-minimization strategy
  - Q pairs of training data $\left\{\mathbf{y}_{(q)}, \mathbf{x}_{(q)}\right\}_{q=1}^{Q}$
  - Different learned parameters for each stage to increase the capacity of the network
  - Loss function : negative PSNR
- Loss-minimization Strategies
  - Greedy approach - stage independent learning : $\mathcal{L}\left(\boldsymbol{\Theta}^t\right) = \sum_{q=1}^{Q} \ell\left(\hat{\mathbf{x}}_{(q)}^t, \mathbf{x}_{(q)}\right)$
  - Joint learning of all stages : $\mathcal{L}\left(\boldsymbol{\Theta}\right) = \sum_{q=1}^{Q} \ell\left(\hat{\mathbf{x}}_{(q)}^S, \mathbf{x}_{(q)}\right)$
- Minimization algorithm — ADAM
  - Variant of SGD that involves adaptive normalization of the learning rate
  - Derivative computation with back-propagation

Trained Models for Image Denoising

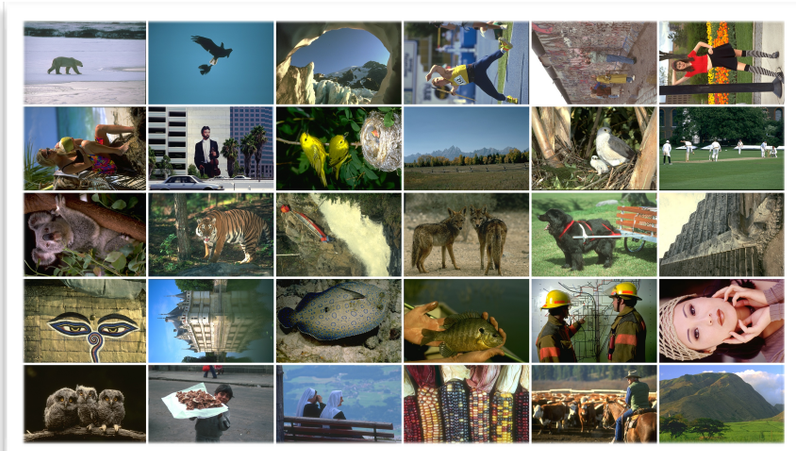
- Deep residual denoising networks for color & grayscale images
  - Grayscale and color local / non-local nets (5-stage feedforward networks)
  - A single network can handle a wide range of noise levels (σ = 5 … 55)
  - In the non-local deep nets similar patches are searched in a window 31 x 31
  - Group of non-local operator with K=8 closest neighbors

- Training data
  - Berkeley database (500 color / grayscale images)
  - For training : 400 images of size 180 x 180
  - For validation : 100 images of size 180 x 180

- Network training
  - Network initialization : greedy training with ADAM (100 epochs per stage)
  - Final network parameters : joint training with ADAM (100 epochs)
  - Training on NVIDIA GeForce 1080 Ti GPU

Comparisons

- Restoration performance evaluation on 68 BSDS images of size 321 x 481
- Comparisons against **state-of-the-art** methods
  - Three patch-based methods : BM3D [Dabov et al. `07], EPLL [Zoran & Weiss `11], WNNM [Gu et al. `14]
  - ML method : DCNN [Zhang et al. `17] : Deep network with approximately **14x** (**7x** for color images) more parameters and **2x** more deep
- Grayscale / color results



Methods	Noise level - σ (std.) 5	10	15	20	25	30	35	40	45	50	55	avg.
BM3D [8]	37.57	33.30	31.06	29.60	28.55	27.74	27.07	26.45	25.99	25.60	25.26	28.93
EPLL [45]	37.55	33.36	31.18	29.73	28.67	27.84	27.16	26.58	26.09	25.71	25.34	29.02
WNMM [15]	37.76	33.55	31.31	29.83	28.73	27.94	27.28	26.72	26.26	25.85	25.49	29.16
DCNN [42]	37.68	33.72	31.60	30.19	29.15	28.33	27.66	27.10	26.62	26.21	25.80	29.46
UNet_5	37.59	33.54	31.38	29.93	28.84	28.01	27.38	26.85	26.38	25.95	25.53	29.22
UNLNet_5	37.62	33.62	31.47	30.04	28.96	28.13	27.50	26.96	26.48	26.04	25.64	29.32
$\text{UNLNet}_5^{\text{orc}}$	37.79	33.97	31.95	30.59	29.51	28.54	27.97	27.47	26.97	26.41	25.80	29.72

Noise σ (std.)	Methods CBM3D [8]	CDCNN [42]	CUNet_5	CUNLNet_5	$\text{CUNLNet}_5^{\text{orc}}$
5	40.24	40.11	40.31	40.39	40.54
10	35.88	36.11	36.08	36.20	36.70
15	33.49	33.88	33.78	33.90	34.58
20	31.88	32.36	32.21	32.34	33.11
25	30.68	31.22	31.03	31.17	31.95
30	29.71	30.31	30.06	30.24	31.06
35	28.86	29.57	29.37	29.53	30.37
40	28.06	28.94	28.77	28.91	29.75
45	27.82	28.39	28.23	28.37	29.19
50	27.36	27.91	27.74	27.89	28.65
55	26.95	27.45	27.27	27.44	28.10
avg.	30.99	31.48	31.35	31.49	32.18

