student-success-analysis

Deskriptivna analiza

Osnove

Učitavamo podatke, provjeravamo kojeg je oblika skup podataka i od kojih se stupaca sastoji.

```
students_org <- readxl::read_excel("student_data.xlsx")

# 370 rows, 39 columns
dim(students_org)

# Show column names
names(students_org)
```

Provjeravamo prvih par redataka podatkovnog skupa

```
# Show first few rows
head(students_org)
```

Saznajemo osnovne podatke za svaki stuapc

```
# Show details for each column
summary(students_org)
```

Provjeravamo koji su stupci kojeg tipa: numerički, kategorički…

```
# Check the class of the column. 'numeric', 'character'...
sapply(students_org, class)
```

Provjeravamo postoje li nevažeći podaci koji prelaze maksimalne vrijednosti specificirane u uputama o podacima. Sve vrijednosti su dobrom intervalu.

```
# Let's check if any columns exceed the maximum or minumum values specified in
# the pdf This makes sense only for numerical values

colMax <- students_org %>%
    select(where(is.numeric)) %>%
    sapply(., max, na.rm = TRUE)
colMax
# Every column has normal maximum value
```

Izbacivanje svih NaN/NA/null vrijednosti iz podatkovnog skupa. Na sreću, takvih vrijednosti nije bilo.

```
# Are there any na values?
students_org %>%
    filter(is.na(.))
sum(apply(students_org, 2, is.nan))
students_org %>%
    filter(is.null(.)) %>%
    summarise(n = n())
```

```r
# Drop these values just in case they show up with another dataset We will
# continue using 'student' variable
students <- students_org %>%
    filter_all(all_vars(!is.na(.) & !is.nan(.) & !is.null(.)))

students_clean <- students
```

Testing

```r
# pronadi char stupce (tj. kategoricke varijable)
charcols <- names(students %>%
    select(where(is_character)))
charcols
```

```
##  [1] "school"     "sex"        "address"    "famsize"    "Pstatus"
##  [6] "Mjob"       "Fjob"       "reason"     "guardian"   "schoolsup"
## [11] "famsup"     "paid_mat"   "paid_por"   "activities" "nursery"
## [16] "higher"     "internet"   "romantic"
```

```r
students_char = students

# pretvori char stupce u faktore (kategoricke varijable)
students_char[charcols] <- lapply(students_char[charcols], function(x) factor(x,
    ordered = TRUE))

# one hot encodeaj kateogricke varijable sad stupci vise nisu vrijednosti char
# nego su 1 ili 0 npr. MALE 0 (zensko) MALE 1 (musko)

students_dummy = dummy_cols(students_char, charcols, remove_selected_columns = TRUE)
# ako ti treba, cor tablica radi!  cor(students_dummy)

summary(lm(students_char$G3_mat ~ students_char$traveltime * students_char$higher))
```

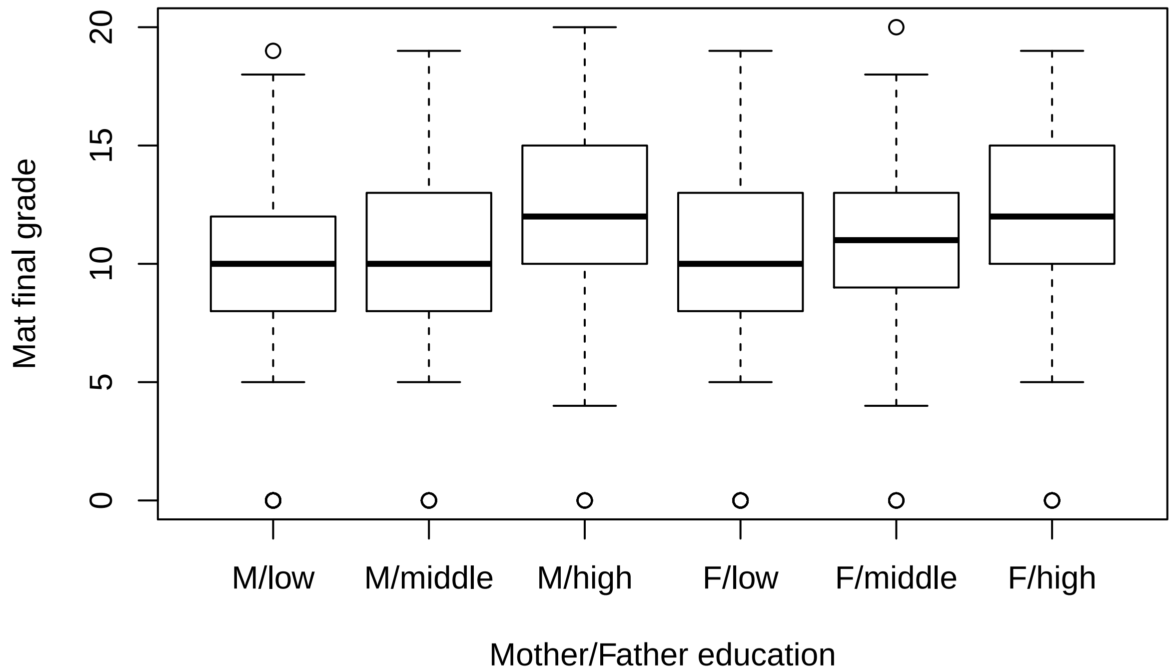
```
##
## Call:
## lm(formula = students_char$G3_mat ~ students_char$traveltime *
##     students_char$higher)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -11.0965  -2.0965  -0.0965   2.9035   8.9035
##
## Coefficients:
##                                                 Estimate Std. Error t value
## (Intercept)                                       7.6655     1.2248   6.259
## students_char$traveltime                          0.2285     0.6721   0.340
## students_char$higher.L                            6.1767     1.7321   3.566
## students_char$traveltime:students_char$higher.L  -1.6477     0.9504  -1.734
##                                                 Pr(>|t|)
## (Intercept)                                     1.09e-09 ***
## students_char$traveltime                         0.73401
## students_char$higher.L                           0.00041 ***
## students_char$traveltime:students_char$higher.L  0.08384 .
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.455 on 366 degrees of freedom
## Multiple R-squared:  0.07311,    Adjusted R-squared:  0.06552
## F-statistic: 9.624 on 3 and 366 DF,  p-value: 3.95e-06
```

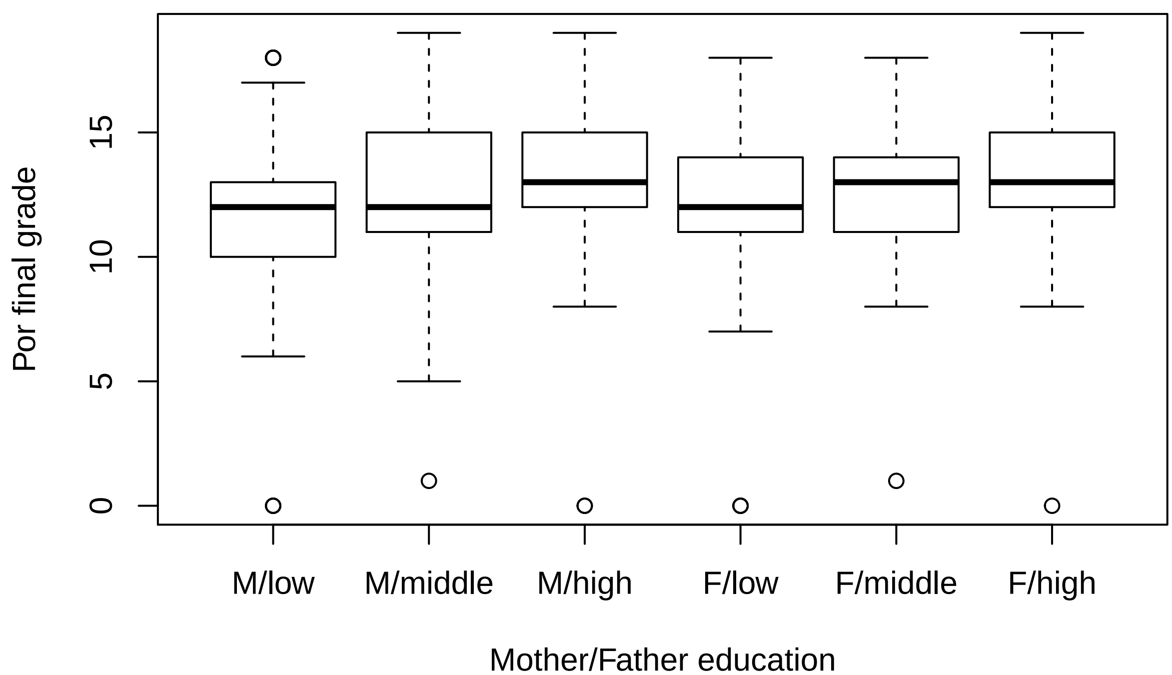
Petar Dragojević

zavisnost izmedu edukacije roditelja i uspješnosti

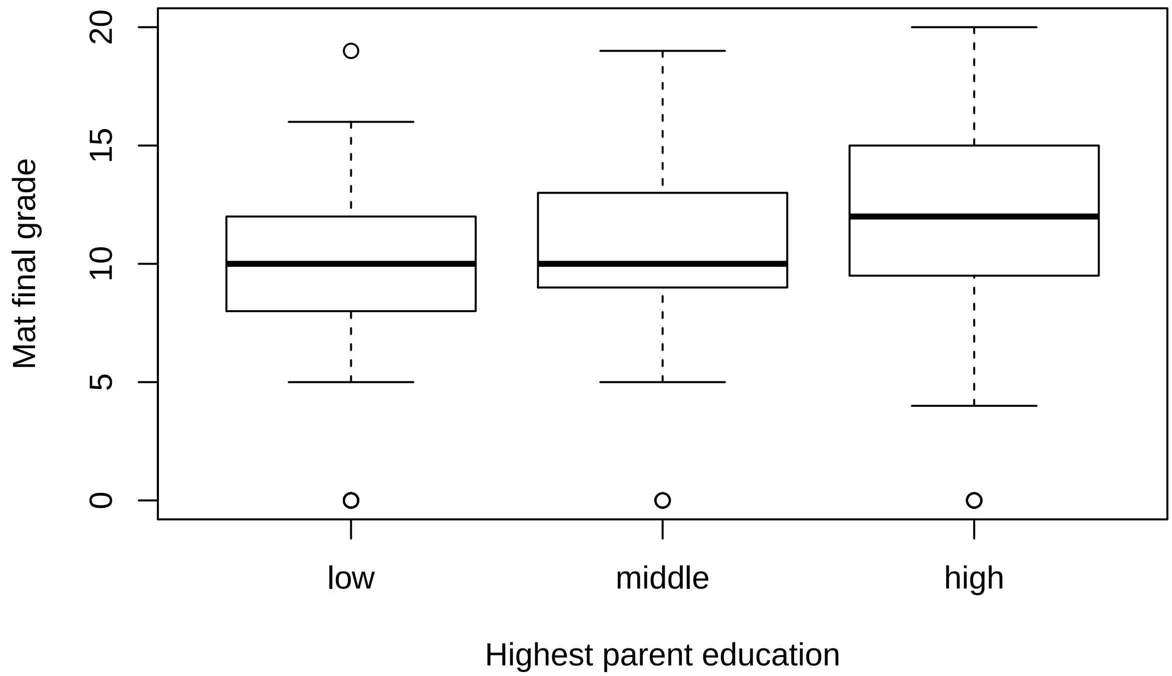
```
# Mat_grade i Por_grade prebacivanje ocjena u engleski način ocjenjivanja
students <- students %>%
    mutate(Mat_grade = case_when(G3_mat < 10 ~ "F", G3_mat >= 10 & G3_mat < 14 ~
        "C", G3_mat >= 14 & G3_mat < 16 ~ "B", G3_mat >= 16 ~ "A"))
students <- students %>%
    mutate(Por_grade = case_when(G3_por < 10 ~ "F", G3_por >= 10 & G3_mat < 14 ~
        "C", G3_por >= 14 & G3_mat < 16 ~ "B", G3_por >= 16 ~ "A"))

# MeduMod i FeduMod grupiranje edukacije roditelja u veće podgrupe
students <- students %>%
    mutate(MeduMod = case_when(Medu == "0" | Medu == "1" | Medu == "2" ~ "0", Medu ==
        "3" ~ "1", Medu == "4" ~ "2"))
students <- students %>%
    mutate(FeduMod = case_when(Fedu == "0" | Medu == "1" | Fedu == "2" ~ "0", Fedu ==
        "3" ~ "1", Fedu == "4" ~ "2"))

# za edukaciju roditelja uzimamo onu koja je veća
students$highestparentedu <- pmax(students$MeduMod, students$FeduMod)

boxplot(students$G3_mat[students$MeduMod == "0"], students$G3_mat[students$MeduMod ==
    "1"], students$G3_mat[students$MeduMod == "2"], students$G3_mat[students$FeduMod ==
    "0"], students$G3_mat[students$FeduMod == "1"], students$G3_mat[students$FeduMod ==
    "2"], names = c("M/low", "M/middle", "M/high", "F/low", "F/middle", "F/high"),
    xlab = "Mother/Father education", ylab = "Mat final grade")
```



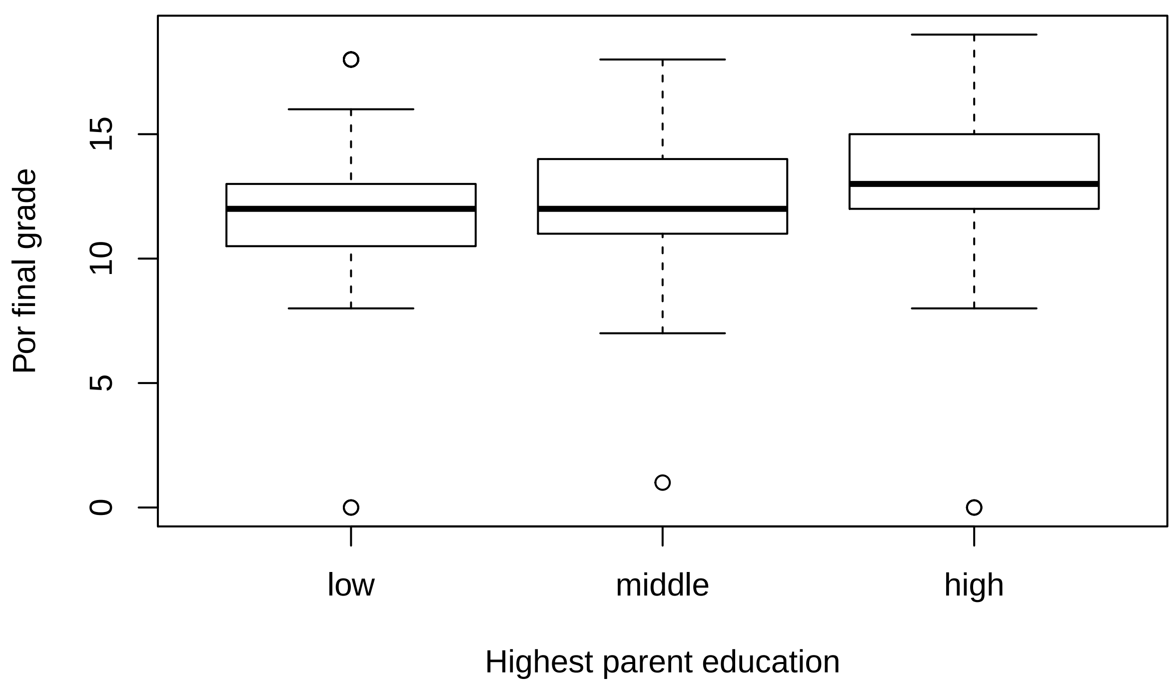

```
boxplot(students$G3_por[students$MeduMod == "0"], students$G3_por[students$MeduMod ==
    "1"], students$G3_por[students$MeduMod == "2"], students$G3_por[students$FeduMod ==
    "0"], students$G3_por[students$FeduMod == "1"], students$G3_por[students$FeduMod ==
    "2"], names = c("M/low", "M/middle", "M/high", "F/low", "F/middle", "F/high"),
    xlab = "Mother/Father education", ylab = "Por final grade")
```




```
boxplot(students$G3_mat[students$highestparentedu == "0"], students$G3_mat[students$highestparentedu ==
    "1"], students$G3_mat[students$highestparentedu == "2"], names = c("low", "middle",
    "high"), xlab = "Highest parent education", ylab = "Mat final grade")
```




```
boxplot(students$G3_por[students$highestparentedu == "0"], students$G3_por[students$highestparentedu ==
    "1"], students$G3_por[students$highestparentedu == "2"], names = c("low", "middle",
    "high"), xlab = "Highest parent education", ylab = "Por final grade")
```




```
# H0: Ocjena iz matematike i edukacija više educiranog roditelja su nezavisna
# obilježja H1: Ocjena iz matematike i edukacija više educiranog roditelja su
# zavisna obilježja

tbl = table(students$highestparentedu, students$Mat_grade)
added_margins_tbl = addmargins(tbl)
print(added_margins_tbl)
```

```
##
##          A   B   C   F Sum
##   0      4   9  48  39 100
##   1      7   8  37  25  77
##   2     25  35  54  38 152
##   Sum   36  52 139 102 329
```

```
chisq.test(tbl, correct = F)
```

```
##
##  Pearson's Chi-squared test
##
## data:  tbl
## X-squared = 25.134, df = 6, p-value = 0.0003224
```

