PES University, Bengaluru

(Established under Karnataka Act 16 of 2013)

END SEMESTER ASSESSMENT (ESA) - JULY - 2023

UE20MA251 - Linear Algebra and Its Applications

Total Marks : 100.0

1.a.

Solve the following system of equations using Gaussian elimination: (6.0 Marks)

$2x + y + 3z = 1,\ 2x + 6y + 8z = 3,\ 6x + 8y + 18z = 5$

1.b. Write down the elementary matrices E, F, G associated with the system of equations $2u + v + 3w = -1,\ 4u + v + 7w = 5,\ -6u - 2v - 12w = -2$. Also find the LU decomposition of A.

(7.0 Marks)

1.c. Use the Gauss – Jordan method to find the inverse of: (7.0 Marks)

$$A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}.$$

(6.0 Marks)

2.a. Find the value(s) of 'h' for which the following set of vector's,

$$\left\{ \mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} h \\ 1 \\ -h \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2h \\ 3h+1 \end{bmatrix} \right\}$$

Is linearly independent.

2.b. Find a basis and the dimension of the subspaces $V = \{(a, b, 0)\colon\ a \text{ and } b \text{ are real numbers}\}$, $W = \{(0, b, c)\colon b \text{ and } c \text{ are real numbers}\}$ and $V \cap W$ in R^3. Geometrically describe the basis of V, W and $V \cap W$.

(7.0 Marks)

(7.0 Marks)

2.c. If $A = \begin{pmatrix} 0 & 1 & 3 \\ -1 & 0 & 1 \\ -1 & 2 & 7 \end{pmatrix}$,

(i) Find the basis for the column space of A.
(ii) Find the basis for the Row space of A.
(iii) Find the basis for the Null space of A.
(iv) Is the vector $w = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ is in Row space of A. (3+1+2+1=7M)

3.a. Let $T: R^3 \to R^3$ given by $T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2x+y+z \\ x-y+2z \\ x+2y-z \end{bmatrix}$, then find Null space of T i.e., N(T).

(6.0 Marks)

3.b. What matrix P projects every point in R^3 onto the line of intersection of the planes $x + y + z = 0$ and $x - z = 0$? What are the column space and row space of matrix P.

(7.0 Marks)

3.c.

Consider the linear system $-x_1 + x_2 = 10, 2x_1 + x_2 = 5,\ x_1 - 2x_2 = 20$. This system is over determined and inconsistent. Find the least approximation to b and the least squares solution $\hat{x}$ to this system and rewrite the linear system as $A\hat{x} = b_1$

(7.0 Marks)

4.a. Find a third column so that the matrix $Q = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{14}} & --- \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{14}} & --- \\ \frac{1}{\sqrt{3}} & \frac{-3}{\sqrt{14}} & --- \end{bmatrix}$ is orthogonal.

(5.0 Marks)

4.b. Find the Eigen Value and Eigen vectors of $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

(7.0 Marks)

4.c. Use Gram Schmidt process to find the orthonormal vectors q_1, q_2, q_3 form the given independent vectors $a = (1,1,1), b = (0,1,1), c = (0,0,1)$, and also factorize A=QR

(8.0 Marks)

5.a. For the semi definite matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$, write $XA^T X$ as the sum of two squares.

(6.0 Marks)

5.b. Let $Q(x) = {x_1}^2 + b{x_2}^2 + 7{x_3}^2 + 4x_1x_2 + 16x_2x_3 + 8x_1x_3$, for what range of numbers b , Q(x) is positive definite.

(4.0 Marks)

5.c. Find the SVD of $A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}_{2x3}$

(10.0 Marks)