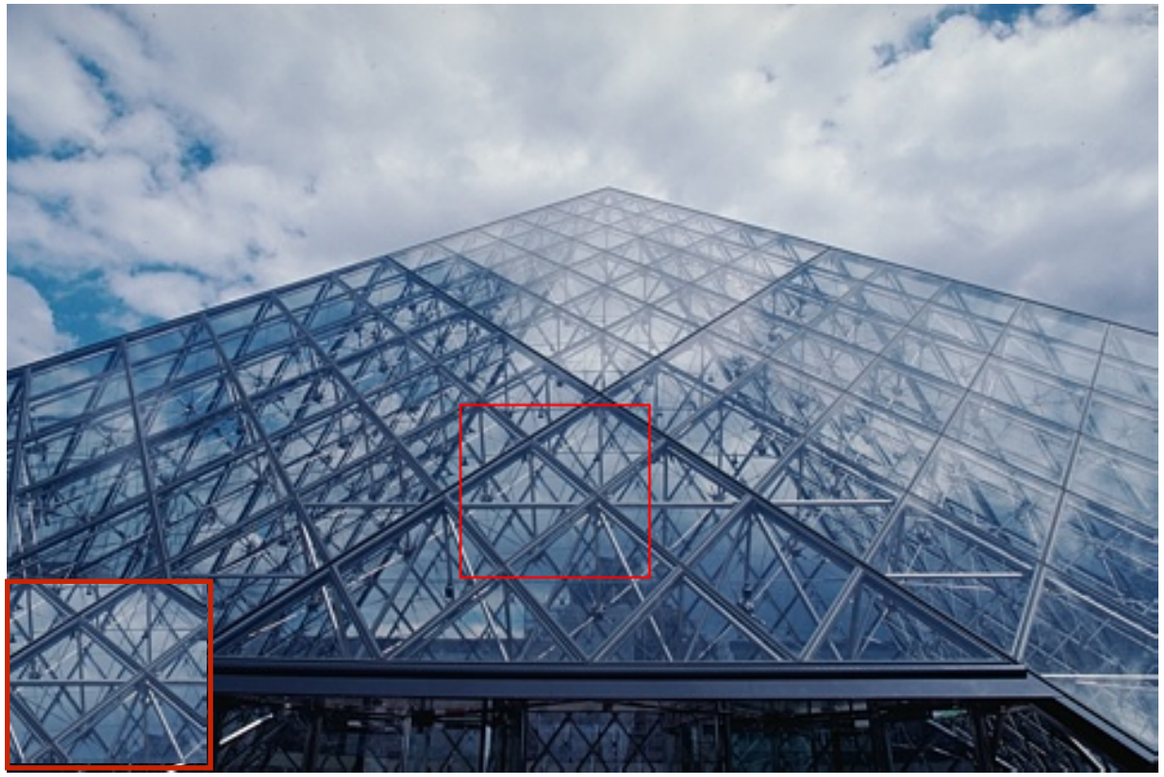
Grayscale Image Denoising

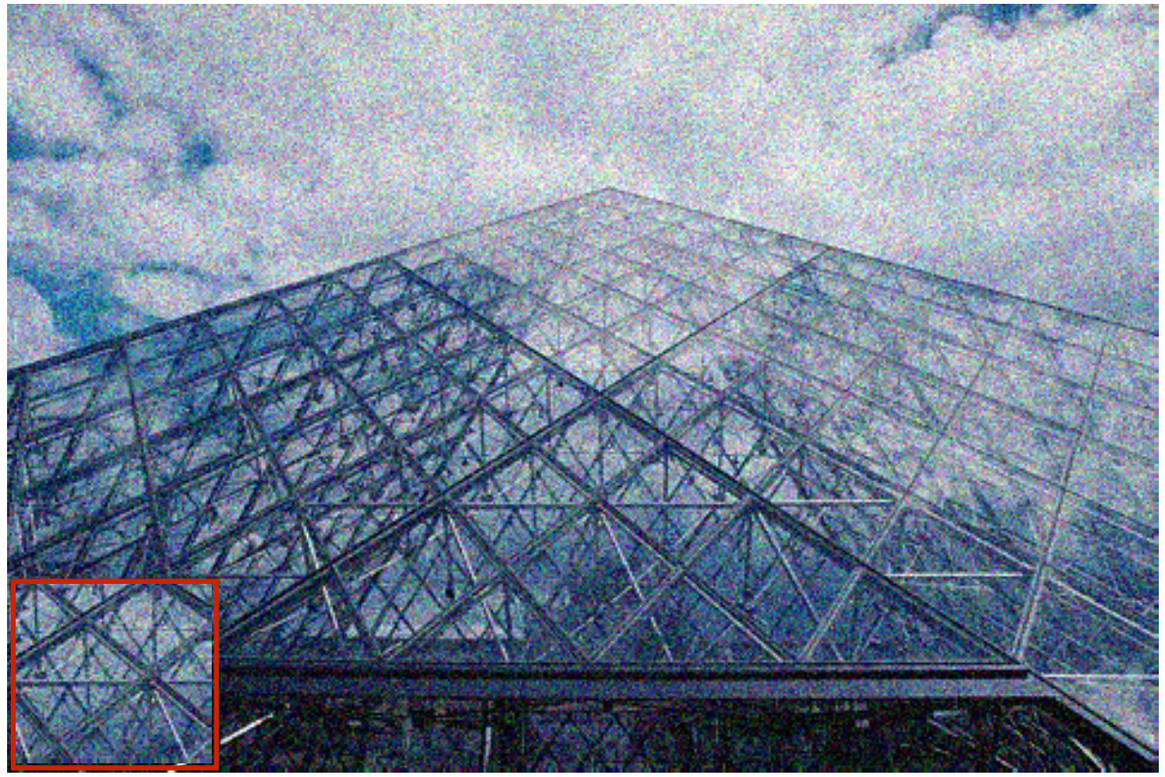
Noisy (std =20 ; PSNR = 22.10 dB)

EPLL (PSNR = 31.54 dB)

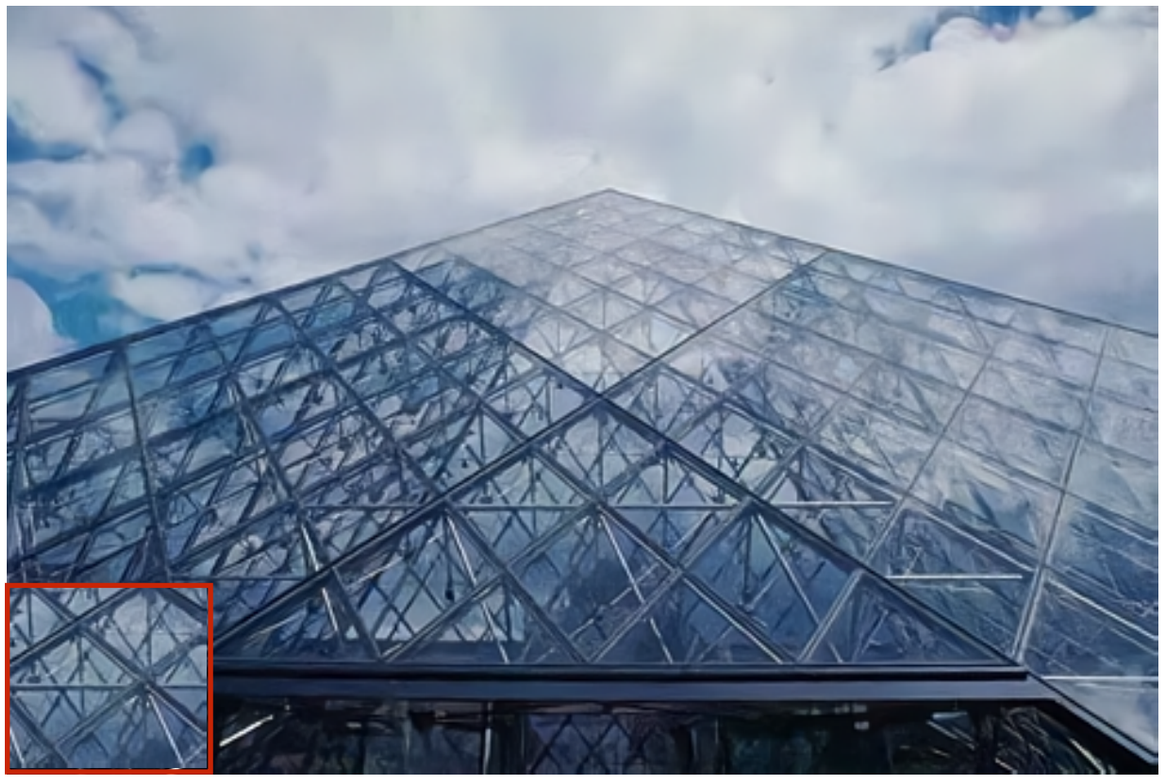
UNet_5 (PSNR = 31.71 dB)

Color Image Denoising

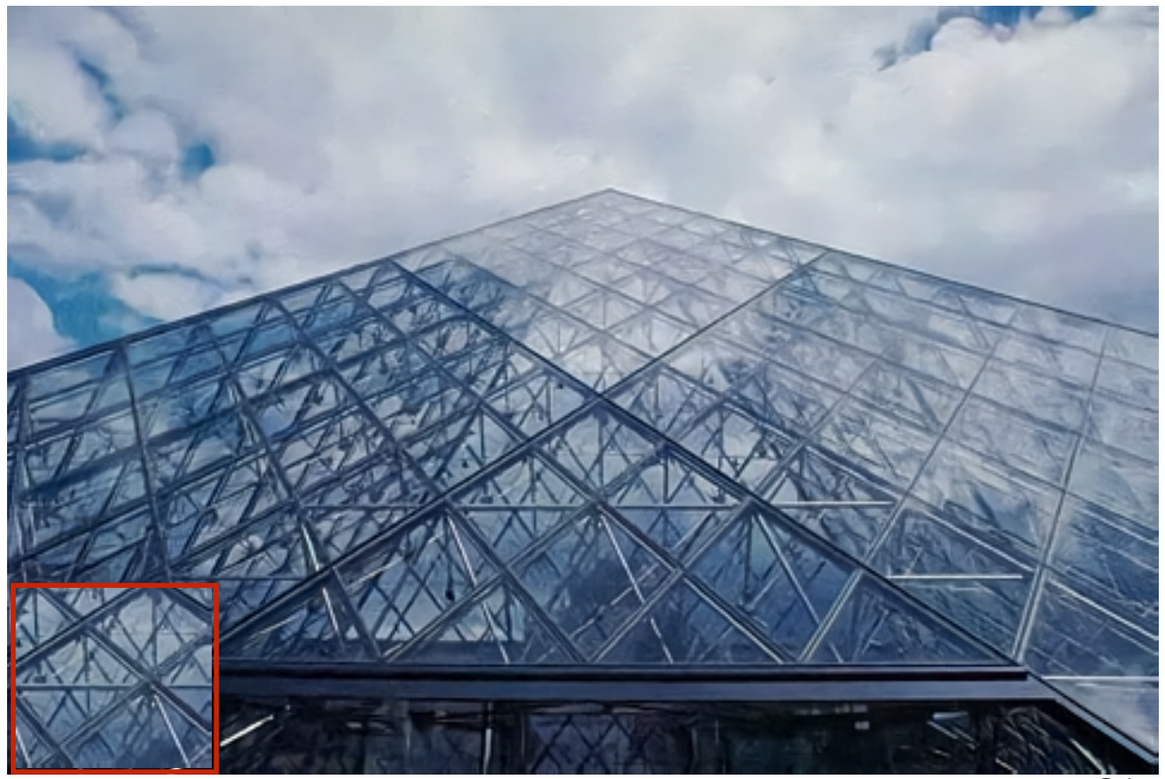




Noisy (std = 30 ; PSNR = 18.57 dB)



DCNN (PSNR = 29.08 dB)



CUNLNet_5 (PSNR = 29.13 dB)

Grayscale Image Denoising (real noise)

Noisy

BM3D

DCNN

UNet_5

Color Image Denoising (real noise)