p-value testa iznosi manje od 0.05 stoga odbacujemo hipotezu H0, te zaključujemo da su edukacija više educiranog roditelja i završna ocjena iz matematike zavisne.

```
# H0: Ocjena iz matematike i edukacija majke su nezavisna obilježja H1: Ocjena
# iz matematike i edukacija majke su zavisna obilježja
tbl = table(students$MeduMod, students$Mat_grade)
added_margins_tbl = addmargins(tbl)
print(added_margins_tbl)
```

```
##
##          A   B   C   F Sum
##   0      7  17  68  56 148
##   1     11  12  37  33  93
##   2     22  30  45  32 129
##   Sum   40  59 150 121 370
```

```
chisq.test(tbl, correct = F)
```

```
##
##  Pearson's Chi-squared test
##
## data:  tbl
## X-squared = 22.482, df = 6, p-value = 0.0009898
```

```
# H0: Ocjena iz matematike i edukacija oca su nezavisna obilježja H1: Ocjena iz
# matematike i edukacija oca su zavisna obilježja
tbl2 = table(students$FeduMod, students$Mat_grade)
added_margins_tbl2 = addmargins(tbl2)
print(added_margins_tbl2)
```

```
##
##          A   B   C   F Sum
##   0     11  19  64  49 143
##   1     11  12  40  30  93
##   2     14  21  35  23  93
##   Sum   36  52 139 102 329
```

```
chisq.test(tbl2, correct = F)
```

```
##
##  Pearson's Chi-squared test
##
## data:  tbl2
```

```
## X-squared = 9.0666, df = 6, p-value = 0.1699
```

p-value Testa nezavisnosti iznosi manje od 0.05 stoga odbacujemo hipotezu H0, te zaključujemo da su edukacija majke i završna ocjena iz matematike zavisne, dok do tog zaključka ne možemo doći u slučaju edukacije oca.

```
# H0: Ocjena iz portugala i edukacija više educiranog roditelja su nezavisna
# obilježja H1: Ocjena iz portugala i edukacija više educiranog roditelja su
# zavisna obilježja
tbl = table(students$highestparentedu, students$Por_grade)
added_margins_tbl = addmargins(tbl)
print(added_margins_tbl)
```

```
##
##          A   B   C   F Sum
##   0      3   3  77  10  93
##   1      3   6  57   5  71
##   2     12  23  85   7 127
##   Sum   18  32 219  22 291
```

```
chisq.test(tbl, correct = F)
```

```
## Warning in chisq.test(tbl, correct = F): Chi-squared approximation may be
## incorrect
```

```
##
##  Pearson's Chi-squared test
##
## data:  tbl
## X-squared = 19.409, df = 6, p-value = 0.003526
```

```
# očekivane frekvencije svih razreda moraju biti veće ili jednake 5
for (col_names in colnames(added_margins_tbl)) {
    for (row_names in rownames(added_margins_tbl)) {
        if (!(row_names == "Sum" | col_names == "Sum")) {
            cat("Očekivane frekvencije za razred ", col_names, "-", row_names, ": ",
                (added_margins_tbl[row_names, "Sum"] * added_margins_tbl["Sum", col_names])/added_margi
                  "Sum"], "\n")
        }
    }
}
```

```
## Očekivane frekvencije za razred  A - 0 :  5.752577
## Očekivane frekvencije za razred  A - 1 :  4.391753
## Očekivane frekvencije za razred  A - 2 :  7.85567
## Očekivane frekvencije za razred  B - 0 :  10.2268
## Očekivane frekvencije za razred  B - 1 :  7.80756
## Očekivane frekvencije za razred  B - 2 :  13.96564
## Očekivane frekvencije za razred  C - 0 :  69.98969
## Očekivane frekvencije za razred  C - 1 :  53.43299
## Očekivane frekvencije za razred  C - 2 :  95.57732
## Očekivane frekvencije za razred  F - 0 :  7.030928
## Očekivane frekvencije za razred  F - 1 :  5.367698
## Očekivane frekvencije za razred  F - 2 :  9.601375
```

```
# Vidimo da postoje očekivane frekvencije manje od 5 pa koristimo fisher.test()
# umjesto chiq.test()
fisher.test(tbl)
```

```
##
##  Fisher's Exact Test for Count Data
##
## data:  tbl
## p-value = 0.003003
## alternative hypothesis: two.sided
```

chisq.test je nepouzdan pošto su očekivane frekvencije pojedinih razreda manje od 5, radi toga koristimo fisher.test. p-value Fesherovog testa iznosi manje od 0.05 stoga odbacujemo hipotezu H0, te zaključujemo da su edukacija više educiranog roditelja i završna ocjena iz portigala zavisne.

```
# H0: Ocjena iz portugala i edukacija majke su nezavisna obilježja H1: Ocjena
# iz portugala i edukacija majke su zavisna obilježja
tbl = table(students$MeduMod, students$Por_grade)
added_margins_tbl = addmargins(tbl)
print(added_margins_tbl)
```

```
##
##         A   B   C   F Sum
##   0     5   9 108  16 138
##   1     6   9  62   8  85
##   2     9  19  72   5 105
##   Sum  20  37 242  29 328
```

```
chisq.test(tbl, correct = F)
```

```
##
##  Pearson's Chi-squared test
##
## data:  tbl
## X-squared = 13.658, df = 6, p-value = 0.0337
```

```
# H0: Ocjena iz portugala i edukacija oca su nezavisna obilježja H1: Ocjena iz
# portugala i edukacija oca su zavisna obilježja
tbl2 = table(students$FeduMod, students$Por_grade)
added_margins_tbl2 = addmargins(tbl2)
print(added_margins_tbl2)
```

```
##
##         A   B   C   F Sum
##   0     8  10  99  14 131
##   1     3   7  67   3  80
##   2     7  15  53   5  80
##   Sum  18  32 219  22 291
```

```
chisq.test(tbl2, correct = F)
```

```
## Warning in chisq.test(tbl2, correct = F): Chi-squared approximation may be
## incorrect
```

```
##
##  Pearson's Chi-squared test
##
## data:  tbl2
## X-squared = 12.75, df = 6, p-value = 0.04719
```

```
# očekivane frekvencije svih razreda moraju biti veće ili jednake 5
for (col_names in colnames(added_margins_tbl2)) {
```

```
    for (row_names in rownames(added_margins_tbl2)) {
        if (!(row_names == "Sum" | col_names == "Sum")) {
            cat("Očekivane frekvencije za razred ", col_names, "-", row_names, ": ",
                (added_margins_tbl2[row_names, "Sum"] * added_margins_tbl2["Sum",
                  col_names])/added_margins_tbl2["Sum", "Sum"], "\n")
        }
    }
}
```

```
## Očekivane frekvencije za razred  A - 0 :  8.103093
## Očekivane frekvencije za razred  A - 1 :  4.948454
## Očekivane frekvencije za razred  A - 2 :  4.948454
## Očekivane frekvencije za razred  B - 0 :  14.4055
## Očekivane frekvencije za razred  B - 1 :  8.797251
## Očekivane frekvencije za razred  B - 2 :  8.797251
## Očekivane frekvencije za razred  C - 0 :  98.58763
## Očekivane frekvencije za razred  C - 1 :  60.20619
## Očekivane frekvencije za razred  C - 2 :  60.20619
## Očekivane frekvencije za razred  F - 0 :  9.90378
## Očekivane frekvencije za razred  F - 1 :  6.04811
## Očekivane frekvencije za razred  F - 2 :  6.04811
```

```
# Vidimo da postoje očekivane frekvencije manje od 5 pa koristimo fisher.test()
# umjesto chiq.test()
fisher.test(tbl)
```

```
##
##  Fisher's Exact Test for Count Data
##
## data:  tbl
## p-value = 0.03133
## alternative hypothesis: two.sided
```

p-value Testa nezavisnosti iznosi manje od 0.05 stoga odbacujemo hipotezu H0, te zaključujemo da su edukacija majke i završna ocjena iz portugala zavisne, kod usporedbe s edukacijom oca koristimo Fisherov test gdje je p-value manji od 0.05 pa odbacujemo H0 i zaključujemo da su edukacija oca i završna ocjena iz portugala zavisne.

Tomislav Prhat

1. Jesu li učenici uspješniji u matematici ili glavnom jeziku?

```
students_org %>%
    summarise(Mean.G3_mat = mean(G3_mat), Mean.G3_por = mean(G3_por), ) -> summary.result1
summary.result1
```

```
## # A tibble: 1 x 2
##   Mean.G3_mat Mean.G3_por
##         <dbl>       <dbl>
## 1        10.5        12.6
```

```
students_org %>%
    summarise(Med.G3_mat = median(G3_mat), Med.G3_por = median(G3_por), ) -> summary.result2
summary.result2
```

```
## # A tibble: 1 x 2
##   Med.G3_mat Med.G3_por
```

```
##        <dbl>      <dbl>
## 1         11         13
```

```
students_org %>%
    summarise(Mean.G3_mat = mean(G3_mat, trim = 0.1), Mean.G3_por = mean(G3_por,
        trim = 0.1), ) -> summary.result3
summary.result3
```

```
## # A tibble: 1 x 2
##   Mean.G3_mat Mean.G3_por
##         <dbl>       <dbl>
## 1        10.9        12.6
```

```
(1 - summary.result3/summary.result1) * 100
```

```
##   Mean.G3_mat Mean.G3_por
## 1   -4.016012  -0.7265877
```

Kao što je vidljivo iz podataka, učenici su malo uspješniji u glavnom jeziku (portugalskom), ali ako gleda prema samoj ocjeni obje skupine spadaju u ocjenu "C". Čak i ako uzmemo podrezanu srednju vrijednost (10%), rezultat se promijeni za ~1%.

```
students_org %>%
    summarise(IQR.G3_mat = IQR(G3_mat), IQR.G3_por = IQR(G3_por), ) -> summary.result4
summary.result4
```

```
## # A tibble: 1 x 2
##   IQR.G3_mat IQR.G3_por
##        <dbl>      <dbl>
## 1          6          3
```

```
students_org %>%
    summarise(Var.G3_mat = var(G3_mat), Var.G3_por = var(G3_por), ) -> summary.result5
summary.result5
```

```
## # A tibble: 1 x 2
##   Var.G3_mat Var.G3_por
##        <dbl>      <dbl>
## 1       21.2       8.67
```

```
students_org %>%
    summarise(sd.G3_mat = sd(G3_mat), sd.G3_por = sd(G3_por), ) -> summary.result6
summary.result6
```

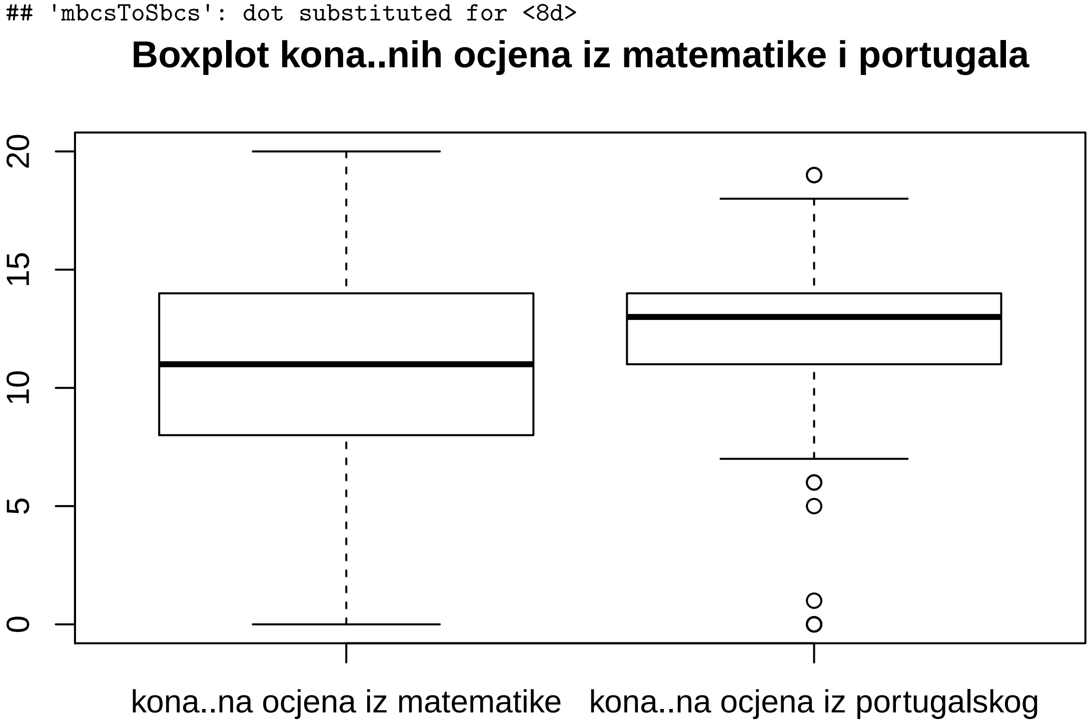
```
## # A tibble: 1 x 2
##   sd.G3_mat sd.G3_por
##       <dbl>     <dbl>
## 1      4.61      2.94
```

Ako gledamo raspršenost varijabli vidimo da ocjene iz portugalskog jezika imaju manje sve tri mjere (IQR, varijanca i standardna devijacija) vidimo da se ocjene iz portugalskog manje manje odmiču od srednje vrijednosti nego ocjene iz matematike.

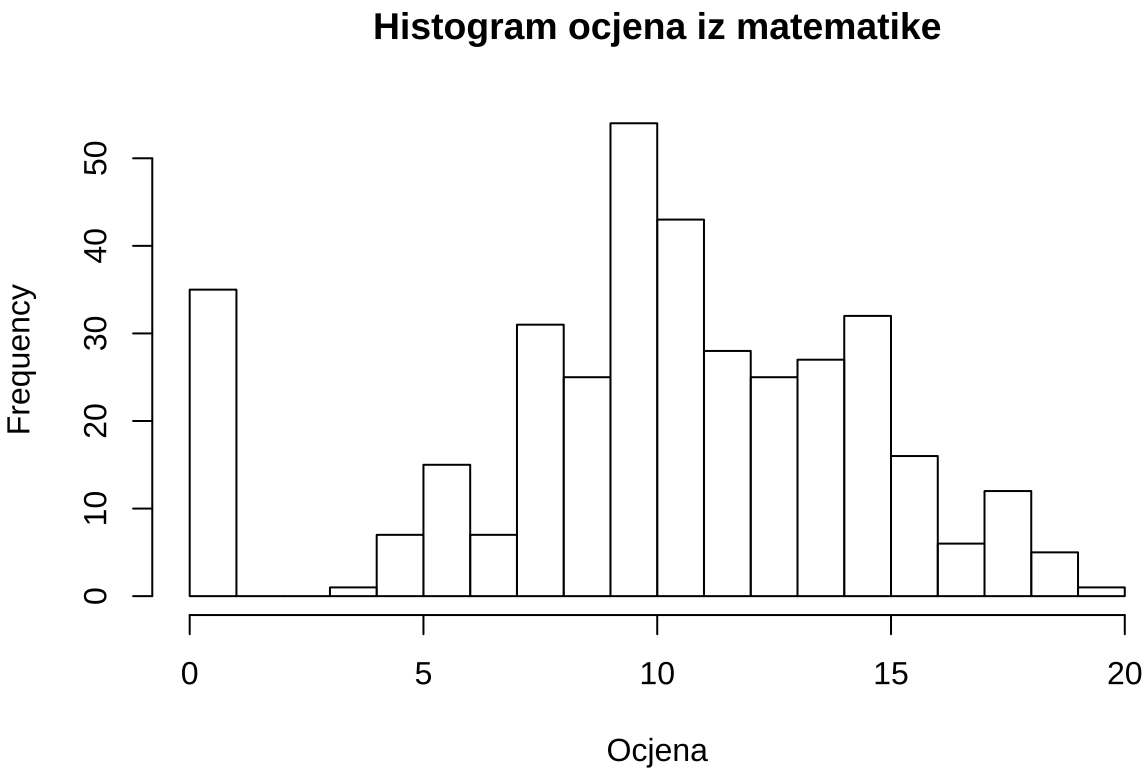
```
boxplot(students_org$G3_mat, students_org$G3_por, names = c("konačna ocjena iz matematike",
    "konačna ocjena iz portugalskog"), main = "Boxplot konačnih ocjena iz matematike i portugala")
```

```
## Warning in axis(side = 1, at = 1:2, labels = c("konačna ocjena iz matematike", :
## conversion failure on 'konačna ocjena iz matematike' in 'mbcsToSbcs': dot
## substituted for <c4>
```

