PES University, Bengaluru

(Established under Karnataka Act 16 of 2013)

END SEMESTER ASSESSMENT (ESA) - JULY - 2023

UE17MA251 - Linear Algebra and Its Applications

Total Marks : 100.0

1.a. Solve: $x+y+z=6,\ 2x-y+z=3,\ x+z=4$ by using Gauss elimination method.

(6.0 Marks)

(7.0 Marks)

1.b. Find LU factorization for $A=\begin{bmatrix} 2 & -3 & -1 & 2 & 3 \\ 4 & -4 & -1 & 4 & 11 \\ 2 & -5 & -2 & 2 & -1 \\ 0 & 2 & 1 & 0 & 4 \end{bmatrix}$

(7.0 Marks)

1.c. Find the inverse of $A=\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$, using Gauss-Jordon method.

2.a.

i) Define free and pivot variables.

ii) For every c, find R and special solutions to $Ax = 0$, where $A = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & 2 & 4 & 4 \\ 1 & c & 2 & 2 \end{bmatrix}$

(7.0 Marks)

2.b.

i) Define basis and dimension for the vector space V with example.

ii) If V is the subspace spanned by $(1,1,1)$ and $(2,1,0)$, find a matrix A that has V as its row space, find a matrix B that has V as its null space.

(6.0 Marks)

2.c. Find the four fundamental subspaces of the matrix $A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$

(7.0 Marks)

3.a.

I. Define linear transformation.

II. Find the differentiation matrix A_{diff} which differentiate the fourth degree polynomials.

(7.0 Marks)

3.b.

Let $T_1 : R^2 \to R^2$ be the reflection about the line $y = x$ and $T_2 : R^2 \to R^2$ be the projection on $x-$axis. Find the standard matrix of $T_2 \cdot T_1$. Is this matrix same as the standard matrix of $T_1 \cdot T_2$? Illustrate your result graphically.

(7.0 Marks)

3.c.

Use the method least squares, find the equation of the line that runs through four points $(1,-1),(4,11),(-1,-9)$ and $(-2,-13)$.

(6.0 Marks)

4.a.

Find the matrices Q and R such that $QR = A$, where A has columns $(1,1,-2),(1,2,-3),(0,1,1)$.

(8.0 Marks)

4.b. Calculate the four iterations of the power method to find the largest Eigen value and corresponding Eigen vector of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$, use (1, 1, 1) as initial approximation.

(5.0 Marks)

(7.0 Marks)

4.c. Diagonalize the matrix $A = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$ and hence find A^{50}.

5.a. Define positive definite and positive semi definite of the matrix. And test the following matrices for positive or positive semi definite

i) $\begin{bmatrix} 5 & 2 & 1 \\ 2 & 2 & 2 \\ 1 & 2 & 5 \end{bmatrix}$ ii) $\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$

(10.0 Marks)

5.b. Find Singular Value Decomposition (SVD) of the matrix $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & 2 \end{bmatrix}$ (10.0 Marks)