Noisy

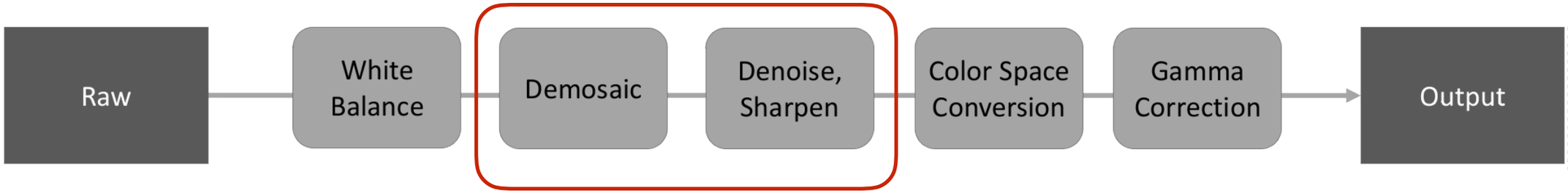
CBM3D

Noise Clinic

CUNLNet_5

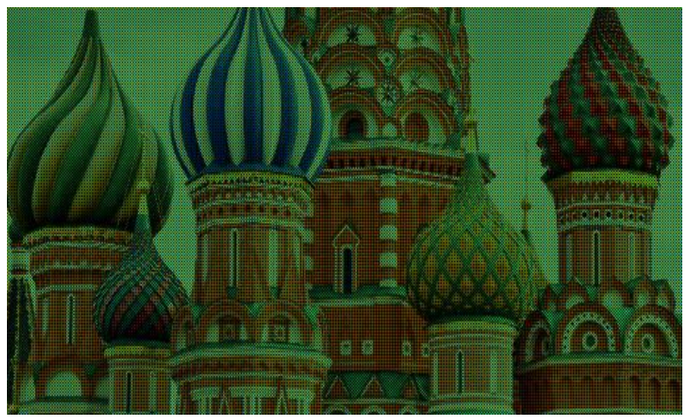
Image Demosaicking - Denoising

- First steps in the camera ISP :
  - Conversion of intensity readings to color images



- **Challenging** problem
  - Only one third of the intensities are known
  - Measurements are further distorted by noise

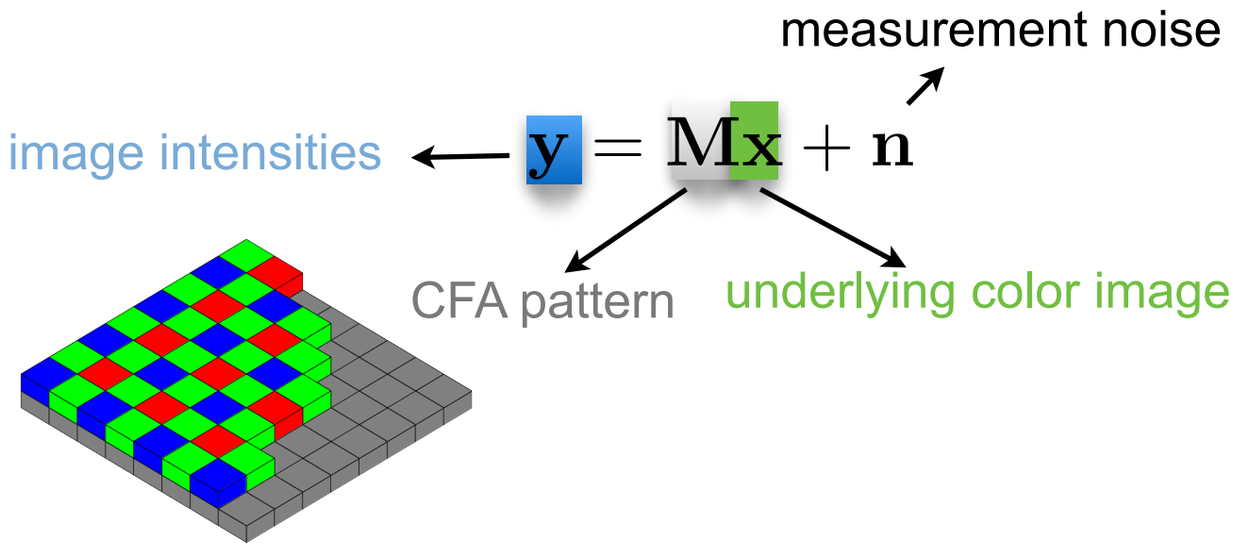
Noisy Light Intensity Measurements



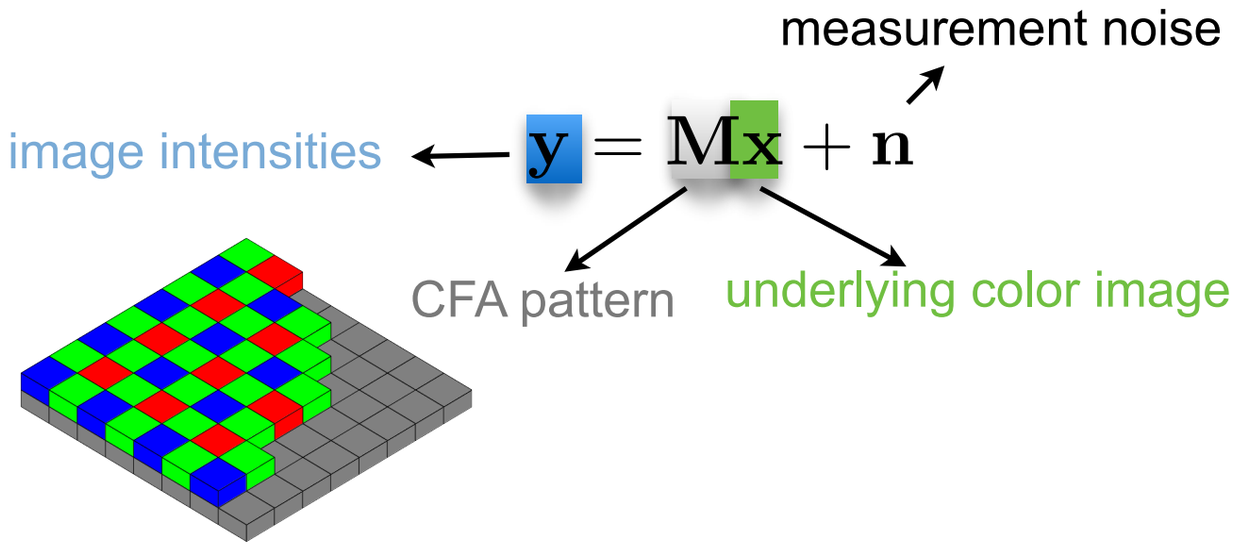
Demosaick & Denoise

End Result

Problem Formulation



Problem Formulation



- Solution of a minimization problem

$$\mathbf{x}^{\star} = \arg\min_{\mathbf{x}} \underbrace{\frac{1}{2\sigma^2} \|\mathbf{y} - \mathbf{M}\mathbf{x}\|_2^2 + r(\mathbf{x})}_{Q(\mathbf{x})}$$

- Challenges to deal with :
  - coupling of the CFA pattern with the solution
  - non-trivial way to minimize the objective function
  - choice of an appropriate regularizer

Majorization-Minimization framework

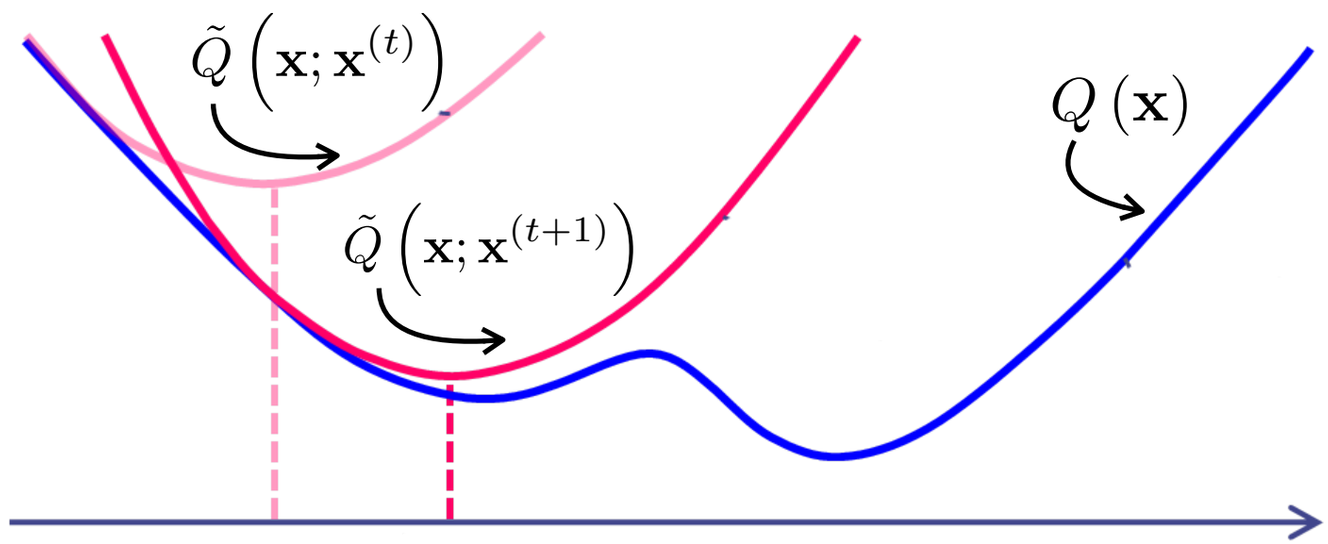
- Iterative minimization approach of the form :

$$\mathbf{x}^{(t+1)} = \arg\min_{\mathbf{x}} \tilde{Q}\left(\mathbf{x}; \mathbf{x}^{(t)}\right)$$