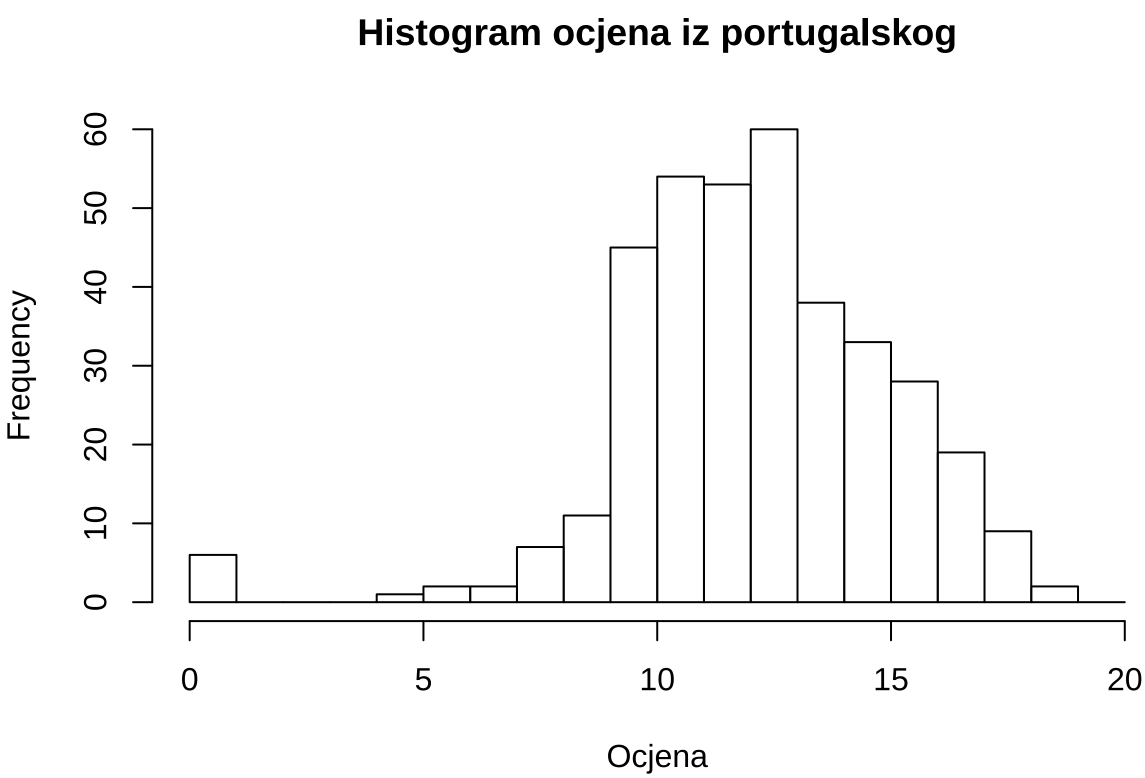
```
## Warning in axis(side = 1, at = 1:2, labels = c("konačna ocjena iz matematike", :
## conversion failure on 'konačna ocjena iz matematike' in 'mbcsToSbcs': dot
## substituted for <8d>

## Warning in axis(side = 1, at = 1:2, labels = c("konačna ocjena iz matematike", :
## conversion failure on 'konačna ocjena iz matematike' in 'mbcsToSbcs': dot
## substituted for <c4>

## Warning in axis(side = 1, at = 1:2, labels = c("konačna ocjena iz matematike", :
## conversion failure on 'konačna ocjena iz matematike' in 'mbcsToSbcs': dot
## substituted for <8d>

## Warning in axis(side = 1, at = 1:2, labels = c("konačna ocjena iz matematike", :
## conversion failure on 'konačna ocjena iz portugalskog' in 'mbcsToSbcs': dot
## substituted for <c4>

## Warning in axis(side = 1, at = 1:2, labels = c("konačna ocjena iz matematike", :
## conversion failure on 'konačna ocjena iz portugalskog' in 'mbcsToSbcs': dot
## substituted for <8d>

## Warning in axis(side = 1, at = 1:2, labels = c("konačna ocjena iz matematike", :
## conversion failure on 'konačna ocjena iz portugalskog' in 'mbcsToSbcs': dot
## substituted for <c4>

## Warning in axis(side = 1, at = 1:2, labels = c("konačna ocjena iz matematike", :
## conversion failure on 'konačna ocjena iz portugalskog' in 'mbcsToSbcs': dot
## substituted for <8d>

## Warning in (function (main = NULL, sub = NULL, xlab = NULL, ylab = NULL, :
## conversion failure on 'Boxplot konačnih ocjena iz matematike i portugala' in
## 'mbcsToSbcs': dot substituted for <c4>

## Warning in (function (main = NULL, sub = NULL, xlab = NULL, ylab = NULL, :
## conversion failure on 'Boxplot konačnih ocjena iz matematike i portugala' in
## 'mbcsToSbcs': dot substituted for <8d>
```



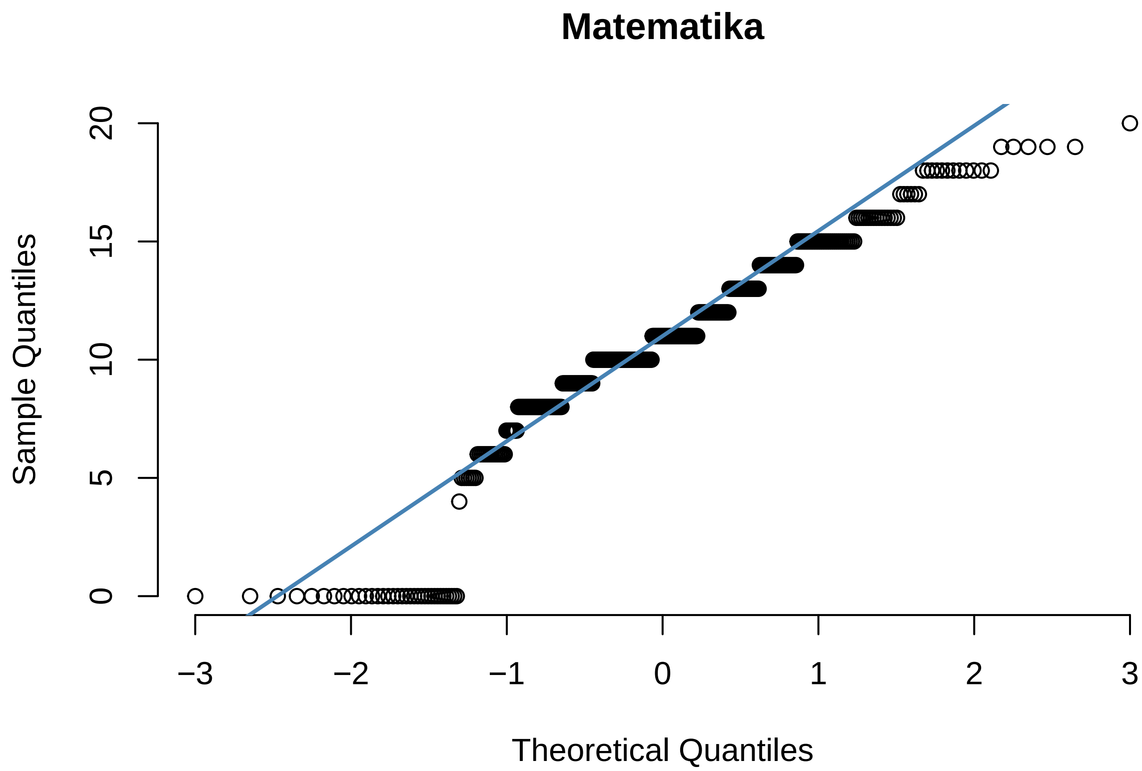
```r
hist(students_org$G3_mat, breaks = seq(0, 20), main = "Histogram ocjena iz matematike",
    xlab = "Ocjena")
```



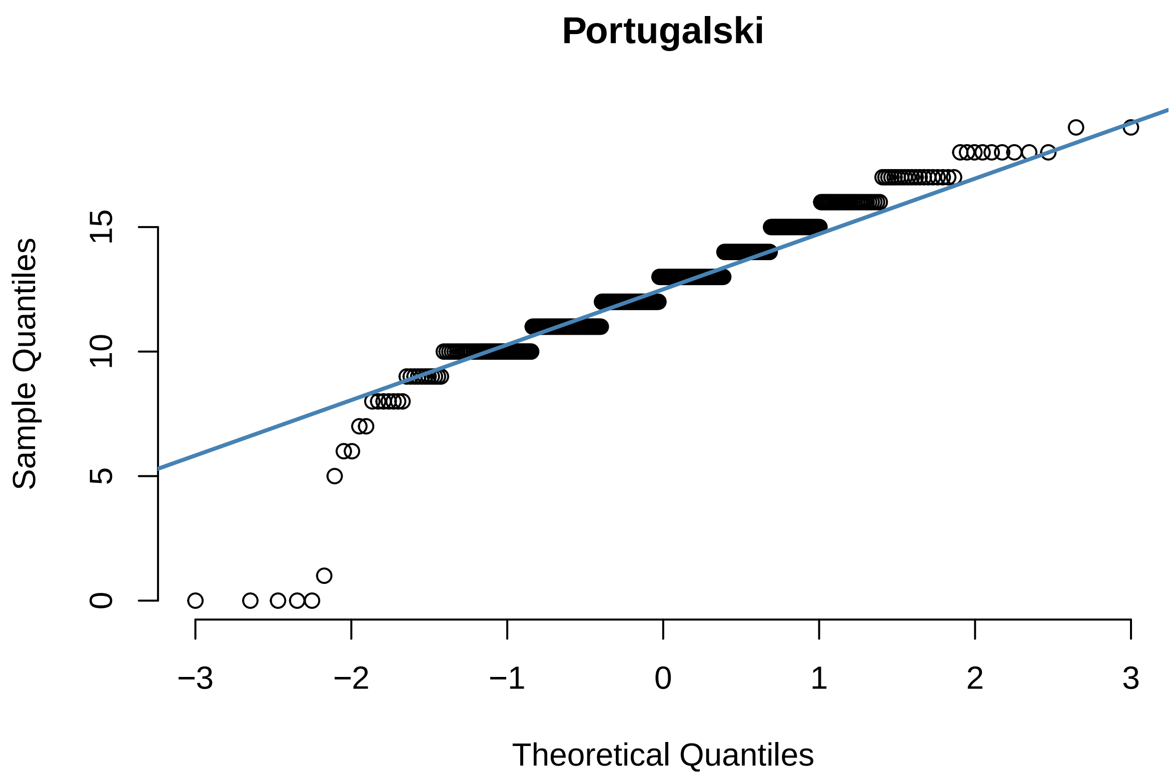

```r
hist(students_org$G3_por, breaks = seq(0, 20), main = "Histogram ocjena iz portugalskog",
    xlab = "Ocjena")
```



```r
qqnorm(students_org$G3_mat, pch = 1, frame = FALSE, main = "Matematika")
qqline(students_org$G3_mat, col = "steelblue", lwd = 2)
```




```r
qqnorm(students_org$G3_por, pch = 1, frame = FALSE, main = "Portugalski")
qqline(students_org$G3_por, col = "steelblue", lwd = 2)
```




```
var(students_org$G3_mat)
```

```
## [1] 21.24131
```

```
var(students_org$G3_por)
```

```
## [1] 8.665092
```

```
var.test(students_org$G3_mat, students_org$G3_por)
```

```
##
##  F test to compare two variances
##
## data:  students_org$G3_mat and students_org$G3_por
## F = 2.4514, num df = 369, denom df = 369, p-value < 2.2e-16
## alternative hypothesis: true ratio of variances is not equal to 1
## 95 percent confidence interval:
##  1.998239 3.007245
## sample estimates:
## ratio of variances
##           2.451366
```

Zbog jako male vrijednosti odbacujemo hipotezu H0 da su varijance dva uzorka jednake.

```
t.test(students_org$G3_por, students_org$G3_mat, alternative = "greater", var.equal = FALSE)
```

```
##
##  Welch Two Sample t-test
##
## data:  students_org$G3_por and students_org$G3_mat
## t = 7.3485, df = 627.1, p-value = 3.137e-13
## alternative hypothesis: true difference in means is greater than 0
```

```
## 95 percent confidence interval:
##  1.620861      Inf
## sample estimates:
## mean of x mean of y
##  12.55405  10.46486
```

Zbog jako male p-vrijednosti odbacujemo hipotezu H0 da su prosjeci ocjena jednaki u korist hipoteze H1 da je prosjek ocjena iz portugalskog značajno veći od prosjeka ocjena iz matematike.

Matej Ciglenečki

Kako vrijeme putovanja do škole utjeće na uspjeh učenika?

Na ovo pitanje odgovirit ćemo ANOVA-om. Pretpostavke ANOVA-e su:

- nezavisnost pojedinih podataka u uzorcima
- normalna razdioba podataka
- homogenost varijanci među populacijama

Postavljamo hipotezu H0 koja glasi, srednja vrijednost grupa su podjednake.

$$H_0 : \mu_1 = \mu_2 = \ldots = \mu_k$$

$$H_1 : \neg H_0$$

S obizrom da se radi o različitim školama i različitim predmetima možemo pretpostaviti nezavisnost ocjena.

Ukoliko nakon provedbe ANOVA-e odbacimo H0 hipotezu možemo zaključiti da su srednje vrijednosti međusobno različite, tj. da vrijeme putovanje utječe na uspjeh učenika.

Obrada kategoričkih stupaca

Kao grupe koristiti će se vrijednosti iz stupca `traveltime` Prvo je potrebno pretvoriti stupac `traveltime` u kategoričke podatke (s poretkom). `traveltime` se sastoji od 4 mogućih vrijednosti koje definiraju potrebno vrijeme od škole do doma:

- < 15min
- 15 - 30 min
- 30 - 60 min
- > 60 min

Nadalje, zadnju kategoriju (60min+) spojiti ćemo sa predzadnjom kategorijom (30-60min) zbog toga što se u zadnjoj kategoriji nalaze samo 8 podataka dok se u preostalim kategorijama nalazi puno veći broj podataka.

```
count(students, students$traveltime)
```

```
## # A tibble: 4 x 2
##   `students$traveltime`     n
##                   <dbl> <int>
## 1                     1   242
## 2                     2    99
## 3                     3    21
## 4                     4     8
```

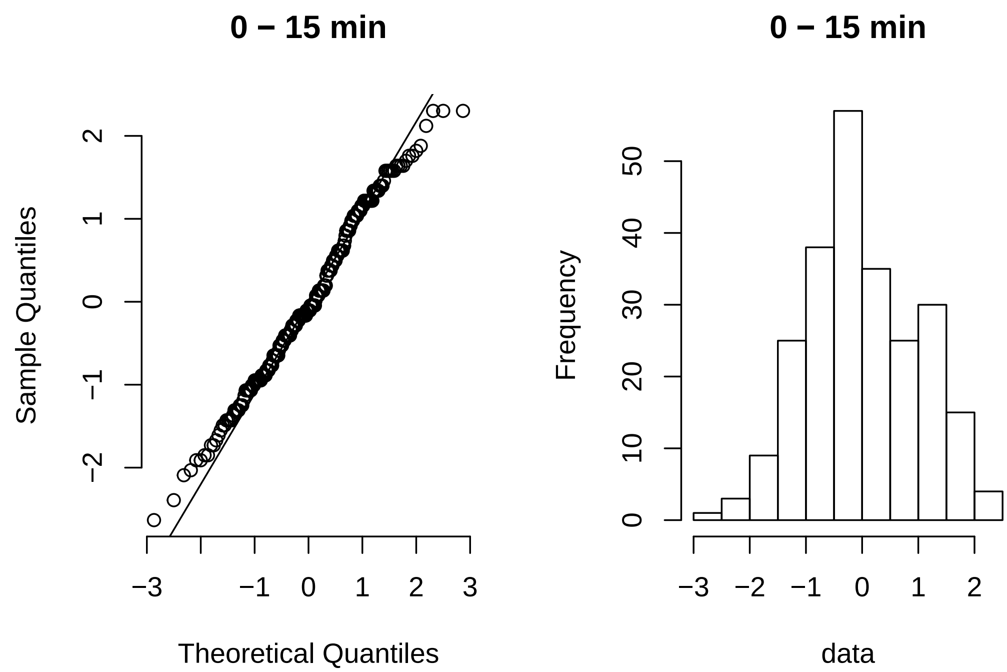
```
students <- students_clean
students$traveltime <- factor(students$traveltime, ordered = TRUE, labels = c("0 - 15 min",
    "15 - 30 min", "> 30 min", "> 30 min"))
```

Za uspjeh koristiti ćemo zboj varijabli `G[1,2,3]_mat` i `G[1,2,3]_por` koji ćemo spremitit u novu varijablu `G_total`.

```
students$G3_total <- students$G3_mat + students$G3_por
students$G2_total <- students$G2_mat + students$G2_por
students$G1_total <- students$G1_mat + students$G1_por
students$G_total <- students$G1_total + students$G2_total + students$G3_total
```

ANOVA je robustna na blaga odstupanja što se tiče normalnosti. Svejedno, testirati ćemo normalnost varijable `G_total` nad cijelim podatkovnim skupom, a zatim nad `G_total` za svaku pojedinu grupu `traveltime`-a.

```
model = lm(students$G_total ~ students$traveltime)

par(mfrow = c(1, 2))  # 2 plots in 1 row

timeperiod = "0 - 15 min"
data <- rstandard(model)[students$traveltime == timeperiod]
qqnorm(data, pch = 1, frame = FALSE, main = timeperiod)
qqline(data)
hist(data, main = timeperiod)
```




```
lillie.test(data)["p.value"]
```

```
## $p.value
## [1] 0.008983716
```

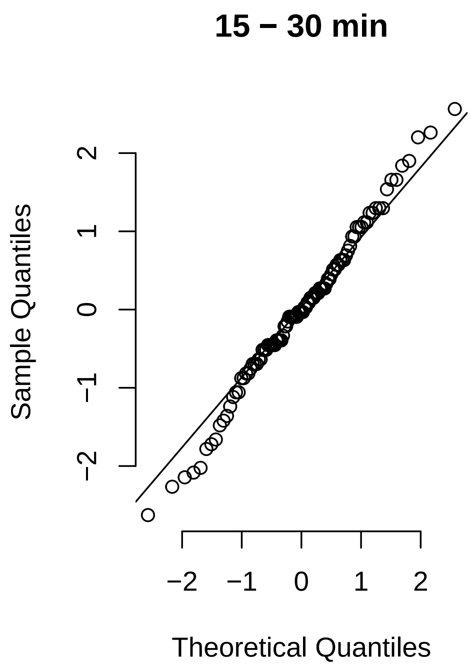
```
ks.test(data, "pnorm", mean = mean(data), sd = sd(data))["p.value"]
```

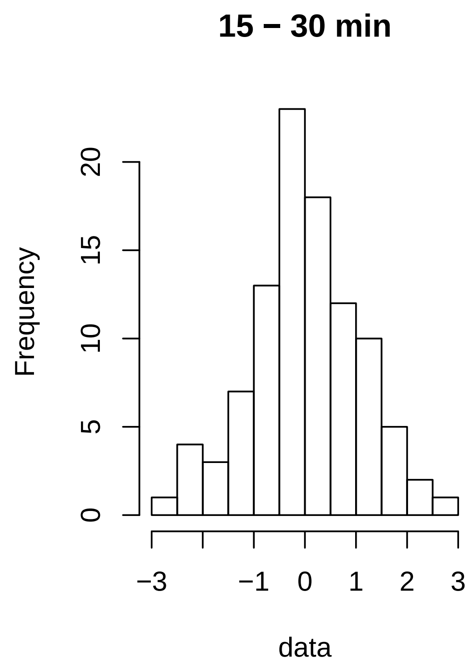
```
## Warning in ks.test(data, "pnorm", mean = mean(data), sd = sd(data)): ties should
## not be present for the Kolmogorov-Smirnov test
```

```
## $p.value
## [1] 0.2157153
```

```
timeperiod = "15 - 30 min"
data <- rstandard(model)[students$traveltime == timeperiod]
qqnorm(data, pch = 1, frame = FALSE, main = timeperiod)
qqline(data)
```

```r
hist(data, main = timeperiod)
```






```r
lillie.test(data)["p.value"]
```

