**PES University, Bengaluru**

(Established under Karnataka Act 16 of 2013)

**END SEMESTER ASSESSMENT (ESA) - JULY - 2023**

**UE19MA251 - Linear Algebra and Its Applications**

**Total Marks : 100.0**

1.a. Determine the values of a and b for which the system of equations
x + y + a z = 2b ,
x + 3y + ( 2 + 2a )z = 7b
3x + y + ( 3 + 3a ) z = 11b Will have (i) unique nontrivial solution (ii)trivial solution ( iii )no solution(iv) infinity of solutions. (5.0 Marks)

1.b. A boy is walking along the path y = ax^2 + bx + c through the points ( - 6, 8),(2, 12) , and (3,8) .
He wants to meet his friend at P(7, 60) .
Will he meet his friend? (Use Gaussian elimination method.) (5.0 Marks)

1.c. Solve the system of equation by using LU Decomposition.
2x + 3y + z = 9,
x + 2y + 3z = 6,
3x + y + 2z = 8. (5.0 Marks)

1.d. . Solve the following system by using the Gauss-Jordan elimination method.
$2x + y + z = 0$
$4x - 6y = 0$
$-2x + 7y + 2z = 0$ (5.0 Marks)

2.a. Examine if the following set of vectors are linearly independent. When the set is dependent find a relation between the vectors:
{(1, 3, 1, 2), (2, 5, -1, 2), (1, 3, 7, -2)} (5.0 Marks)

2.b. Reduce the following matrice to Row Reduced Echelon form and determine their ranks,
Identify the pivot variables and free variables. Find the special solutions to Ax=0. (5.0 Marks)

$$\begin{pmatrix} 2 & -4 & 4 & -2 \\ 4 & -9 & 7 & -3 \\ 1 & -4 & 8 & 0 \end{pmatrix}$$

2.c. Find the Column space and Null space for the following matrices: (5.0 Marks)

$$\begin{pmatrix} 2 & 4 & -2 & 2 \\ -2 & 5 & 7 & 3 \\ -3 & 6 & -8 & 6 \end{pmatrix}$$

2.d. For which vector ($b_1$,$b_2$,$b_3$, $b_4$) is this system solvable? (5.0 Marks)

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 2 & 5 & 7 \\ 3 & 9 & 12 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$$

3.a. Find the matrix of the linear transformation T on $\mathbb{R}^3$ defined by T(x,y,z)=(x+2y-z, y+z, x+y-2z) with respect to
the non-standard basis (1,1,1),(1,1,0), ((1,0,0) (5.0 Marks)

3.b. Project b = ( 1, 0, 0 ) onto the lines through a1 = ( -1 , 2, 2 ) , a2 = ( 2, 2, -1 ) and a3 = ( 2 , -1 , 2 ). Add the three projections to get the sum b. Also find the corresponding projection matrices P1, P2 and P3. Check that their sum is I and the product is 0. (5.0 Marks)

3.c. Find the point on the plane 2x+5y+z=0 that is nearest to (2,0,1) (5.0 Marks)

3.d. An ice- cream vendor records the number of hours of sun shine (x) versus the number of ice- creams sold in an hour ( y ) at his shop from Monday to Friday and found the following data :

x: 2 3 5 7 9

y: 4 5 7 10 15

Find the best values of m and c that suit the equation y = mx + c. If there is a weather forecast that says there would be 8 hours of sun shine the next day, estimate the number of ice- creams that he expects to sell on that day. (5.0 Marks)

4.a. If W is a subspace spanned by the orthogonal vectors ( 2, 5, -1 ) and ( -2 , 1, 1 ) find the point in W that is closest to ( 1, 2, 3 ). (5.0 Marks)

4.b. Find the eigenvalues and the corresponding eigenvectors of ( 2 0 1, 0 2 0, 1 0 2 ). (5.0 Marks)

4.c.

Find the matrix A whose eigen values are 2 and 5, and whose eigen vectors are $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}1\\1\end{pmatrix}$ respectively using $S\Delta S^{-1}$.

(5.0 Marks)

4.d. Find the matrices S and $S^{-1}$ to diagonalize $A = \begin{pmatrix} 0.6 & 0.4 \\ 0.4 & 0.6 \end{pmatrix}$

What are limits of $\Delta^k$ and $S\Delta^k S^{-1}$ as $k \to \infty$ ?

(5.0 Marks)

5.a. For which a are the matrix A have all

lambda > 0 and are therefore positive definite. $A = \begin{pmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{pmatrix}$

(5.0 Marks)

5.b.

Compute the quadratic form $x^T Ax$ for $A = \begin{pmatrix} 4 & 3 & 0 \\ 3 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ and

$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

(5.0 Marks)

5.c. Find SVD of the following matrix $\begin{pmatrix} 4 & 4 \\ -3 & 3 \end{pmatrix}$

(5.0 Marks)

5.d. The following table lists the weights and heights of 5 boys:

| Boy | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Weight(lbs) | 120 | 125 | 125 | 135 | 145 |
| Height(in) | 61 | 60 | 64 | 68 | 72 |

Find the covariance matrix for the above data (5.0 Marks)