- $\tilde{Q}\left(\mathbf{x}; \mathbf{x}^{(t)}\right)$ is a majorizer of the objective function that satisfies the conditions:

$$(\alpha) \quad \tilde{Q}\left(\mathbf{x}; \mathbf{x}^{(t)}\right) > Q\left(\mathbf{x}\right), \forall \mathbf{x} \neq \mathbf{x}^{(t)}$$

$$(\beta) \quad \tilde{Q}\left(\mathbf{x}^{(t)}; \mathbf{x}^{(t)}\right) = Q\left(\mathbf{x}^{(t)}\right)$$



MM for Demosaicking-Denoising

- Derive a majorizer by majoring the data fidelity term

$$\tilde{d}\left(\mathbf{x};\mathbf{x}^{(t)}\right) = \underbrace{\frac{1}{2\sigma^2}\left\|\mathbf{y}-\mathbf{M}\mathbf{x}\right\|_2^2}_{d(\mathbf{x})} + g\left(\mathbf{x},\mathbf{x}^{(t)}\right)$$

$$g\left(\mathbf{x},\mathbf{x}^{(t)}\right) = \frac{1}{2\sigma^2}\left(\mathbf{x}-\mathbf{x}^{(t)}\right)^T\left[\alpha\mathbf{I}-\mathbf{M}^T\mathbf{M}\right]\left(\mathbf{x}-\mathbf{x}^{(t)}\right)$$

- $d\left(\cdot,\cdot\right)$ is a valid majorizer iff $g\left(\mathbf{x},\mathbf{y}\right) > 0\,, \forall \mathbf{x} \neq \mathbf{y}$ and $g\left(\mathbf{x},\mathbf{x}\right) = 0$
  - $\alpha\mathbf{I}-\mathbf{M}^T\mathbf{M} = \alpha\mathbf{I}-\mathbf{M}$ is positive definite $\Rightarrow \;\; \alpha > \left\|\mathbf{M}\right\| = 1$

MM for Demosaicking-Denoising

- Derive a majorizer by majoring the data fidelity term

$$\tilde{d}\left(\mathbf{x};\mathbf{x}^{(t)}\right)=\underbrace{\frac{1}{2\sigma^2}\left\|\mathbf{y}-\mathbf{M}\mathbf{x}\right\|_2^2}_{d(\mathbf{x})}+g\left(\mathbf{x},\mathbf{x}^{(t)}\right)$$

$$g\left(\mathbf{x},\mathbf{x}^{(t)}\right)=\frac{1}{2\sigma^2}\left(\mathbf{x}-\mathbf{x}^{(t)}\right)^T\left[\alpha\mathbf{I}-\mathbf{M}^T\mathbf{M}\right]\left(\mathbf{x}-\mathbf{x}^{(t)}\right)$$

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  - $\alpha\mathbf{I}-\mathbf{M}^T\mathbf{M}=\alpha\mathbf{I}-\mathbf{M}$ is positive definite $\Rightarrow\ \alpha>\left\|\mathbf{M}\right\|=1$
- Overall majorizer is of the form :

$$\begin{aligned}\tilde{Q}\left(\mathbf{x};\mathbf{x}^{(t)}\right)&=\tilde{d}\left(\mathbf{x};\mathbf{x}^{(t)}\right)+r\left(\mathbf{x}\right)\\&=\frac{1}{2\left(\sigma\sqrt{\alpha}\right)^2}\left\|\mathbf{x}-\mathbf{z}\right\|_2^2+r\left(\mathbf{x}\right)+\text{const.}\end{aligned}$$

$$\mathbf{z}=\mathbf{y}+(\mathbf{I}-\mathbf{M})\mathbf{x}^{(t)}$$

MM for Demosaicking-Denoising

- Derive a majorizer by majoring the data fidelity term

$$\tilde{d}\left(\mathbf{x};\mathbf{x}^{(t)}\right)=\underbrace{\frac{1}{2\sigma^2}\left\|\mathbf{y}-\mathbf{M}\mathbf{x}\right\|_2^2}_{d(\mathbf{x})}+g\left(\mathbf{x},\mathbf{x}^{(t)}\right)$$

$$g\left(\mathbf{x},\mathbf{x}^{(t)}\right)=\frac{1}{2\sigma^2}\left(\mathbf{x}-\mathbf{x}^{(t)}\right)^T\left[\alpha\mathbf{I}-\mathbf{M}^T\mathbf{M}\right]\left(\mathbf{x}-\mathbf{x}^{(t)}\right)$$

- $d\left(\cdot,\cdot\right)$ is a valid majorizer iff $g\left(\mathbf{x},\mathbf{y}\right)>0\,,\forall\mathbf{x}\neq\mathbf{y}$ and $g\left(\mathbf{x},\mathbf{x}\right)=0$
  - $\alpha\mathbf{I}-\mathbf{M}^T\mathbf{M}=\alpha\mathbf{I}-\mathbf{M}$ is positive definite $\Rightarrow\ \alpha>\left\|\mathbf{M}\right\|=1$
- Overall majorizer is of the form :

$$\tilde{Q}\left(\mathbf{x};\mathbf{x}^{(t)}\right)=\tilde{d}\left(\mathbf{x};\mathbf{x}^{(t)}\right)+r\left(\mathbf{x}\right)$$

denoising objective function

$$=\frac{1}{2\left(\sigma\sqrt{\alpha}\right)^2}\left\|\mathbf{x}-\mathbf{z}\right\|_2^2+r\left(\mathbf{x}\right)+\text{const.}$$

$$\mathbf{z}=\mathbf{y}+(\mathbf{I}-\mathbf{M})\mathbf{x}^{(t)}$$

Residual Denoising Network

- Recast the problem of image demosaicking to a series of **image denoising problems**

$$\tilde{Q}\left(\mathbf{x};\mathbf{x}^{(t)}\right)=\frac{1}{2\left(\sigma\sqrt{\alpha}\right)^{2}}\left\|\mathbf{x}-\mathbf{z}\right\|_{2}^{2}+r\left(\mathbf{x}\right)+\text{const.}$$

- In each iteration the estimate of the demosaicked image is refined

Residual Denoising Network

- Recast the problem of image demosaicking to a series of **image denoising problems**

$$\tilde{Q}\left(\mathbf{x};\mathbf{x}^{(t)}\right)=\frac{1}{2\left(\sigma\sqrt{\alpha}\right)^{2}}\left\|\mathbf{x}-\mathbf{z}\right\|_{2}^{2}+r\left(\mathbf{x}\right)+\text{const.}$$

- In each iteration the estimate of the demosaicked image is refined
- Instead of solving an optimization problem we employ a denoising network

Residual Denoising Network

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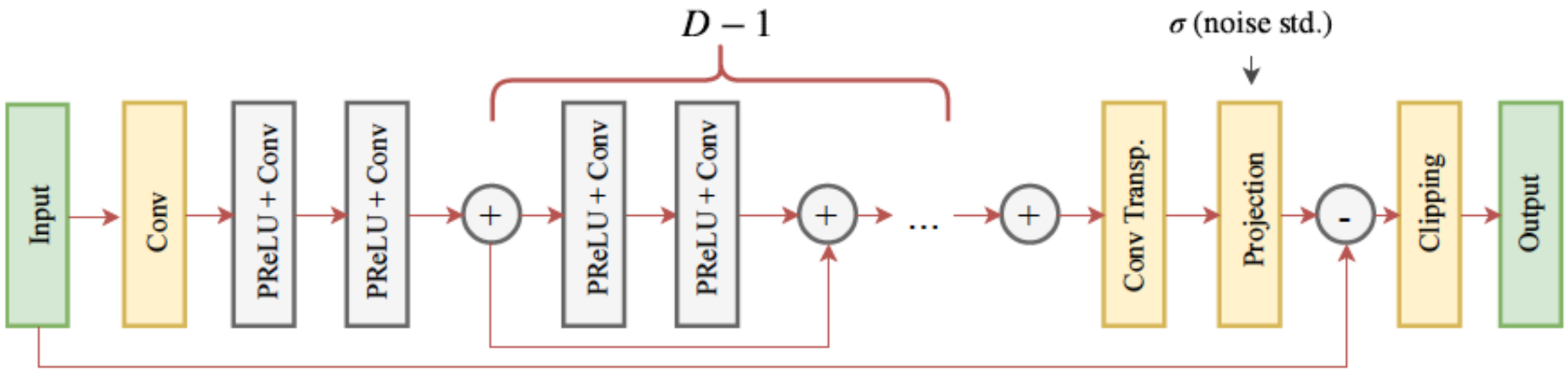
- In each iteration the estimate of the demosaicked image is refined
- Instead of solving an optimization problem we employ a denoising network



Iterative Joint Demosaicking-Denoising

Algorithm 1: The proposed demosaicking network described as an iterative process. The ResDNet parameters are shared across all iterations.