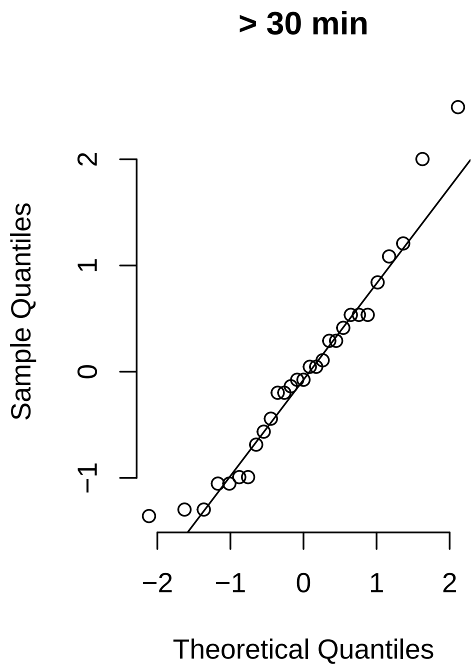
```
## $p.value
## [1] 0.5782076
```

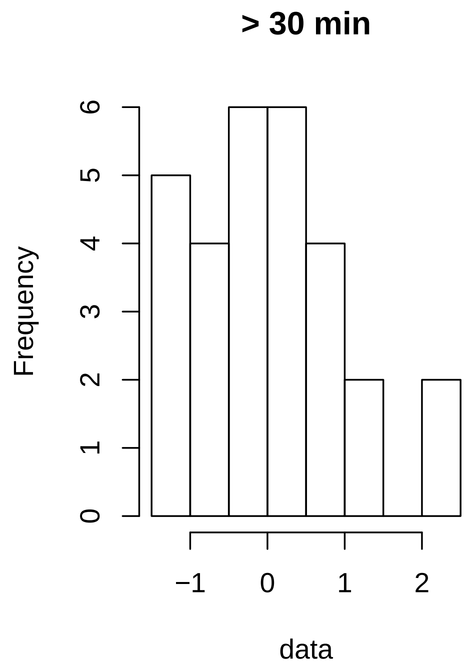
```r
ks.test(data, "pnorm", mean = mean(data), sd = sd(data))["p.value"]
```

```
## Warning in ks.test(data, "pnorm", mean = mean(data), sd = sd(data)): ties should
## not be present for the Kolmogorov-Smirnov test
```

```
## $p.value
## [1] 0.897279
```

```r
timeperiod = "> 30 min"
data <- rstandard(model)[students$traveltime == timeperiod]
qqnorm(data, pch = 1, frame = FALSE, main = timeperiod)
qqline(data)
hist(data, main = timeperiod)
```






```r
lillie.test(data)["p.value"]
```

```
## $p.value
## [1] 0.4329395
```

```r
ks.test(data, "pnorm", mean = mean(data), sd = sd(data))["p.value"]
```

```
## Warning in ks.test(data, "pnorm", mean = mean(data), sd = sd(data)): ties should
## not be present for the Kolmogorov-Smirnov test

## $p.value
## [1] 0.8440515
```

Na svakom grafu možemo vidjeti da podaci uglavnom prate normalnu distribuciju uz manji broj stršećih vrijednosti (lijevi rep). Nadalje, `p` vrijednosti Lillieforsovog testa nisu uvijek iznad 0.05 međutim za sve Kolmogorov–Smirnov testove `p` vrijednosti su iznad 0.05.

Lilliefors koristimo ako nam nije poznata varijanca i srednja vrijednost populacije, što je s ovim podacima i slučaj. Poznato je da Lilliefors konzervativniji i da odbacuje hipotezu H0 češće nego Kolmogorov–Smirnov.

S obzirom na manja odstupanja, ne toliko male `p` vrijednosti i grafički izgled `qqnorm`-a i histograma pretpostaviti ćemo da su podaci uzrokovani iz normalne distribucije.

Homogenost varijanci - Bartlettov test

Prvo je potrebno postaviti hipoteze H0 i H1:

$$H_0 : \sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2$$

$$H_1 : \neg H_0$$

```r
var(students$G_total[students$traveltime == "> 30 min"])
```

```
## [1] 241.6897
```

```r
var(students$G_total[students$traveltime == "15 - 30 min"])
```

```
## [1] 296.1703
```

```r
var(students$G_total[students$traveltime == "> 30 min"])
```

```
## [1] 241.6897
```

```r
bartlett.test(students$G_total ~ students$traveltime)
```

```
##
##  Bartlett test of homogeneity of variances
##
## data:  students$G_total by students$traveltime
## Bartlett's K-squared = 0.48546, df = 2, p-value = 0.7845
```

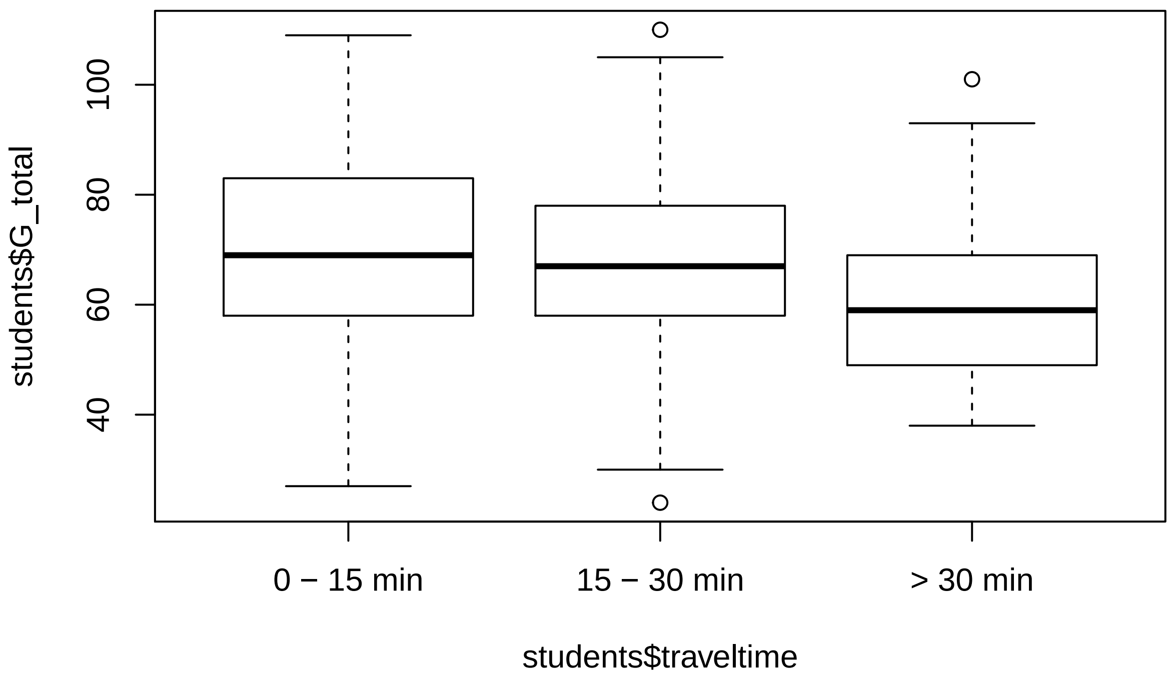
Vidimo da su vrijednosti varijance slične. S obzirom da je p vrijednost testa veća od 0.05 ne odbacujemo H0 čime zadovoljavamo ANOVA pretpostavku o homogenosti varijanca.

ANOVA - Jesu li srednje vrijednosti za različite grupe drugačije?

$$H_0 : \mu_1 = \mu_2 = \ldots = \mu_k$$

$$H_1 : \neg H_0$$

```r
boxplot(students$G_total ~ students$traveltime)
```



Grafički možemo pretpostaviti da se vrijeme putovanja utječe na uspjeh učenika. Naravno, ANOVA-om je potrebno provjeritit koliko je ta razlika statistički značajna.

```r
model = lm(students$G_total ~ students$traveltime)
anova(model)
```

```
## Analysis of Variance Table
##
## Response: students$G_total
##                      Df Sum Sq Mean Sq F value   Pr(>F)
## students$traveltime   2   3185 1592.35  5.7419 0.003504 **
## Residuals           367 101777  277.32
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

ANOVA nam govori da postoji razlika između grupa `traveltime`. Iako nije strogo značajna i dalje se radi o značajnoj `p` vrijednosti koja se nalazi između `0.001` i `0.01`. Možemo zaključiti da za različite grupe vremena putovanja imaju utjecaj na učenikov uspjeh.

Koja škola je bolja u matematici a koja u portugalskom?

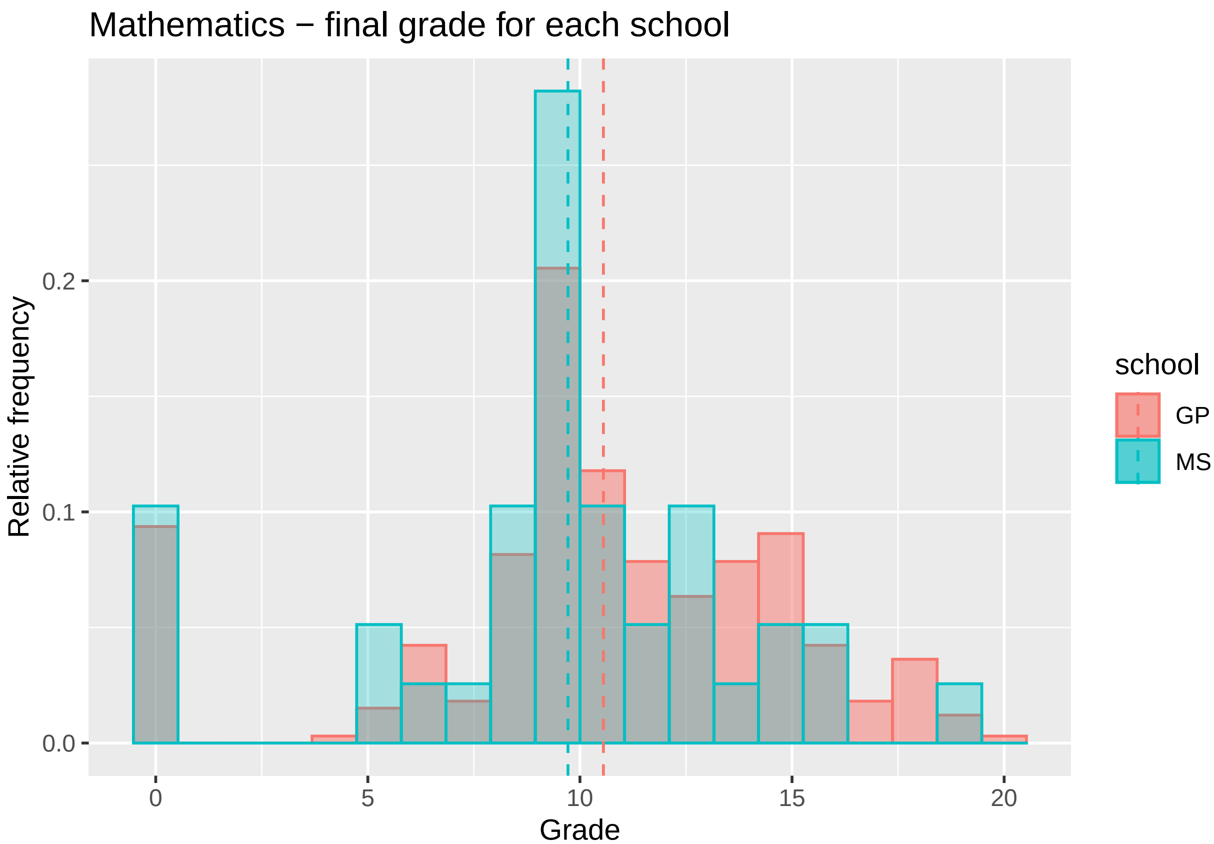
Na ovo pitanje odgovoriti ćemo provedbom t-testa koristeći 4 različita podatkovna skupa. Razdvajanje podatkovnog skupa na dvije škole (GP, MS) te na dva predmeta (matematika i portugalski) dobivamo sljedeće podatkovne skupove: `gp_mat`, `gp_por`, `ms_mat`, `ms_por`

```r
# Show average grade for all schools
schools <- students %>%
    select("school") %>%
    distinct(.)
schools  # [GP, MS]
subject_final_grade_names <- names(students)[grepl("G3", names(students))]

# all_of Note: Using an external vector in selections is ambiguous. Use
# `all_of(vars)` instead of `vars` to silence this message.
students_final_grade <- students %>%
    select("school", all_of(subject_final_grade_names))

# Select only the subject grade and school
gp_mat <- students_final_grade %>%
    filter(school == "GP") %>%
    select(G3_mat, school)
gp_por <- students_final_grade %>%
    filter(school == "GP") %>%
    select(G3_por, school)
ms_mat <- students_final_grade %>%
    filter(school == "MS") %>%
    select(G3_mat, school)
ms_por <- students_final_grade %>%
    filter(school == "MS") %>%
    select(G3_por, school)
```

Prikaz relativnih frekvencija predmeta i škola

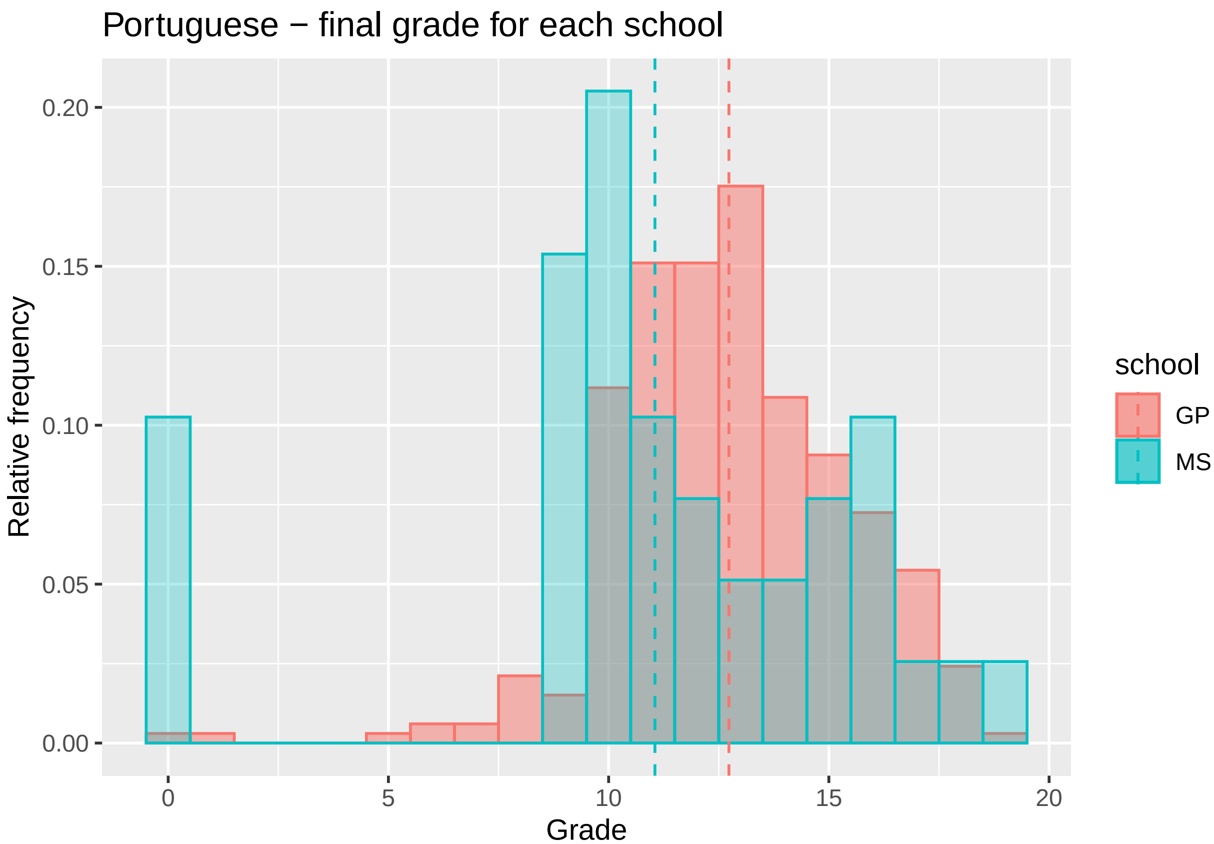
Iz grafa relativne frekvencije možemo usporediti vertikalne crte koje određuju srednju vrijednost ocjene za pojedinu školu i također dobiti osjećaj za normalnost podataka. Konstruirati ćemo jednosmjerni T-test a alternativa će ići u korist škole koja ima veću srednju vrijednost čime ćemo provjeriti je li ta škola statistički značajno bolja u matematici/portugalskom.

Matematika - prikaz relativnih frekvencija i srednjih vrijednosti



Na grafu za matematiku vidi se da škola `GP` ima veću srednju vrijednost od škole `MS`

Portugalski - prikaz relativnih frekvencija i srednjih vrijednosti



Na grafu za portugalski vidi se da škola `GP` ima veću srednju vrijednost od škole `MS`

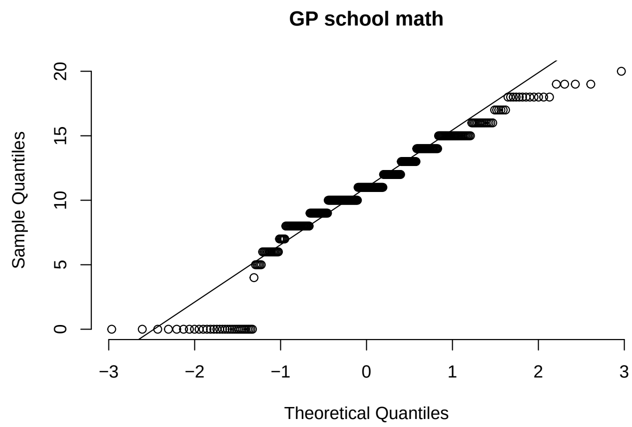
Provjera normalnosti

Normalnost se provjerva na više načina. U sljedećim koracima biti će prikazani `qqnorm` grafovi i provedeni Lilliefors i Kolmogorov-Smirnov testovi na temelju kojih će se pretpostaviti (ne)normalnost.