Input: M: CFA, y: input, K: iterations, $w \in \mathbb{R}^K$: extrapolation weights, σ: estimated noise, $\gamma \in \mathbb{R}^K$: projection parameters

$\mathbf{x}^{(0)} = \mathbf{0}$;

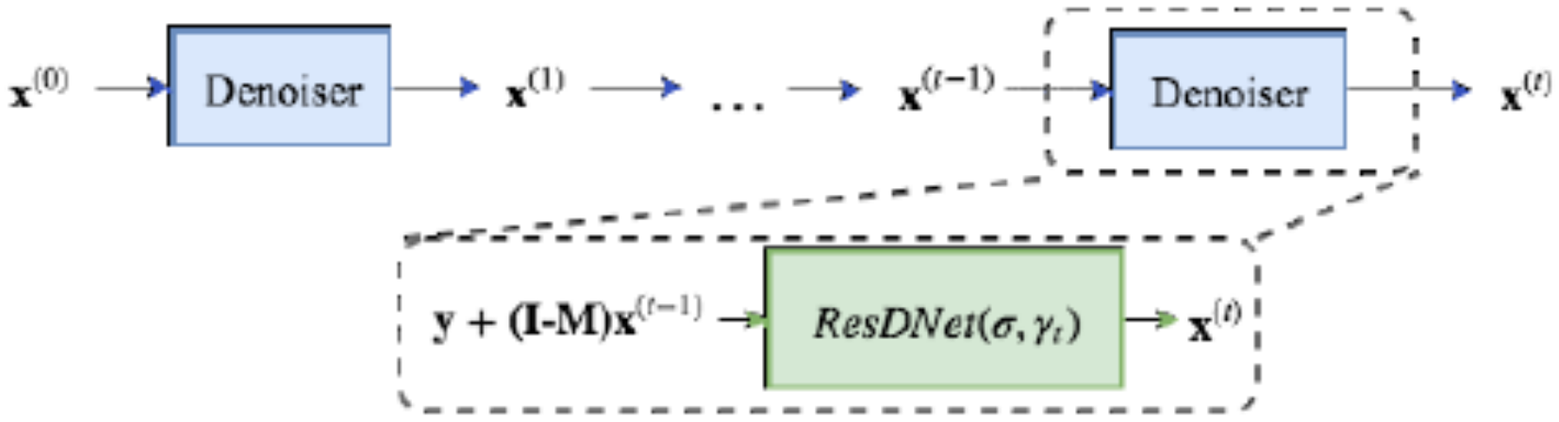
Initialize $\mathbf{x}^{(1)}$ using y;

for $i \leftarrow 1$ **to** K **do**

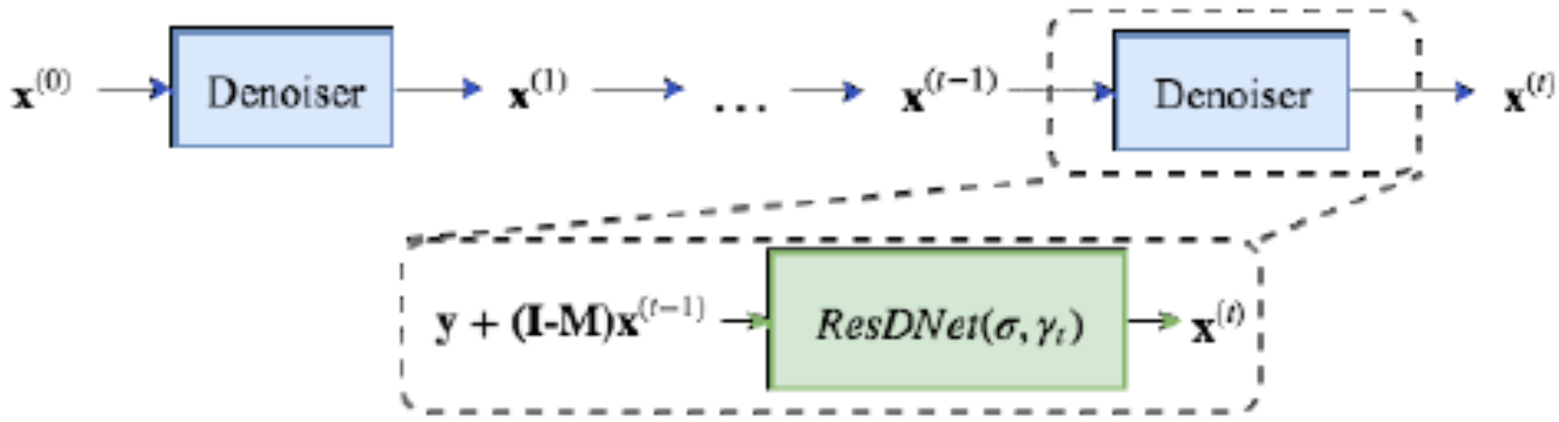
  $\mathbf{u} = \mathbf{x}^{(i)} + w_i(\mathbf{x}^{(i)} - \mathbf{x}^{(i-1)})$;

  $\mathbf{x}^{(i+1)} = \text{ResDNet}((\mathbf{I} - \mathbf{M})\mathbf{u} + \mathbf{y}, \sigma, \gamma_i)$;

end

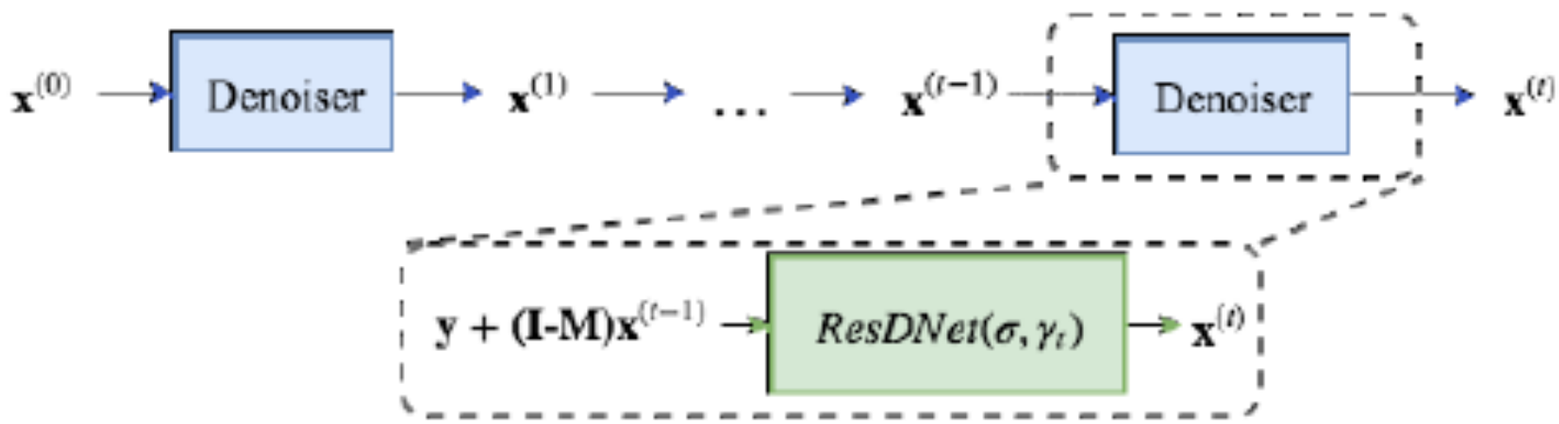


Iterative Joint Demosaicking-Denoising



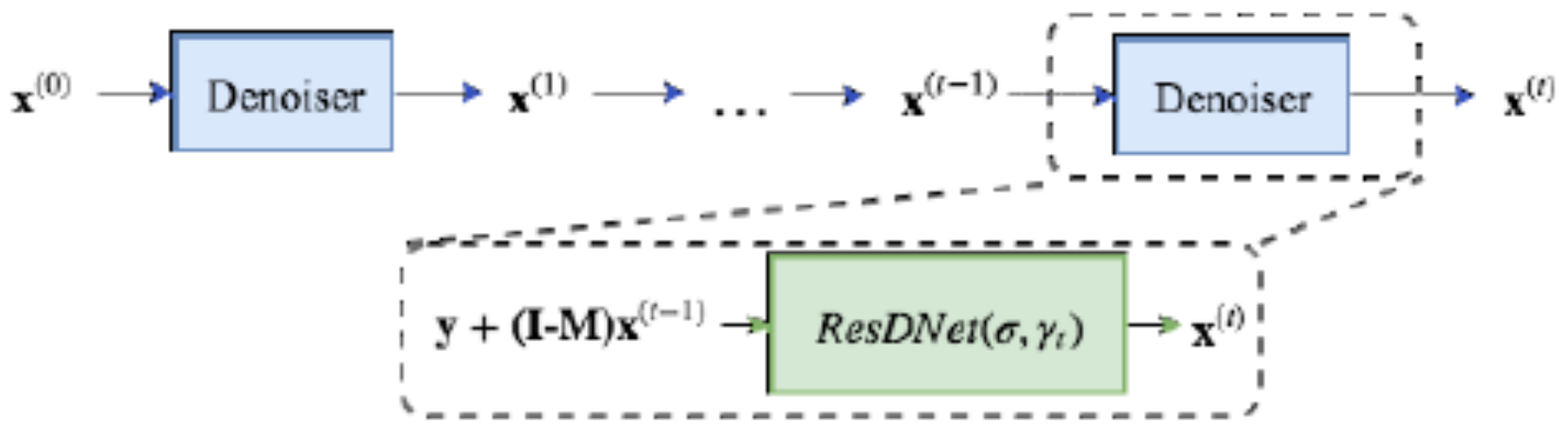
- The network parameters are **shared** across iterations
  - ✓ small memory footprint of the network

Iterative Joint Demosaicking-Denoising

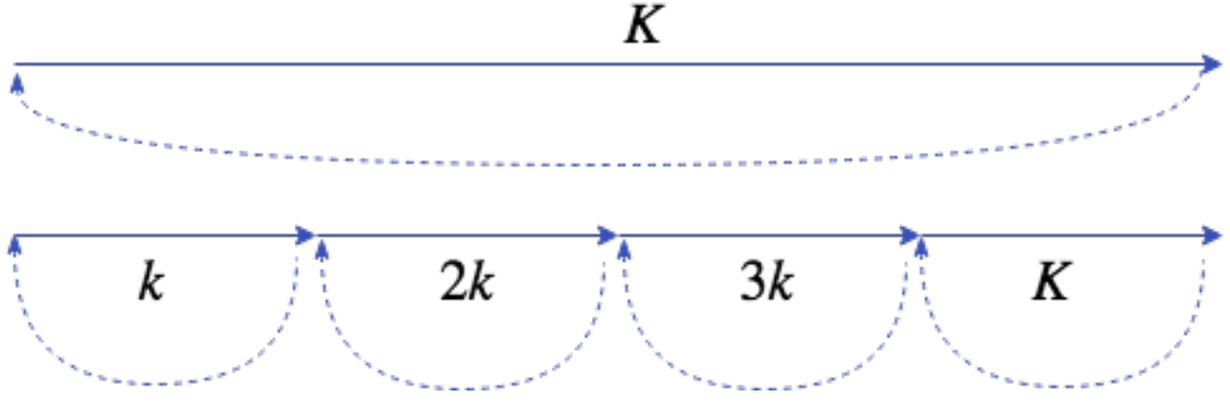


- The network parameters are **shared** across iterations
  - ✓ small memory footprint of the network
- Challenge: How to efficiently train the network by back-propagation?
  - Back propagation through time (BPTT) is **extremely memory demanding**

Iterative Joint Demosaicking-Denoising



- The network parameters are **shared** across iterations
  - ✓ small memory footprint of the network
- Challenge: How to efficiently train the network by back-propagation?
  - Back propagation through time (BPTT) is **extremely memory demanding**
- Use instead Truncated BPTT
  - adopted approach in natural language processing (NLP)
  - first to use in image restoration or computer vision problems
  - ✓ We can train arbitrarily deep demosaicking networks



Comparisons

Noise-free scenario on synthetic data

	Kodak	McM	VDP	Moire
Bilinear	32.9	32.5	25.2	27.6
Zhang (NLM) [2]	37.9	36.3	30.1	31.9
Hirakawa [49]	36.5	33.9	30.0	32.1
Getreuer [50]	38.1	36.1	30.8	32.5
Heide [5]	40.0	38.6	27.1	34.9
Klatzer [22]	35.3	30.8	28.0	30.3
Gharbi [15]	41.2	39.5	34.3	37.0
Kokkinos [23]	41.5	**39.7**	**34.5**	37.0
Tan [7]	**42.0**	39.0	-	-
Henz [16]	41.9	39.5	34.3	36.3
$MMNet_{10}$ (ours)	**42.0**	**39.7**	**34.5**	**37.1**

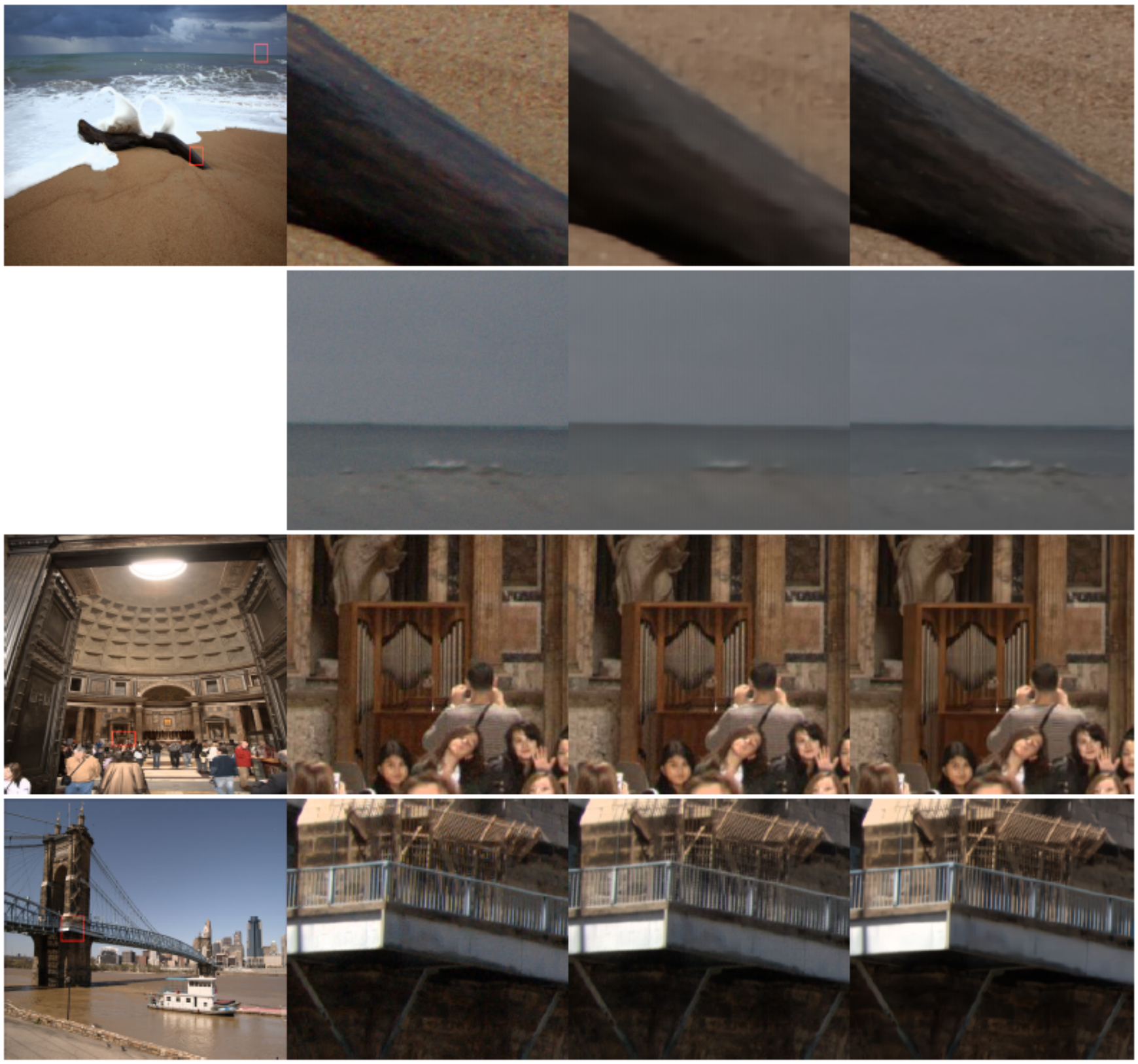
real-world data (MSR dataset) / Bayer CFA

	noisy		noise-free	
	linRGB	sRGB	linRGB	sRGB
Bilinear	-	-	30.9	24.9
Zhang(NLM) [2]	-	-	38.4	32.1
Hirakawa [50]	-	-	37.2	31.3
Heide [5]	-	-	40.0	33.8
Khasabi [11]	37.8	31.5	39.4	32.6
Klatzer [23]	38.8	32.6	40.9	34.6
Gharbi [15]	38.6	32.6	42.7	35.9
Kokkinos [24]	39.2	33.3	41.0	34.6
$MMNet_2$ (ours)	39.2	33.3	42.1	35.6
$MMNet_{20}$ (ours)	**40.1**	**34.2**	**42.8**	**36.4**