```
nrow(gp_mat)
nrow(gp_por)
nrow(ms_mat)
nrow(ms_por)
```

`n` - broj podataka za matematiku je `331` a za portugalski `39`

```
qqnorm(gp_mat$grade, pch = 1, frame = FALSE, main = "GP school math")
qqline(gp_mat$grade)
```




```
lillie.test(gp_mat$grade)["p.value"]
```

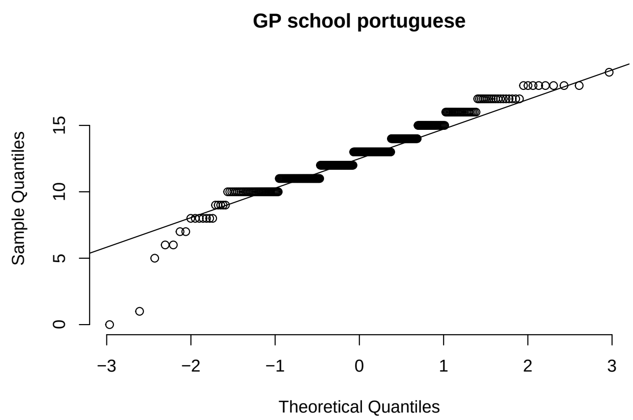
```
## $p.value
## [1] 7.814771e-14
```

```
ks.test(gp_mat$grade, "pnorm", mean(gp_mat$grade), sd(gp_mat$grade))["p.value"]
```

```
## Warning in ks.test(gp_mat$grade, "pnorm", mean(gp_mat$grade), sd(gp_mat$grade)):
## ties should not be present for the Kolmogorov-Smirnov test
```

```
## $p.value
## [1] 5.330255e-05
```

```
qqnorm(gp_por$grade, pch = 1, frame = FALSE, main = "GP school portuguese")
qqline(gp_por$grade)
```




```
lillie.test(gp_por$grade)["p.value"]
```

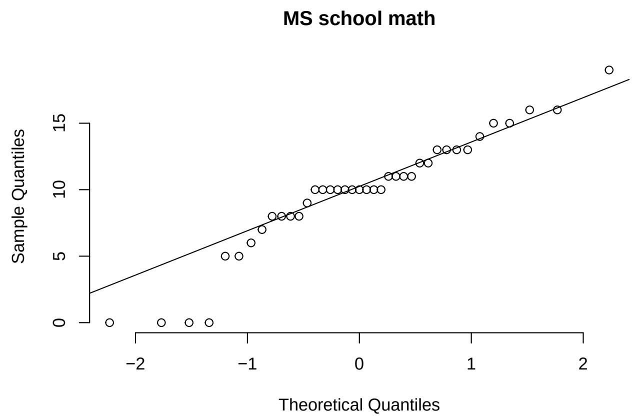
```
## $p.value
## [1] 1.673428e-09
```

```
ks.test(gp_por$grade, "pnorm", mean(gp_por$grade), sd(gp_por$grade))["p.value"]
```

```
## Warning in ks.test(gp_por$grade, "pnorm", mean(gp_por$grade), sd(gp_por$grade)):
## ties should not be present for the Kolmogorov-Smirnov test
```

```
## $p.value
## [1] 0.001247681
```

```
qqnorm(ms_mat$grade, pch = 1, frame = FALSE, main = "MS school math")
qqline(ms_mat$grade)
```




```
lillie.test(ms_mat$grade)["p.value"]
```

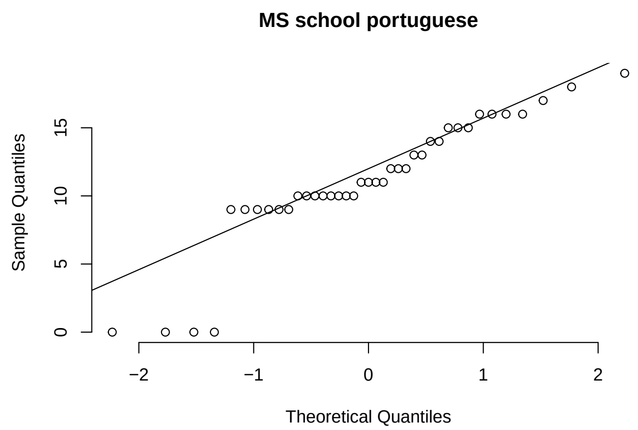
```
## $p.value
## [1] 0.0009170632
```

```
ks.test(ms_mat$grade, "pnorm", mean(ms_mat$grade), sd(ms_mat$grade))["p.value"]
```

```
## Warning in ks.test(ms_mat$grade, "pnorm", mean(ms_mat$grade), sd(ms_mat$grade)):
## ties should not be present for the Kolmogorov-Smirnov test
```

```
## $p.value
## [1] 0.1131777
```

```
qqnorm(ms_por$grade, pch = 1, frame = FALSE, main = "MS school portuguese")
qqline(ms_por$grade)
```




```
lillie.test(ms_por$grade)["p.value"]
```

```
## $p.value
## [1] 1.951046e-05
```

```
ks.test(ms_por$grade, "pnorm", mean(ms_por$grade), sd(ms_por$grade))["p.value"]
```

```
## Warning in ks.test(ms_por$grade, "pnorm", mean(ms_por$grade), sd(ms_por$grade)):
## ties should not be present for the Kolmogorov-Smirnov test
```

```
## $p.value
## [1] 0.03355273
```

Repovi su prisutni na lijevoj strani podataka zbog čega je `p` vrijednost skoro uvijek manja od 0.05 za Kolmogorov-Smirnov i Lillieforsov test. Grafički, na temelju rezultata određujemo da za sve skupove vrijedi da proizlaze iz normalne distribucije ali s opaskom da postoje stršeće vrijednosti na lijeovj strani distribucije.

F-test - test o jednakosti varijanca

Važno je napomenuti da je test o varijanci iznimno osjetljiv na normalnost. Test će biti proveden zbog vježbe ali njegov **rezultat se neće uzeti u obzir** jer podaci nisu normalno distribuirani.

$p -$ vjerojatnost da pod H0 dobijemo vrijednost koja je jednako ili više ekstremna nego vrijednost koji bi dobili izračunom iz uzorka kojeg imamo

Ako je $p < \alpha$, odbacujemo hipotezu $H0$ u korist hipoteze $H1$:

- pada u desni ili lijevi rep => odbacivanje

$$H_0 : \sigma_1^2 = \sigma_2^2$$
$$H_1 : \neg H_0$$

Poredak argumenata za `var.test` nije bitna ali generalno vrijedi:

$$\frac{\sigma_1^2}{\sigma_2^2}, \quad \sigma_1^2 > \sigma_2^2$$

```
cat("Mathematics variances", var(gp_mat$grade), var(ms_mat$grade))
```

```
## Mathematics variances 21.38735 19.89204
```

```
cat("Portugeuse variances", var(gp_por$grade), var(ms_por$grade))
```

```
## Portugeuse variances 6.839605 22.1552
```

Na prvi pogled čini se da će H0 hipoteza za portugalski biti odbačena zbog toga što su varijance značajno drugačije. Potrebno je provesti f-test da se uvjerimo da se radi o statistički značajnoj razlici varijanci.

Konstruirajmo i provedimo testove o varijanci:

```
alpha <- 0.05

# H0 - Variance of GP_MAT and MS_MAT are equal H1 - not H0
mat_f_test <- var.test(gp_mat$grade, ms_mat$grade, alternative = "two.sided")  # F = 1.0752, p = 0.817

# H0 - Variance of GP_POR and MS_MAT are equal H1 - not H0
por_f_test <- var.test(gp_por$grade, ms_por$grade, alternative = "two.sided")  # F = 0.30871, p = 1.217
```

```
var_equal_mat <- if (mat_f_test$p.value < alpha) FALSE else TRUE
cat_reject_h0("Matematika - test o jednakosti varijanca:", !var_equal_mat)
```

```
## Matematika - test o jednakosti varijanca:
##  Ne odbacujemo hipotezu H0
```

```
var_equal_por <- if (por_f_test$p.value < alpha) FALSE else TRUE
cat_reject_h0("Portugalski - test o jednakosti varijanca:", !var_equal_por)
```

```
## Portugalski - test o jednakosti varijanca:
##  Odbacujemo hipotezu H0 u korist hipoteze H1
```

T-test - testiranje jednakosti srednje vrijednosti ocjena za dvije škole uz nepoznate varijance

Uz to što je `n` veći od 30 za oba podatkovna skupa i uz činjenicu da je t-test robustan na (ne)normalnost provodimo t-test srednje vrijednosti za oba predmeta.

Zbog prethodno dobivenih srednje vrijednosti o ocjenama (koje idu u korist škole `GP`) postavljena je jednosmjerna alternativa hipoteza.

Ponovno, zbog toga što test o varijanci nije robustan na nenormalnost pretpostaviti ćemo da vraijance uzoraka nisu jednake.

```
# H0 - GP school has equal grades to in mathematics to MS (GP=MS) H1 - GP>MS
mat_t_test <- t.test(gp_mat$grade, ms_mat$grade, alt = "greater", var.equal = FALSE)
is_gp_mat_better <- if (mat_t_test$p.value < alpha) TRUE else FALSE
cat_reject_h0("Matematika - t-test:", is_gp_mat_better)
## Matematika - t-test:
##  Ne odbacujemo hipotezu H0

# H0 - GP school has equal grades to in Portuguese to MS (GP=MS) H1 - GP>MS
por_t_test <- t.test(gp_por$grade, ms_por$grade, alt = "greater", var.equal = FALSE)
is_gp_por_better <- if (por_t_test$p.value < alpha) TRUE else FALSE
cat_reject_h0("Portugalski t-test:", is_gp_por_better)
## Portugalski t-test:
##  Odbacujemo hipotezu H0 u korist hipoteze H1
```

Za matematiku, nismo odbacili hipotezu H0 i zbog čega ne možemo zaključiti da škola GP ima bolje ocjene iz matematike od škole MS.

Za portugalski, odbacujemo hipotezu H0 u korist hipoteze H1 i zaključujemo da je škola `GP` ima bolje ocjene iz portgualskog od škole `MS`.

#Predviđanje uspjeha na kraju školske godine drugim varijablama iz skupa podataka

Transformirajmo kategoričke varijable u dummy varijable.

```
require(fastDummies)
students_dummies = dummy_cols(students, remove_first_dummy = TRUE, remove_selected_columns = TRUE)

students_dummies
## # A tibble: 370 x 52
##      age  Medu  Fedu traveltime  studytime failures_mat failures_por famrel
##    <dbl> <dbl> <dbl> <ord>           <dbl>        <dbl>        <dbl>  <dbl>
##  1    18     4     4 15 - 30 min         2            0            0      4
##  2    17     1     1 0 - 15 min          2            0            0      5
##  3    15     1     1 0 - 15 min          2            3            0      4
##  4    15     4     2 0 - 15 min          3            0            0      3
##  5    16     3     3 0 - 15 min          2            0            0      4
##  6    16     4     3 0 - 15 min          2            0            0      5
##  7    16     2     2 0 - 15 min          2            0            0      4
##  8    17     4     4 15 - 30 min         2            0            0      4
##  9    15     3     2 0 - 15 min          2            0            0      4
## 10    15     3     4 0 - 15 min          2            0            0      5
## # ... with 360 more rows, and 44 more variables: freetime <dbl>, goout <dbl>,
## #   Dalc <dbl>, Walc <dbl>, health <dbl>, absences_mat <dbl>,
## #   absences_por <dbl>, G1_mat <dbl>, G2_mat <dbl>, G3_mat <dbl>, G1_por <dbl>,
## #   G2_por <dbl>, G3_por <dbl>, G3_total <dbl>, G2_total <dbl>, G1_total <dbl>,
```

```
## #   G_total <dbl>, school_MS <int>, sex_M <int>, address_U <int>,
## #   famsize_LE3 <int>, Pstatus_T <int>, Mjob_health <int>, Mjob_other <int>,
## #   Mjob_services <int>, Mjob_teacher <int>, Fjob_health <int>, ...
```

Sada provodimo individualne jednostavne linearne regresije G3_mat i G3_por ovisno o svakoj od varijabli iz skupa, te spremamo R^2 vrijednosti i p-vrijednosti F-testova za jednostavnu linearnu regresiju u tablice modelsMat i modelsPor

```
varName = c()
rSquaredM = c()
pValueofFM = c()
rSquaredP = c()
pValueofFP = c()

for (i in 1:ncol(students_dummies)) {
    if (i != 18 && i != 21) {
        colName = colnames(students_dummies)[i]
        names(students_dummies)[i] = "tempx"

        modelMat = lm(formula = G3_mat ~ tempx, data = students_dummies)
        modelPor = lm(formula = G3_por ~ tempx, data = students_dummies)
        names(students_dummies)[i] = colName
        m = summary(modelMat)
        p = summary(modelPor)
        varName = append(varName, colName)
        rSquaredM = append(rSquaredM, m$r.squared)
        pValueofFM = append(pValueofFM, pf(m$fstatistic[1], m$fstatistic[2], m$fstatistic[3],
            lower.tail = FALSE))
        rSquaredP = append(rSquaredP, p$r.squared)
        pValueofFP = append(pValueofFP, pf(p$fstatistic[1], p$fstatistic[2], p$fstatistic[3],
            lower.tail = FALSE))
    }

    modelsMat = data.frame(varName, rSquaredM, pValueofFM)
    modelsPor = data.frame(varName, rSquaredP, pValueofFP)
}
```

##Predviđanje konačne ocjene iz matematike Pogledajmo koje su se varijable ispostavile najboljim prediktorima za G3_mat (poredano po R^2 vrijednostima)

```
modelsMat[order(-modelsMat$rSquaredM), ]
##              varName    rSquaredM    pValueofFM
## 20          G3_total 8.485015e-01 7.068668e-153
## 17            G2_mat 8.220203e-01 5.360474e-140
## 23           G_total 7.768376e-01 6.599137e-122
## 21          G2_total 7.361574e-01 1.630294e-108
## 16            G1_mat 6.482165e-01  1.686172e-85
## 22          G1_total 5.791317e-01  3.796506e-71
## 19            G2_por 3.036584e-01  8.998815e-31
## 18            G1_por 2.620521e-01  4.197705e-26
## 6       failures_mat 1.392004e-01  1.153185e-13
## 48        higher_yes 5.127693e-02  1.091204e-05
## 2               Medu 4.442857e-02  4.374826e-05
## 1                age 3.341011e-02  4.095595e-04
## 15      absences_por 2.982017e-02  8.514064e-04
```

```
## 3               Fedu 2.468901e-02  2.437230e-03
## 50       romantic_yes 2.212492e-02  4.138021e-03
## 4          traveltime 1.763917e-02  3.817115e-02
## 25              sex_M 1.724362e-02  1.146151e-02
## 41     guardian_other 1.572560e-02  1.579910e-02
## 10              goout 1.524576e-02  1.749543e-02
## 29        Mjob_health 1.177072e-02  3.697833e-02
## 39 reason_reputation 1.117669e-02  4.211523e-02
## 7        failures_por 1.110634e-02  4.277112e-02
## 26          address_U 1.093968e-02  4.436825e-02
## 27        famsize_LE3 1.082667e-02  4.548664e-02
## 31      Mjob_services 9.438170e-03  6.192712e-02
## 36       Fjob_teacher 9.040613e-03  6.771572e-02
## 30         Mjob_other 8.462013e-03  7.719322e-02
## 44       paid_mat_yes 8.230946e-03  8.136636e-02
## 42      schoolsup_yes 7.140813e-03  1.046198e-01
## 49       internet_yes 6.747007e-03  1.147224e-01
## 5           studytime 5.772488e-03  1.446729e-01
## 32       Mjob_teacher 3.525534e-03  2.545926e-01
## 13             health 3.454390e-03  2.594503e-01
## 47        nursery_yes 3.256085e-03  2.736086e-01
## 24          school_MS 3.102947e-03  2.852075e-01
## 34         Fjob_other 2.863524e-03  3.046207e-01
## 28          Pstatus_T 2.595037e-03  3.284709e-01
## 33        Fjob_health 2.576309e-03  3.302248e-01
## 43         famsup_yes 2.517837e-03  3.357810e-01
## 8              famrel 1.991185e-03  3.920770e-01
## 38       reason_other 1.856206e-03  4.086276e-01
## 11               Dalc 1.774706e-03  4.191188e-01
## 12               Walc 1.350276e-03  4.810145e-01
## 46     activities_yes 1.260180e-03  4.960353e-01
## 40    guardian_mother 7.775170e-04  5.928930e-01
## 37        reason_home 6.119266e-04  6.352923e-01
## 14       absences_mat 3.779776e-04  7.093439e-01
## 9            freetime 4.639719e-05  8.961068e-01
## 35      Fjob_services 4.089570e-05  9.024280e-01
## 45       paid_por_yes 3.761621e-05  9.064032e-01
```

Razmotrit ćemo prvih 10 najboljih prediktora. Najprije provjerimo jesu li neke od tih varijabli visoko korelirane:

```
cor(cbind(students_dummies$G2_mat, students_dummies$G1_mat, students_dummies$G2_por,
    students_dummies$G1_por, students_dummies$failures_mat, students_dummies$higher_yes,
    students_dummies$Medu, students_dummies$age, students_dummies$absences_por, students_dummies$Fedu))
##             [,1]       [,2]        [,3]       [,4]       [,5]       [,6]
##  [1,]  1.0000000  0.8567705  0.57804548  0.5382023 -0.3686201  0.2136061
##  [2,]  0.8567705  1.0000000  0.59826670  0.5810192 -0.3863053  0.2238001
##  [3,]  0.5780455  0.5982667  1.00000000  0.8874806 -0.3508856  0.2985681
##  [4,]  0.5382023  0.5810192  0.88748063  1.0000000 -0.2912844  0.2771680
##  [5,] -0.3686201 -0.3863053 -0.35088560 -0.2912844  1.0000000 -0.3659806
##  [6,]  0.2136061  0.2238001  0.29856805  0.2771680 -0.3659806  1.0000000
##  [7,]  0.2129388  0.2220518  0.21506989  0.2009143 -0.2231552  0.1571828
##  [8,] -0.1654850 -0.1042716 -0.03497507 -0.1023385  0.1476343 -0.2455488
##  [9,] -0.1800028 -0.1502991 -0.18243277 -0.1472927  0.1689433 -0.1154271
```

```
## [10,]  0.1681930  0.2053066  0.17844395  0.1487680 -0.2259357  0.1703762
##              [,7]        [,8]        [,9]       [,10]
##  [1,]  0.21293884 -0.16548500 -0.18000282  0.16819303
##  [2,]  0.22205178 -0.10427157 -0.15029911  0.20530662
##  [3,]  0.21506989 -0.03497507 -0.18243277  0.17844395
##  [4,]  0.20091426 -0.10233849 -0.14729266  0.14876797
##  [5,] -0.22315525  0.14763426  0.16894333 -0.22593571
##  [6,]  0.15718279 -0.24554880 -0.11542706  0.17037620
##  [7,]  1.00000000 -0.11524237  0.02165622  0.63603508
##  [8,] -0.11524237  1.00000000  0.09332766 -0.12381596
##  [9,]  0.02165622  0.09332766  1.00000000  0.02517965
## [10,]  0.63603508 -0.12381596  0.02517965  1.00000000
```

Kao i očekivano ocjene G2_mat i G1_mat visoko su korelirane, isto kao i G2_por i G1_por, a značajna je i korelacija između ocjena matematike i portugala. Osim toga uočimo koreliranost razina edukacije majke i oca.