real-world data (MSR dataset) / Fuji XTrans CFA

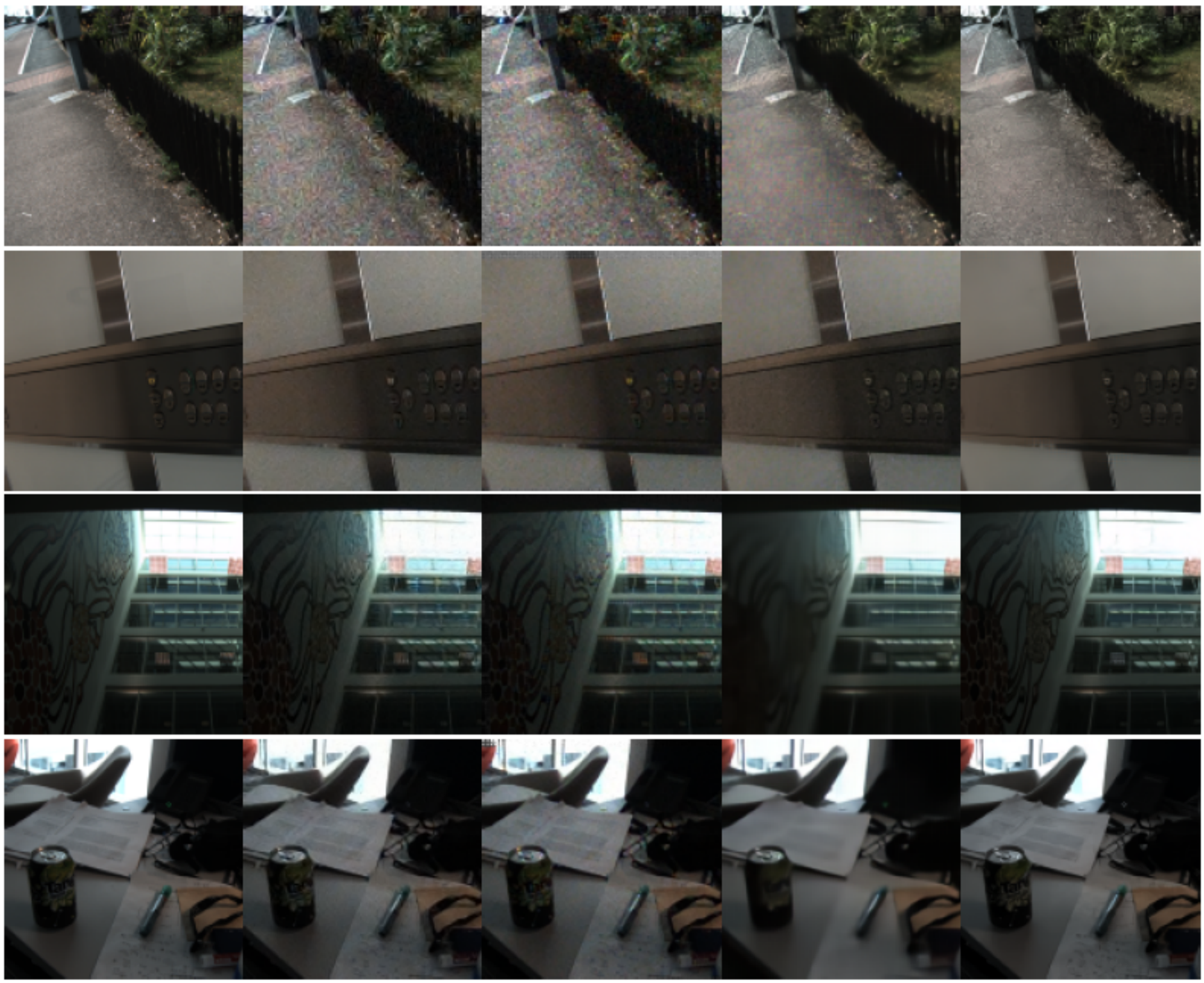
	noise-free	
	linear	sRGB
Trained on MSR Dataset:		
Khashabi [11]	36.9	30.6
Klatzer [23]	39.6	33.1
Gharbi [15]	39.7	33.2
Kokkinos [24]	39.9	33.7
$MMNet_8$ (ours)	40.2	34.0
$MMNet_{20}$ (ours)	**40.6**	**34.4**