```
cor(students_dummies$Medu, students_dummies$Fedu)
## [1] 0.6360351
```

Zasad nećemo eliminirati nijedan regresor. Izgradimo linearni model od gore izdvojenih varijabli za G3_mat:

```
multiMat = lm(G3_mat ~ G2_mat + G1_mat + G2_por + G1_por + failures_mat + higher_yes +
    Medu + age + absences_por + Fedu, data = students_dummies)
summary(multiMat)
##
## Call:
## lm(formula = G3_mat ~ G2_mat + G1_mat + G2_por + G1_por + failures_mat +
##     higher_yes + Medu + age + absences_por + Fedu, data = students_dummies)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -9.3643 -0.3881  0.2843  0.9220  3.3666
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   0.40447    1.77472   0.228   0.8199
## G2_mat        0.95943    0.05325  18.018   <2e-16 ***
## G1_mat        0.13853    0.06182   2.241   0.0257 *
## G2_por        0.07276    0.09549   0.762   0.4466
## G1_por       -0.04011    0.08842  -0.454   0.6504
## failures_mat -0.17299    0.16603  -1.042   0.2981
## higher_yes    0.26655    0.55720   0.478   0.6327
## Medu          0.08720    0.12252   0.712   0.4771
## age          -0.14087    0.09154  -1.539   0.1247
## absences_por -0.00179    0.02163  -0.083   0.9341
## Fedu         -0.10921    0.12210  -0.894   0.3717
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.936 on 359 degrees of freedom
## Multiple R-squared:  0.8283, Adjusted R-squared:  0.8235
## F-statistic: 173.1 on 10 and 359 DF,  p-value: < 2.2e-16
```

Pojednostavimo sad model, uzevši 5 varijabli s najnižim p-vrijednostima

```r
multiMat2 = lm(data = students_dummies, G3_mat ~ G2_mat + G1_mat + age + failures_mat +
    G2_por)
summary(multiMat2)
##
## Call:
## lm(formula = G3_mat ~ G2_mat + G1_mat + age + failures_mat +
##     G2_por, data = students_dummies)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -9.3354 -0.3486  0.2398  0.9426  3.3867
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   0.53966    1.53698   0.351   0.7257
## G2_mat        0.96419    0.05249  18.369   <2e-16 ***
## G1_mat        0.13056    0.06045   2.160   0.0314 *
## age          -0.14085    0.08711  -1.617   0.1068
## failures_mat -0.19456    0.15636  -1.244   0.2142
## G2_por        0.04284    0.05217   0.821   0.4121
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.926 on 364 degrees of freedom
## Multiple R-squared:  0.8277, Adjusted R-squared:  0.8253
## F-statistic: 349.6 on 5 and 364 DF,  p-value: < 2.2e-16
```

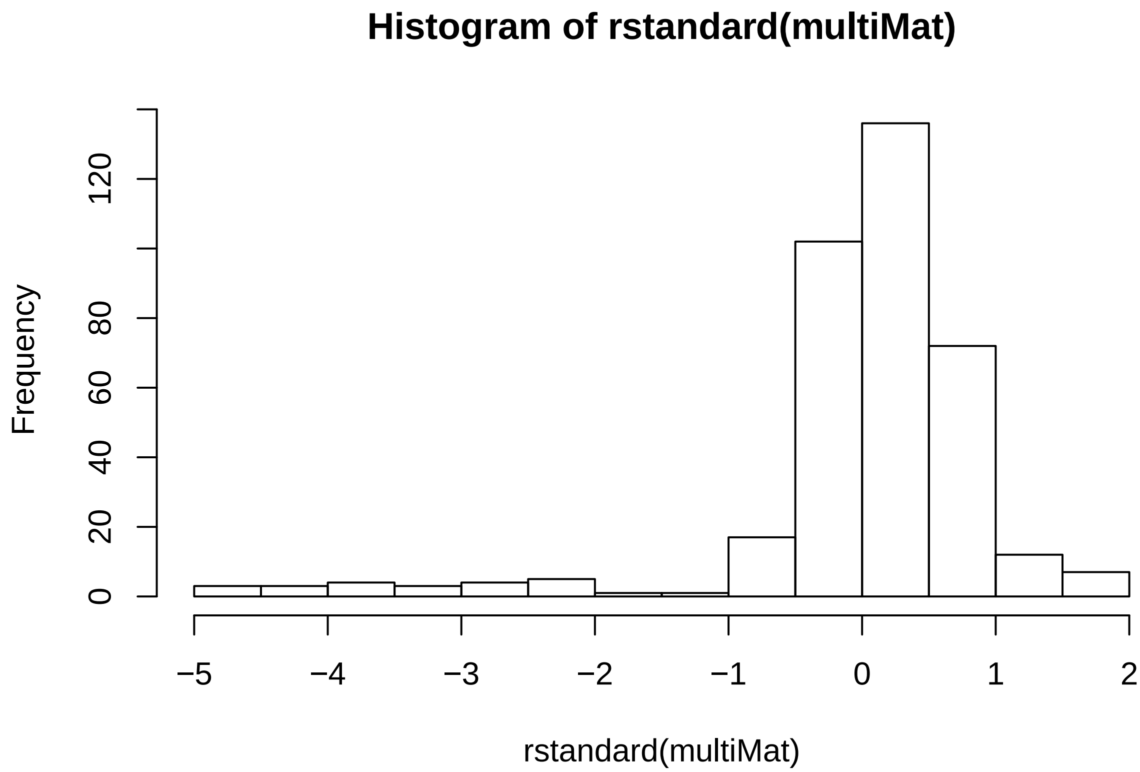
Nešto nam se smanjio R^2, no prilagođeni R^2 se uvećao-indikacija da smo eliminirali neke nepotrebne regresore.

Dodatno pojednostavljenje modela smanjuje prilagođeni R^2

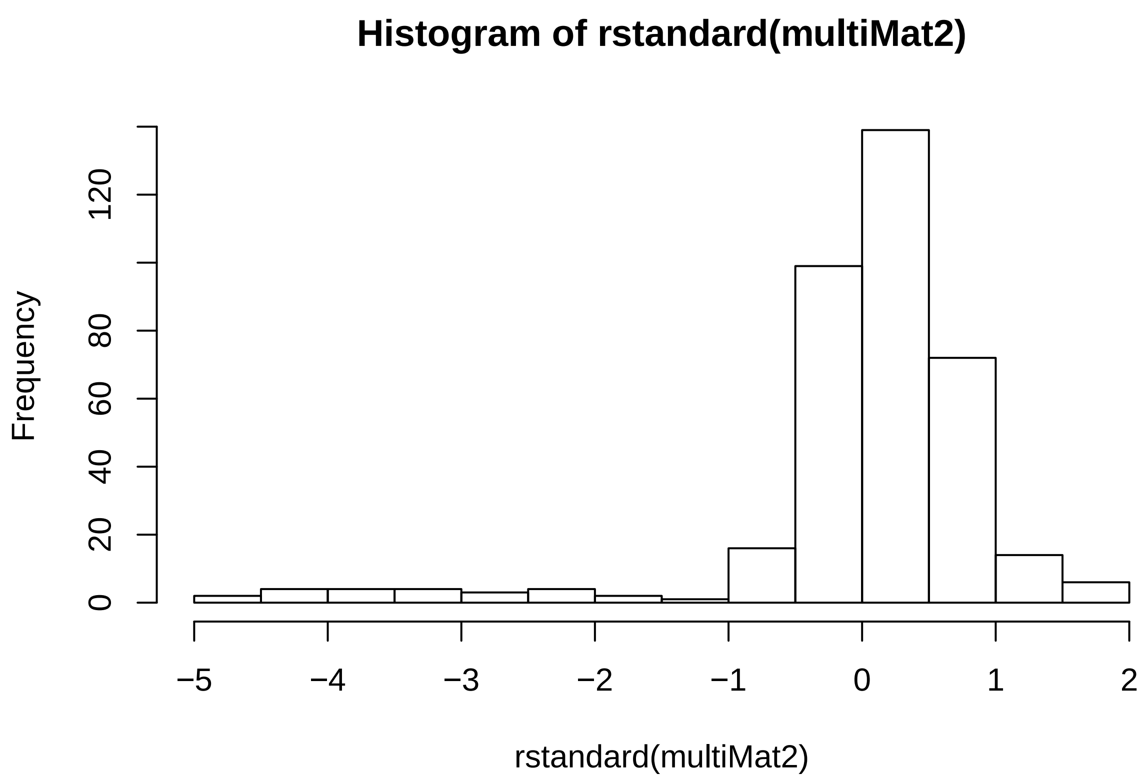
```r
multiMat3 = lm(data = students_dummies, G3_mat ~ G2_mat + G1_mat + age)
summary(multiMat3)
##
## Call:
## lm(formula = G3_mat ~ G2_mat + G1_mat + age, data = students_dummies)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -9.3833 -0.3680  0.2611  0.9880  3.4312
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.75584    1.50771   0.501   0.6164
## G2_mat       0.97551    0.05178  18.841   <2e-16 ***
## G1_mat       0.15411    0.05833   2.642   0.0086 **
## age         -0.14827    0.08643  -1.715   0.0871 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.928 on 366 degrees of freedom
## Multiple R-squared:  0.8264, Adjusted R-squared:  0.825
## F-statistic: 580.9 on 3 and 366 DF,  p-value: < 2.2e-16
```

Provjerimo normalnost reziduala

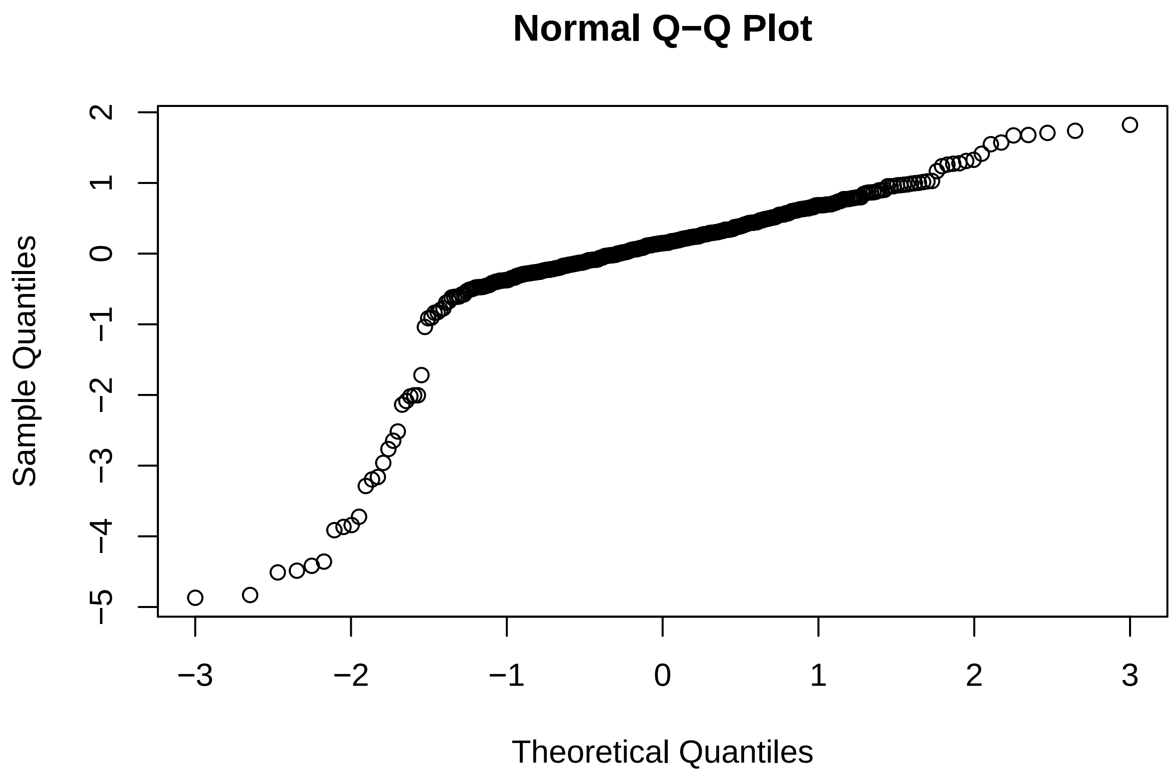
```
hist(rstandard(multiMat))
```



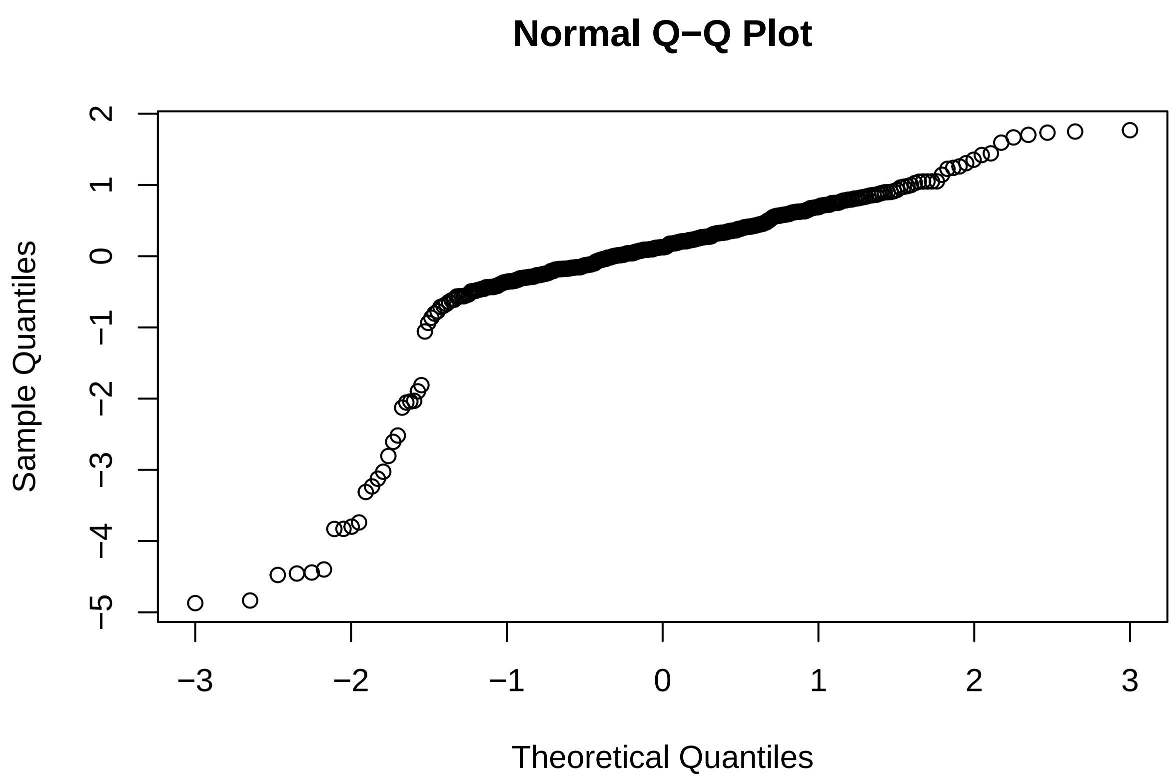