Demosaicking of real raw data



Ours (full) | Hirakawa [50] (dcraw) | Gharbi et al. [15] | ours

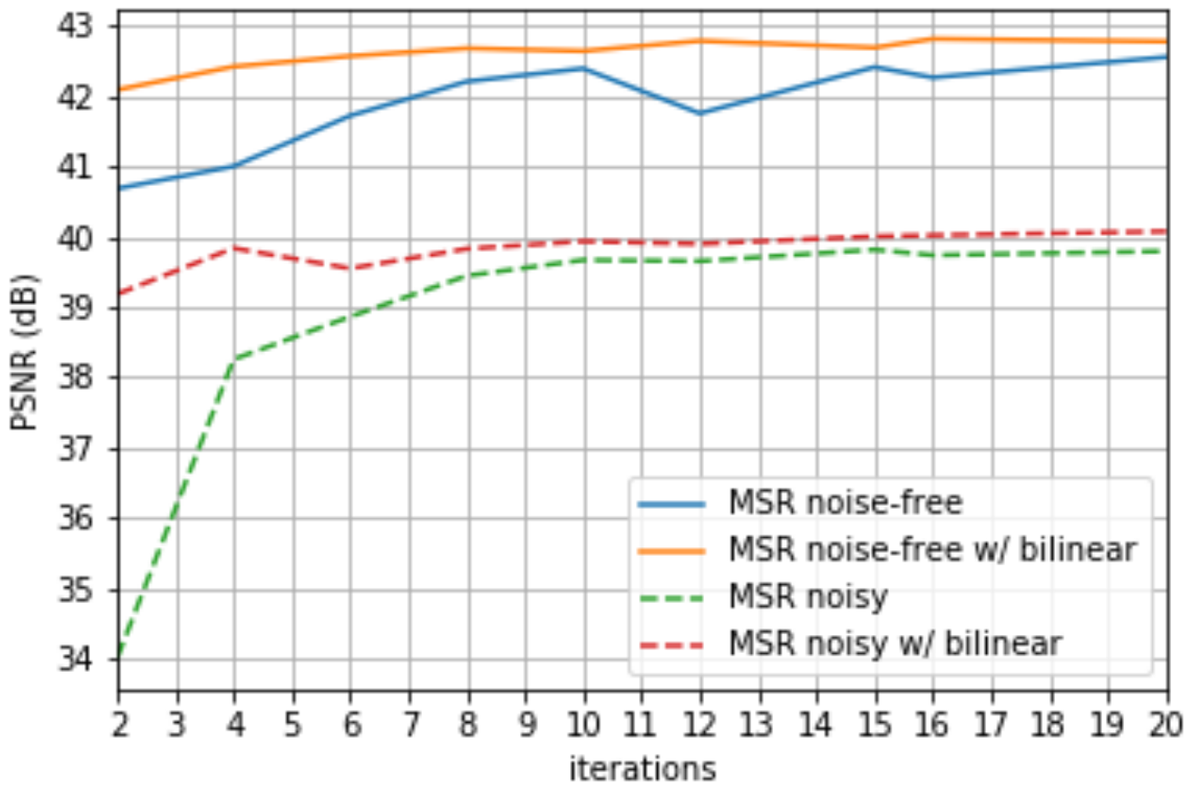
Demosaicking of MSR raw data

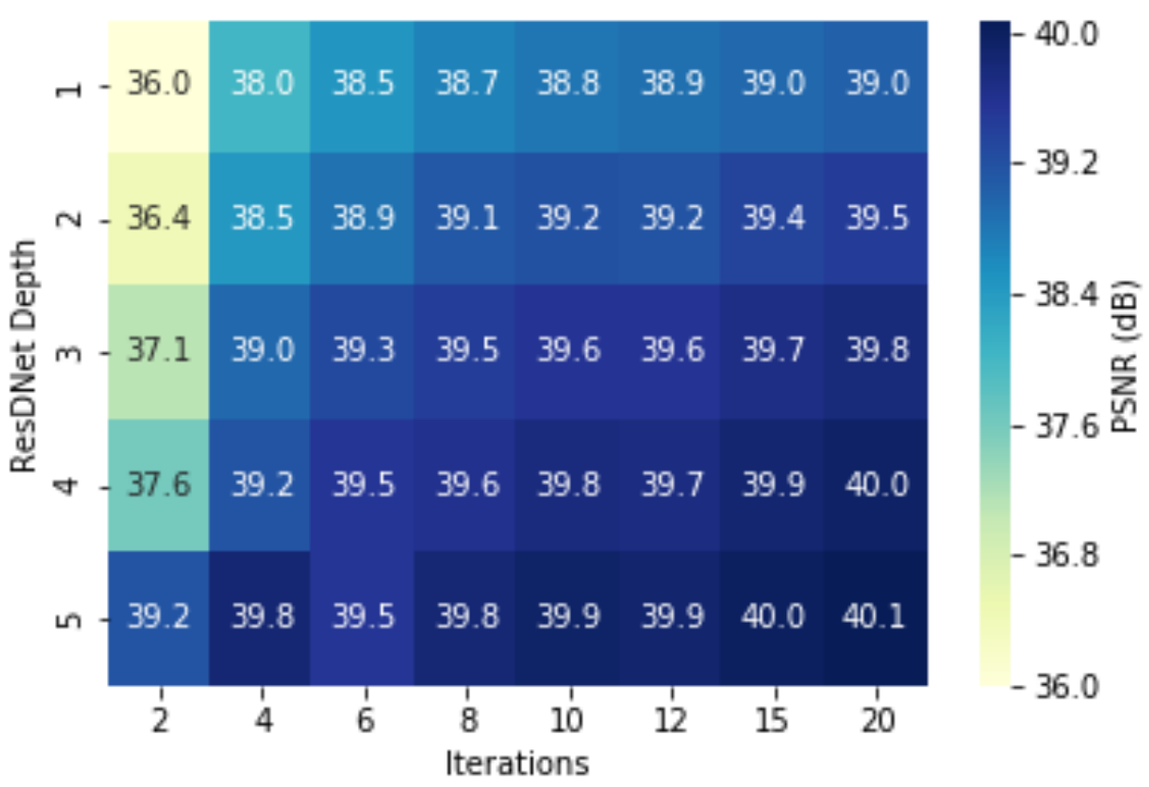


Reference | Hirakawa (dcraw)[50] | Zhang(NLM)[2] | Gharbi et al.[15] | ours

Ablation Study

- Our iterative approach provides a balance between the number of network parameters and its depth (iterations). Specifically we are able :
  - ✓ to retrieve the same performance when reducing the number of parameters by increasing the number of iterations
  - ✓ to be robust to the initialization by using a deep network architecture





Summary and Future Research Directions

- **Deep Learning for inverse imaging problems**
  - Inspiration from variational methods to design a deep network
  - Data-driven approach for learning the potential function and regularization operator
  - Non-local deep models : exploit the self-similarity property of natural images
  - A single set of learned parameters that handles a wide range of noise levels
  - Computational efficiency and increased robustness to degradation factors
  - Good generalization ability of the developed networks to real noise
  - **State-of-the-art** image restoration results using a considerably smaller network than the previous best-performing network
  - The proposed networks can serve as sub-solvers for dealing with other general inverse problems (**state-of-the-art** results in image **demosaicking**)

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  - The proposed networks can serve as sub-solvers for dealing with other general inverse problems (**state-of-the-art** results in image **demosaicking**)

- **Broad spectrum of applications**
  - Computational photography : deblurring, superresolution, demosaicking, …
  - Medical imaging : (P)MRI reconstruction, computed tomography
  - Seismic imaging : seismic data recovery - phase retrieval
  - Computer vision : optical flow estimation, dense stereo reconstruction

Main References

- S. Lefkimmiatis, “Non-Local Color Image Denoising with Convolutional Neural Networks”, IEEE Int. Conference on Computer Vision and Pattern Recognition, 2017
- S. Lefkimmiatis, “Universal Denoising Networks : A Novel CNN Architecture for Image Denoising, IEEE Int. Conference on Computer Vision and Pattern Recognition, 2018
- F. Kokkinos and S. Lefkimmiatis, “Deep Image Demosaicking using a Cascade of Convolutional Residual Denoising Networks”, European Conference on Computer Vision, 2018
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- S. Lefkimmiatis, “Non-Local Color Image Denoising with Convolutional Neural Networks”, IEEE Int. Conference on Computer Vision and Pattern Recognition, 2017
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software and publications available at: http://cig.skoltech.ru

Thank you for your attention

Non-local Regularization Operator

- Construction of the Non-local Operator
  - Patch Extraction : $\mathbf{x}_r = \mathbf{P}_r \mathbf{x}$

    $\mathbf{P}_r \in \mathbb{R}^{P \times N}$: Binary matrix

  - Nearest Neighbors Search : $i_r = \{i_{r,1}, i_{r,2}, \ldots, i_{r,K}\}$, with $r = 1, \ldots, R$

    closest neighbors indices
  - Patch Transform : $\mathbf{f}_r = \mathbf{F}\mathbf{x}_r, \quad \mathbf{F} \in \mathbb{R}^{F \times P}$
  - Grouping : $\mathbf{f}_{i_r} = \begin{bmatrix} \mathbf{f}_{i_{r,1}}^{\mathsf{T}} & \mathbf{f}_{i_{r,2}}^{\mathsf{T}} & \ldots & \mathbf{f}_{i_{r,K}}^{\mathsf{T}} \end{bmatrix}^{\mathsf{T}} \in \mathbb{R}^{F \cdot K}$
  - Collaborative filtering : $\mathbf{z}_r = \mathbf{W}\mathbf{f}_{i_r}, \ \mathbf{W} \in \mathbb{R}^{F \times (F \cdot K)} \qquad \left(\mathbf{z}_r = \sum_{k=1}^{K} w_k \mathbf{f}_{i_{r,k}} \right)$

Non-local Regularization Operator

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  - Nearest Neighbors Search : $i_r = \{i_{r,1}, i_{r,2}, \ldots, i_{r,K}\}$, with $r = 1, \ldots, R$ — closest neighbors indices
  - Patch Transform : $\mathbf{f}_r = \mathbf{F}\mathbf{x}_r, \quad \mathbf{F} \in \mathbb{R}^{F \times P}$
  - Grouping : $\mathbf{f}_{i_r} = \begin{bmatrix} \mathbf{f}_{i_{r,1}}^{\mathsf{T}} & \mathbf{f}_{i_{r,2}}^{\mathsf{T}} & \ldots & \mathbf{f}_{i_{r,K}}^{\mathsf{T}} \end{bmatrix}^{\mathsf{T}} \in \mathbb{R}^{F \cdot K}$
  - Collaborative filtering : $\mathbf{z}_r = \mathbf{W}\mathbf{f}_{i_r}, \ \mathbf{W} \in \mathbb{R}^{F \times (F \cdot K)}$ $\left(\mathbf{z}_r = \sum_{k=1}^{K} w_k \mathbf{f}_{i_{r,k}} \right)$
- Non-local Operator - final form
  - Composition of linear operators : $\mathbf{L}_r \, \mathbf{x} = \left(\mathbf{W}\tilde{\mathbf{F}}\mathbf{P}_{i_r} \right) \mathbf{x}$ — block-diagonal matrix
  - Existence of the adjoint operator : $\mathbf{L}_r^{\mathsf{T}} = \mathbf{P}_{i_r}^{\mathsf{T}} \tilde{\mathbf{F}}^{\mathsf{T}} \mathbf{W}^{\mathsf{T}}$

Non-local Regularization Operator

- Construction of the Non-local Operator
  - Patch Extraction : $\mathbf{x}_r = \mathbf{P}_r \mathbf{x}$

    $\mathbf{P}_r \in \mathbb{R}^{P \times N}$: Binary matrix

  - Nearest Neighbors Search : $i_r = \{i_{r,1}, i_{r,2}, \ldots, i_{r,K}\}, \quad \text{with } r = 1, \ldots, R$

    closest neighbors indices
  - Patch Transform : $\mathbf{f}_r = \mathbf{F}\mathbf{x}_r, \quad \mathbf{F} \in \mathbb{R}^{F \times P}$
  - Grouping : $\mathbf{f}_{i_r} = \begin{bmatrix} \mathbf{f}_{i_{r,1}}^{\mathsf{T}} & \mathbf{f}_{i_{r,2}}^{\mathsf{T}} & \ldots & \mathbf{f}_{i_{r,K}}^{\mathsf{T}} \end{bmatrix}^{\mathsf{T}} \in \mathbb{R}^{F \cdot K}$
  - Collaborative filtering : $\mathbf{z}_r = \mathbf{W}\mathbf{f}_{i_r}, \quad \mathbf{W} \in \mathbb{R}^{F \times (F \cdot K)}$ $\left(\mathbf{z}_r = \sum_{k=1}^{K} w_k \mathbf{f}_{i_{r,k}} \right)$
- Non-local Operator - final form

  block-diagonal matrix
  - Composition of linear operators : $\mathbf{L}_r \, \mathbf{x} = \left(\mathbf{W}\tilde{\mathbf{F}}\mathbf{P}_{i_r} \right) \mathbf{x}$
  - Existence of the adjoint operator : $\mathbf{L}_r^{\mathsf{T}} = \mathbf{P}_{i_r}^{\mathsf{T}} \tilde{\mathbf{F}}^{\mathsf{T}} \mathbf{W}^{\mathsf{T}}$
- Machine-learning objective
  - Learn the patch transform & collaborative filtering coefficients from training data