```
hist(rstandard(multiMat2))
```



```
qqnorm(rstandard(multiMat))
```



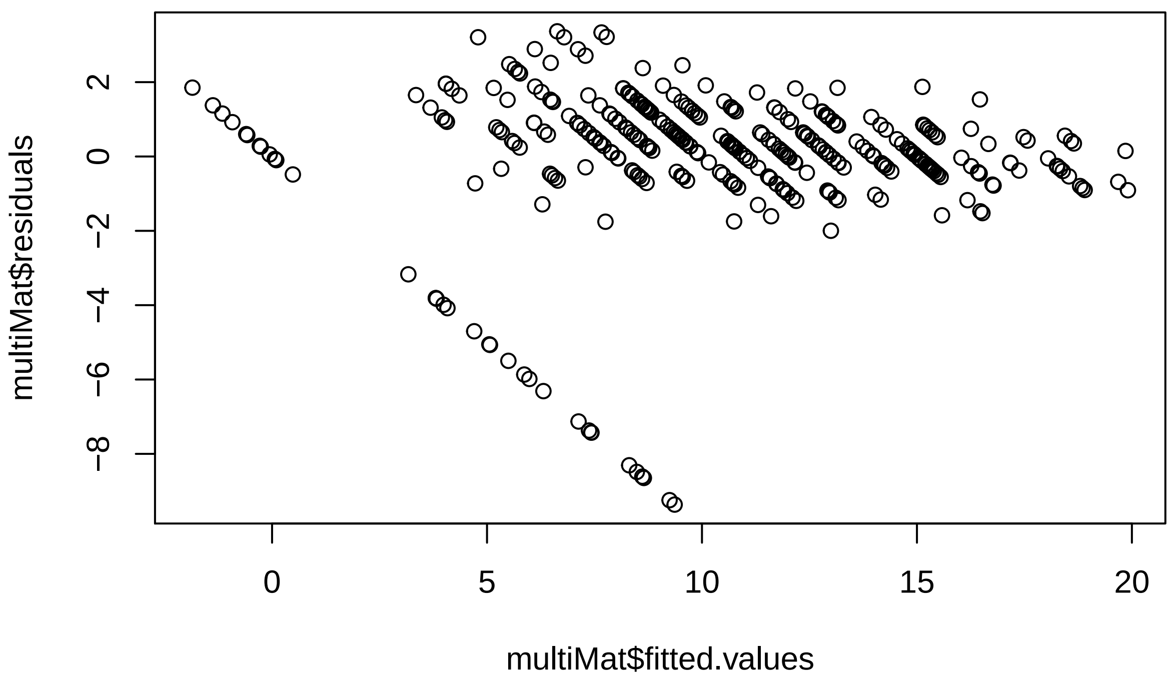

```
qqnorm(rstandard(multiMat2))
```



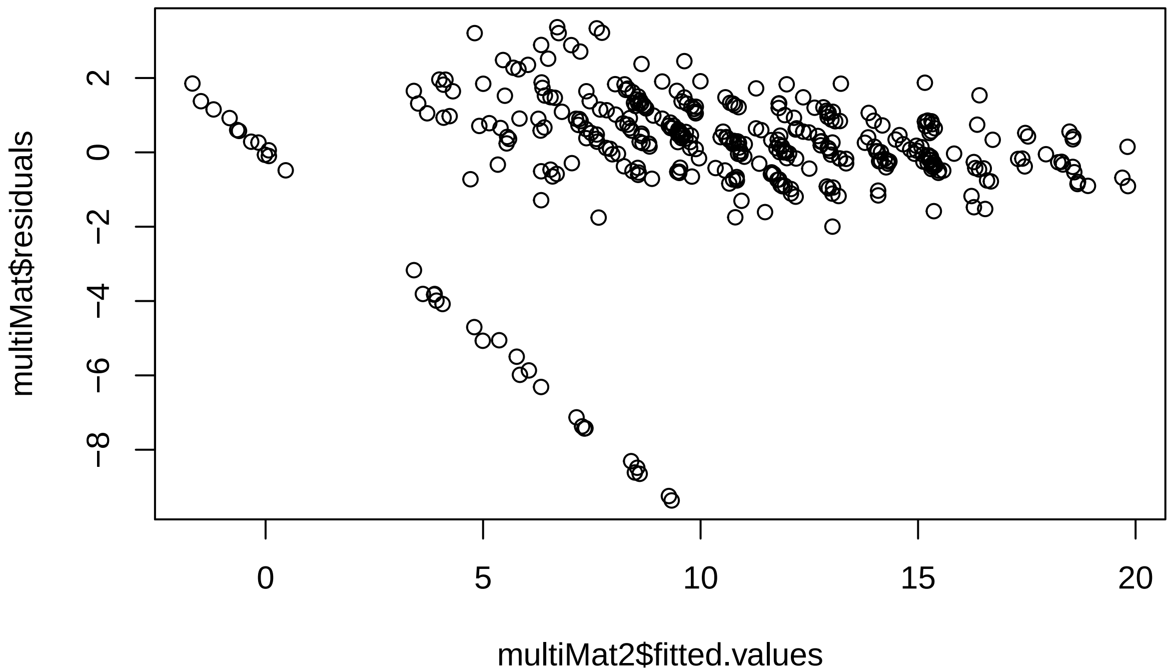
```r
ks.test(rstandard(multiMat), "pnorm")
##
##  One-sample Kolmogorov-Smirnov test
##
## data:  rstandard(multiMat)
## D = 0.20494, p-value = 6.362e-14
## alternative hypothesis: two-sided
ks.test(rstandard(multiMat2), "pnorm")
## Warning in ks.test(rstandard(multiMat2), "pnorm"): ties should not be present
## for the Kolmogorov-Smirnov test
##
##  One-sample Kolmogorov-Smirnov test
##
## data:  rstandard(multiMat2)
## D = 0.204, p-value = 8.438e-14
## alternative hypothesis: two-sided
```

Reziduali ne nalikuju normalnoj distribuciji. Promotrimo ih u ovisnosti o predviđenoj vrijednosti.

```r
plot(multiMat$fitted.values, multiMat$residuals)
```




```r
plot(multiMat2$fitted.values, multiMat$residuals)
```



Zanimljivo je još i pogledati koliko dobro možemo predvidjeti konačnu ocjenu iz matematike bez ikakvog znanja o drugim ocjenama, oslanjajući se na ostalih 6 od 10 najboljih prediktora

```
bezOcjenaMat = lm(data = students_dummies, G3_mat ~ failures_mat + higher_yes + Medu +
    age + absences_por + Fedu)
summary(bezOcjenaMat)
##
## Call:
## lm(formula = G3_mat ~ failures_mat + higher_yes + Medu + age +
##     absences_por + Fedu, data = students_dummies)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -12.2296  -2.1146   0.2624   2.8734  11.1486
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  14.99438    3.67744   4.077 5.60e-05 ***
## failures_mat -1.87569    0.34079  -5.504 7.03e-08 ***
## higher_yes    1.46021    1.18270   1.235   0.2178
## Medu          0.58345    0.26284   2.220   0.0270 *
## age          -0.39114    0.19212  -2.036   0.0425 *
## absences_por -0.10462    0.04618  -2.265   0.0241 *
## Fedu         -0.06830    0.26292  -0.260   0.7952
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.192 on 363 degrees of freedom
## Multiple R-squared:  0.1861, Adjusted R-squared:  0.1727
## F-statistic: 13.84 on 6 and 363 DF,  p-value: 3.545e-14
```

Ovakav model objašnjava svega 18% varijance u promatranoj varijabli. Kao i očekivano same ocjene najbolji su prediktor konačne ocjene, ali i neke druge varijable nisu potpuno irelevantne.

##Predviđanje konačne ocjene iz portugala Pogledajmo koje su se varijable ispostavile najboljim prediktorima za G3_por (poredano po R^2 vrijednostima)

```
modelsPor[order(-modelsPor$rSquaredP), ]
##              varName    rSquaredP    pValueofFP
## 19            G2_por 0.7915470296 2.326282e-127
## 18            G1_por 0.6689900683  2.270811e-90
## 20          G3_total 0.6286216777  3.673507e-81
## 23           G_total 0.6244206858  2.920018e-80
## 22          G1_total 0.5674100983  6.010847e-69
## 21          G2_total 0.5496273344  1.011539e-65
## 16            G1_mat 0.3130606997  7.266864e-32
## 17            G2_mat 0.2654028560  1.805665e-26
## 6       failures_mat 0.1364018388  2.116052e-13
## 7       failures_por 0.1159509571  1.697080e-11
## 48        higher_yes 0.0940955134  1.676684e-09
## 11              Dalc 0.0775925031  5.088939e-08
## 5          studytime 0.0743472469  9.907920e-08
## 12              Walc 0.0568741710  3.502791e-06
## 25             sex_M 0.0410010252  8.765489e-05
## 2               Medu 0.0405516662  9.601895e-05
## 26         address_U 0.0399328014  1.088617e-04
## 24         school_MS 0.0307910847  6.983446e-04
## 13            health 0.0307821478  6.996189e-04
## 3               Fedu 0.0272816595  1.431045e-03
## 4         traveltime 0.0250405704  9.528638e-03
## 30        Mjob_other 0.0218326465  4.396327e-03
## 42     schoolsup_yes 0.0200072403  6.424402e-03
## 10             goout 0.0182977031  9.183912e-03
## 45      paid_por_yes 0.0182200648  9.334687e-03
## 38      reason_other 0.0160759079  1.466799e-02
## 39 reason_reputation 0.0154737362  1.666733e-02
## 32      Mjob_teacher 0.0152059363  1.764436e-02
## 15      absences_por 0.0143263204  2.128856e-02
## 29       Mjob_health 0.0132736154  2.669036e-02
## 46    activities_yes 0.0130844904  2.780209e-02
## 9           freetime 0.0113386701  4.064452e-02
## 14      absences_mat 0.0111220559  4.262370e-02
## 43        famsup_yes 0.0090146763  6.811285e-02
## 44      paid_mat_yes 0.0080125876  8.553334e-02
## 49      internet_yes 0.0068851590  1.110615e-01
## 36      Fjob_teacher 0.0067022942  1.159355e-01
## 28         Pstatus_T 0.0057079539  1.469445e-01
## 47       nursery_yes 0.0054854718  1.550896e-01
## 1                age 0.0038926213  2.312215e-01
## 34        Fjob_other 0.0027946130  3.105208e-01
## 41    guardian_other 0.0016055476  4.422206e-01
## 50      romantic_yes 0.0015392250  4.518122e-01
## 35     Fjob_services 0.0015298339  4.531965e-01
## 8             famrel 0.0014968789  4.581070e-01
## 27       famsize_LE3 0.0012632977  4.955024e-01
## 37       reason_home 0.0009382419  5.569766e-01
## 40   guardian_mother 0.0008942898  5.663672e-01
## 33       Fjob_health 0.0006006413  6.384293e-01
```

```
## 31     Mjob_services 0.0001345507  8.240217e-01
```

Razmotrit ćemo prvih 13 najboljih prediktora. Najprije provjerimo jesu li neke od tih varijabli visoko korelirane:

```
cor(cbind(students_dummies$G2_por, students_dummies$G1_por, students_dummies$G1_mat,
    students_dummies$G2_mat, students_dummies$failures_mat, students_dummies$failures_por,
    students_dummies$higher_yes, students_dummies$Dalc, students_dummies$studytime,
    students_dummies$Walc, students_dummies$sex_M, students_dummies$Medu, students_dummies$address_U))
##             [,1]       [,2]        [,3]        [,4]        [,5]       [,6]
##  [1,]  1.0000000  0.8874806  0.59826670  0.57804548 -0.35088560 -0.2967996
##  [2,]  0.8874806  1.0000000  0.58101916  0.53820225 -0.29128438 -0.2897208
##  [3,]  0.5982667  0.5810192  1.00000000  0.85677052 -0.38630528 -0.1219066
##  [4,]  0.5780455  0.5382023  0.85677052  1.00000000 -0.36862012 -0.1074594
##  [5,] -0.3508856 -0.2912844 -0.38630528 -0.36862012  1.00000000  0.4855861
##  [6,] -0.2967996 -0.2897208 -0.12190662 -0.10745935  0.48558611  1.0000000
##  [7,]  0.2985681  0.2771680  0.22380010  0.21360606 -0.36598055 -0.2956059
##  [8,] -0.2715077 -0.2466711 -0.08283751 -0.05289544  0.14114646  0.1993841
##  [9,]  0.2666218  0.2577811  0.14209034  0.11349573 -0.18644632 -0.1965468
## [10,] -0.2328534 -0.2004985 -0.10744556 -0.07119278  0.16443018  0.1910275
## [11,] -0.1919346 -0.1819224  0.12463492  0.11910003  0.04030618  0.1419387
## [12,]  0.2150699  0.2009143  0.22205178  0.21293884 -0.22315525 -0.1899768
## [13,]  0.1961527  0.1845087  0.06894890  0.13062003 -0.05941857 -0.0571007
##              [,7]        [,8]        [,9]       [,10]       [,11]       [,12]
##  [1,]  0.29856805 -0.27150767  0.26662180 -0.23285336 -0.19193462  0.21506989
##  [2,]  0.27716795 -0.24667108  0.25778111 -0.20049854 -0.18192240  0.20091426
##  [3,]  0.22380010 -0.08283751  0.14209034 -0.10744556  0.12463492  0.22205178
##  [4,]  0.21360606 -0.05289544  0.11349573 -0.07119278  0.11910003  0.21293884
##  [5,] -0.36598055  0.14114646 -0.18644632  0.16443018  0.04030618 -0.22315525
##  [6,] -0.29560592  0.19938413 -0.19654679  0.19102755  0.14193871 -0.18997679
##  [7,]  1.00000000 -0.09292552  0.16777164 -0.11633210 -0.14457663  0.15718279
##  [8,] -0.09292552  1.00000000 -0.19163979  0.65415550  0.25606612  0.04130251
##  [9,]  0.16777164 -0.19163979  1.00000000 -0.25430441 -0.28491033  0.05368487
## [10,] -0.11633210  0.65415550 -0.25430441  1.00000000  0.26642194 -0.02557592
## [11,] -0.14457663  0.25606612 -0.28491033  0.26642194  1.00000000  0.09005189
## [12,]  0.15718279  0.04130251  0.05368487 -0.02557592  0.09005189  1.00000000
## [13,]  0.04811237 -0.10087244 -0.01927545 -0.09195948 -0.02211323  0.13772096
##             [,13]
##  [1,]  0.19615266
##  [2,]  0.18450872
##  [3,]  0.06894890
##  [4,]  0.13062003
##  [5,] -0.05941857
##  [6,] -0.05710070
##  [7,]  0.04811237
##  [8,] -0.10087244
##  [9,] -0.01927545
## [10,] -0.09195948
## [11,] -0.02211323
## [12,]  0.13772096
## [13,]  1.00000000
```

Otprije znamo za visoku koreiranost ocjena, a učimo još i visoku koreliranost razina konzumacija alkohola vikendom i radnim danima.

```
cor(students_dummies$Dalc, students_dummies$Walc)
## [1] 0.6541555
```

Zasad ne odbacujući nijedan regresor izradimo linearni model za prethodno izdvojenih 13 varijabli.

```
multiPor = lm(data = students_dummies, G3_por ~ G2_por + G1_por + G1_mat + G2_mat +
    failures_mat + failures_por + higher_yes + Dalc + studytime + Walc + sex_M +
    Medu + address_U)
summary(multiPor)
##
## Call:
## lm(formula = G3_por ~ G2_por + G1_por + G1_mat + G2_mat + failures_mat +
##     failures_por + higher_yes + Dalc + studytime + Walc + sex_M +
##     Medu + address_U, data = students_dummies)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.5864 -0.4875 -0.0354  0.6254  5.6542
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -0.15002    0.58098  -0.258   0.7964
## G2_por        0.86135    0.06451  13.352   <2e-16 ***
## G1_por        0.12309    0.05982   2.058   0.0403 *
## G1_mat        0.09050    0.04275   2.117   0.0350 *
## G2_mat       -0.05491    0.03605  -1.523   0.1286
## failures_mat -0.11933    0.12386  -0.963   0.3360
## failures_por -0.36065    0.16981  -2.124   0.0344 *
## higher_yes    0.24102    0.37438   0.644   0.5201
## Dalc         -0.07723    0.10418  -0.741   0.4590
## studytime     0.05693    0.08871   0.642   0.5215
## Walc          0.01026    0.07215   0.142   0.8870
## sex_M        -0.15408    0.15683  -0.982   0.3265
## Medu         -0.01481    0.06755  -0.219   0.8266
## address_U     0.23738    0.17234   1.377   0.1693
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.316 on 356 degrees of freedom
## Multiple R-squared:  0.8072, Adjusted R-squared:  0.8001
## F-statistic: 114.6 on 13 and 356 DF,  p-value: < 2.2e-16
```

Pojednostavimo sad uzevši 8 variabli s najnižim p-vrijednostima:

```
multiPor2 = lm(data = students_dummies, G3_por ~ G2_por + failures_por + G1_mat +
    G1_por + G2_mat + address_U + sex_M + failures_mat)
summary(multiPor2)
##
## Call:
## lm(formula = G3_por ~ G2_por + failures_por + G1_mat + G1_por +
##     G2_mat + address_U + sex_M + failures_mat, data = students_dummies)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.6591 -0.4742 -0.0609  0.6474  5.3729
```

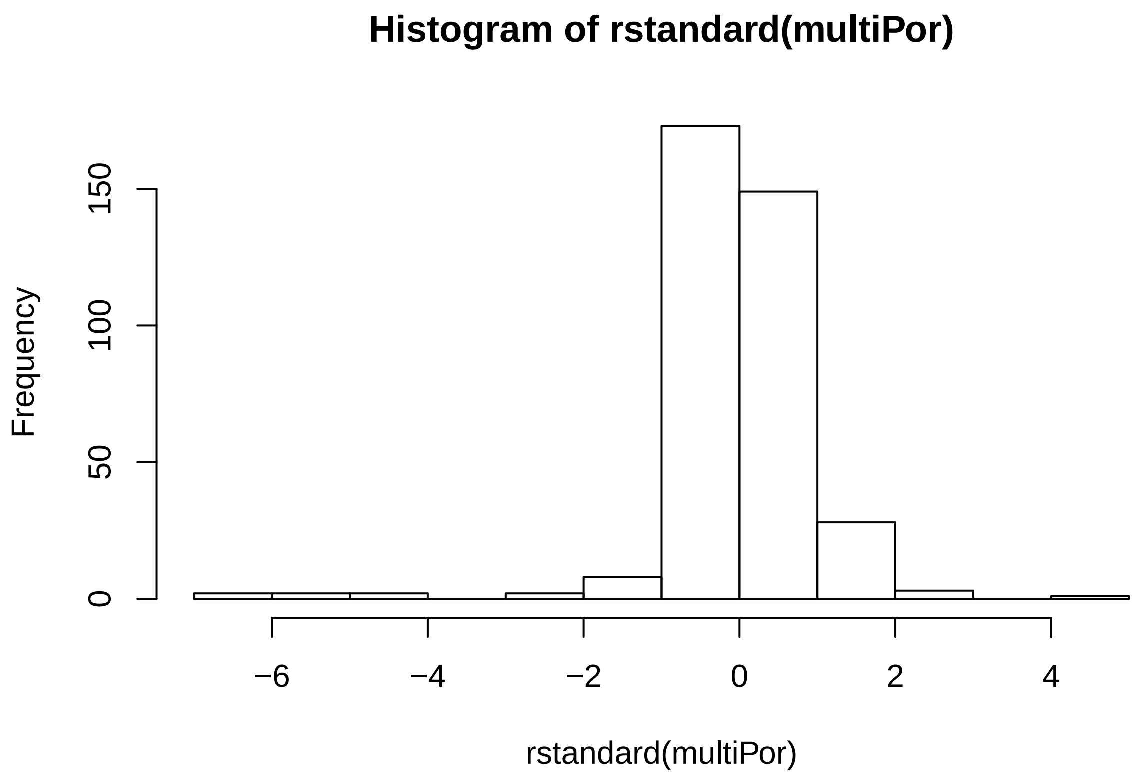
```
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -0.02710    0.41635  -0.065   0.9481
## G2_por        0.86980    0.06369  13.656   <2e-16 ***
## failures_por -0.38679    0.16620  -2.327   0.0205 *
## G1_mat        0.09229    0.04238   2.178   0.0301 *
## G1_por        0.12592    0.05941   2.120   0.0347 *
## G2_mat       -0.05733    0.03576  -1.603   0.1098
## address_U     0.23275    0.16954   1.373   0.1707
## sex_M        -0.21096    0.14718  -1.433   0.1526
## failures_mat -0.14099    0.11997  -1.175   0.2407
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.31 on 361 degrees of freedom
## Multiple R-squared:  0.8063, Adjusted R-squared:  0.802
## F-statistic: 187.8 on 8 and 361 DF,  p-value: < 2.2e-16
```

Ovaj skup varijabli ispostavlja se daje najveći prilagođeni R^2:

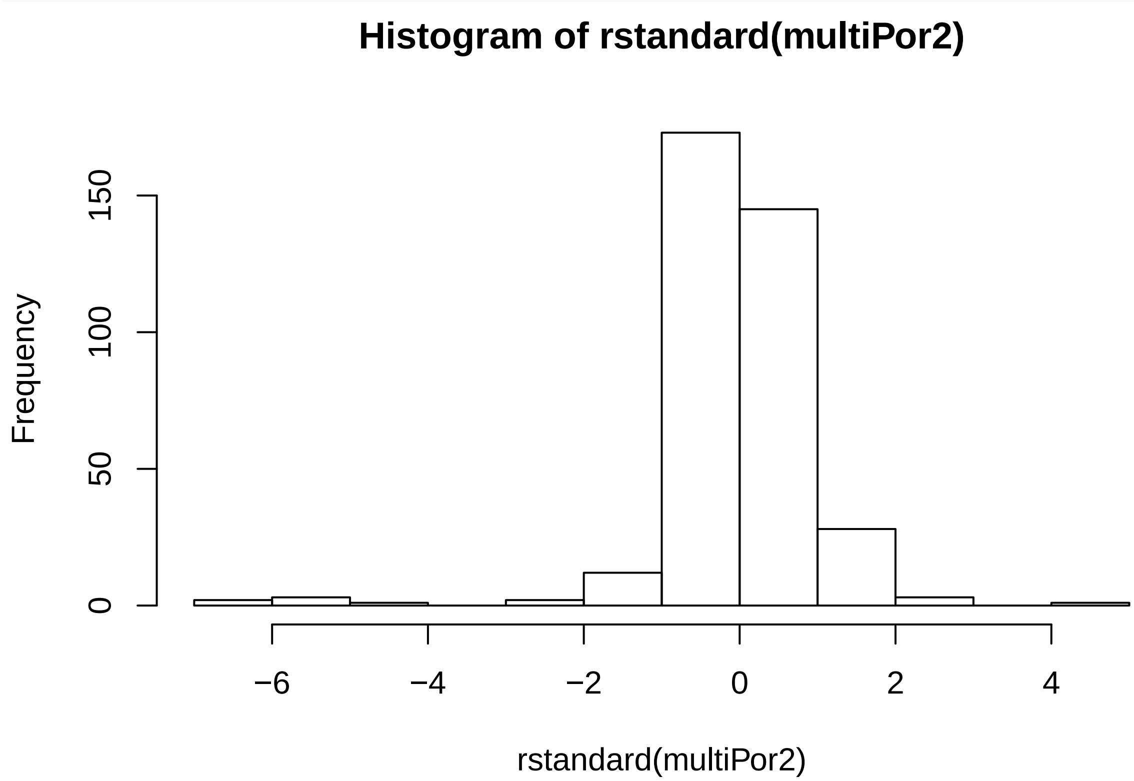
```
# npr ograničavajući na 6 regresora smanjuje se prilagođeni R^2
multiPor3 = lm(data = students_dummies, G3_por ~ G2_por + failures_por + G1_mat +
    G1_por + G2_mat + sex_M)
summary(multiPor3)
##
## Call:
## lm(formula = G3_por ~ G2_por + failures_por + G1_mat + G1_por +
##     G2_mat + sex_M, data = students_dummies)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.6223 -0.4462 -0.0743  0.6620  5.4957
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -0.07026    0.39699  -0.177  0.85963
## G2_por        0.88397    0.06328  13.968  < 2e-16 ***
## failures_por -0.47763    0.14722  -3.244  0.00129 **
## G1_mat        0.09189    0.04163   2.207  0.02792 *
## G1_por        0.12049    0.05899   2.043  0.04182 *
## G2_mat       -0.04862    0.03549  -1.370  0.17156
## sex_M        -0.20984    0.14731  -1.425  0.15515
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.312 on 363 degrees of freedom
## Multiple R-squared:  0.8045, Adjusted R-squared:  0.8013
## F-statistic:   249 on 6 and 363 DF,  p-value: < 2.2e-16
```

Provjerimo još normalnost reziduala:

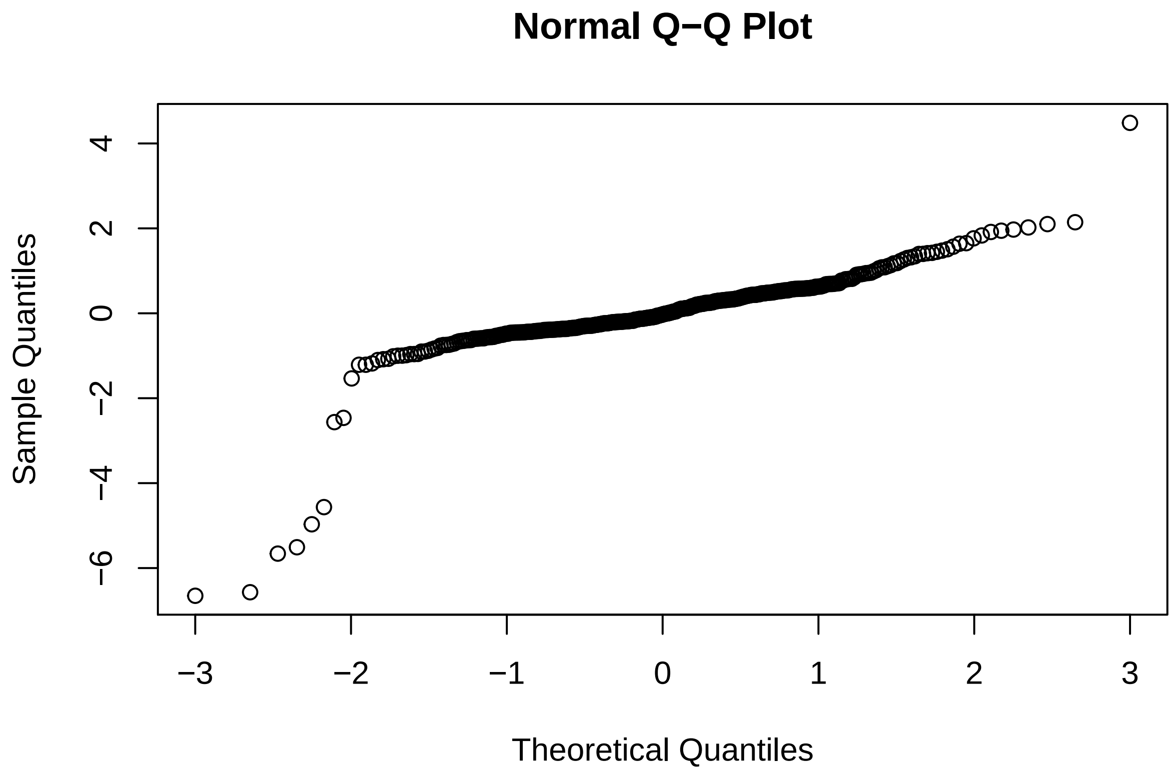
```
hist(rstandard(multiPor))
```



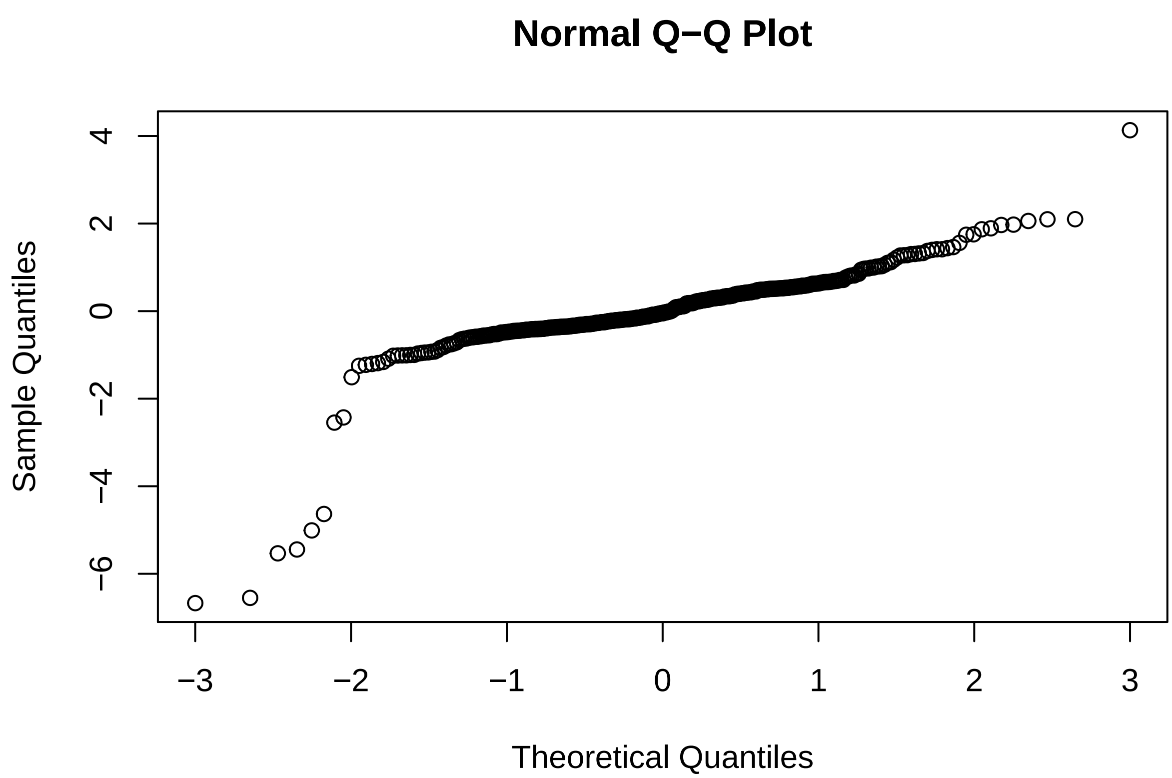

```
hist(rstandard(multiPor2))
```



```
qqnorm(rstandard(multiPor))
```



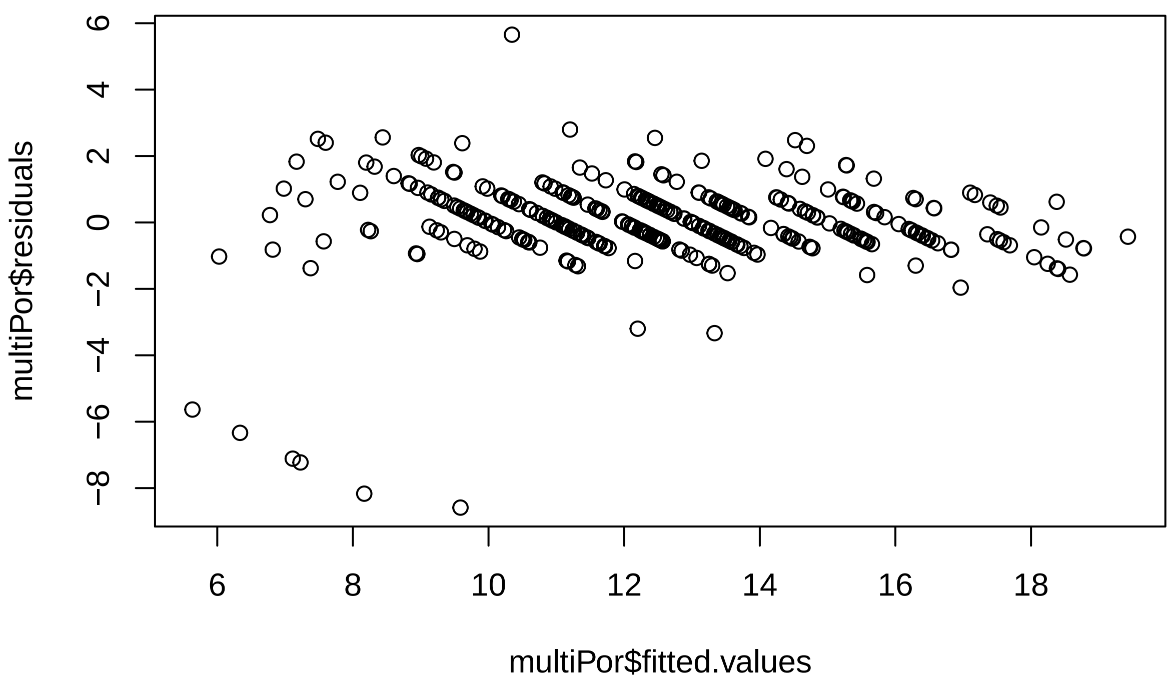

```
qqnorm(rstandard(multiPor2))
```



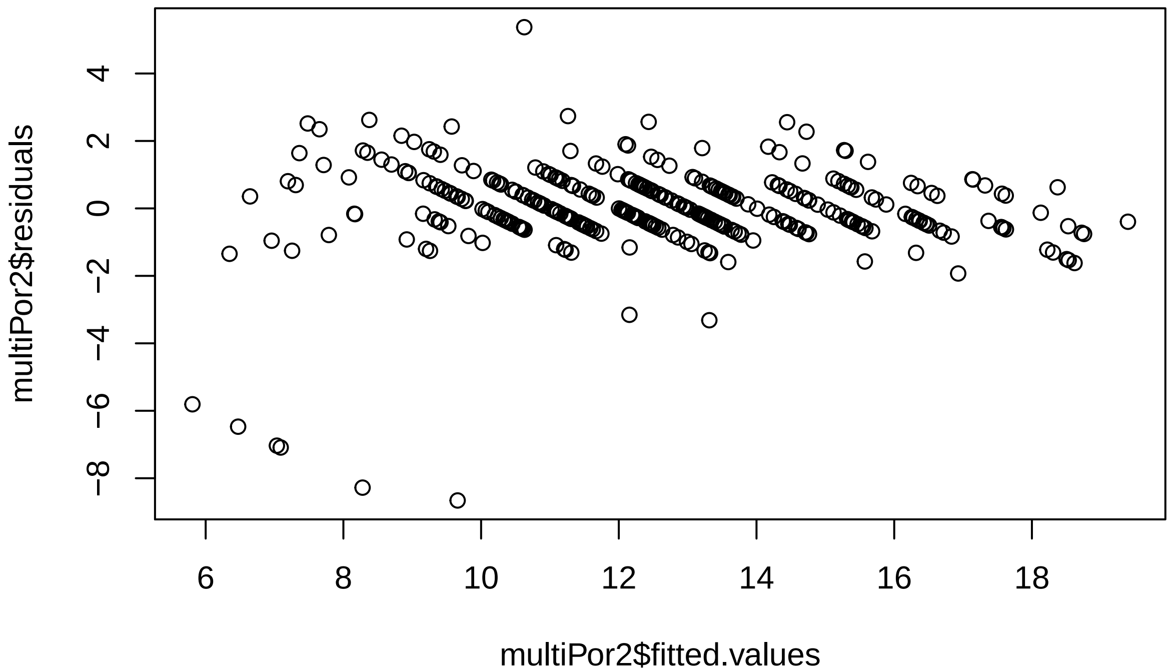
```r
ks.test(rstandard(multiPor), "pnorm")
##
##  One-sample Kolmogorov-Smirnov test
##
## data:  rstandard(multiPor)
## D = 0.16271, p-value = 6.199e-09
## alternative hypothesis: two-sided
ks.test(rstandard(multiPor2), "pnorm")
## Warning in ks.test(rstandard(multiPor2), "pnorm"): ties should not be present
## for the Kolmogorov-Smirnov test
##
##  One-sample Kolmogorov-Smirnov test
##
## data:  rstandard(multiPor2)
## D = 0.16601, p-value = 2.778e-09
## alternative hypothesis: two-sided
```

Reziduali nalikuju normalnoj distribuciji nešto više nego kod modela za konačnu ocjenu iz matematike, ali i dalje ne osobito. Promotrimo ih u ovisnosti o predviđenoj vrijednosti.

```r
plot(multiPor$fitted.values, multiPor$residuals)
```




```r
plot(multiPor2$fitted.values, multiPor2$residuals)
```



Promotrimo još koliko dobro možemo predvidjeti konačnu ocjenu iz portugalskog bez znanja o drugim ocjenama, oslanjajući se na ostalih 9/13 najboljih prediktora:

```
bezOcjenaPor = lm(data = students_dummies, G3_por ~ failures_mat + failures_por +
    higher_yes + Dalc + studytime + Walc + sex_M + Medu + address_U)
summary(bezOcjenaPor)
##
## Call:
## lm(formula = G3_por ~ failures_mat + failures_por + higher_yes +
##     Dalc + studytime + Walc + sex_M + Medu + address_U, data = students_dummies)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -11.4097  -1.3974  -0.0087   1.5525   6.9747
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    9.4176     0.9259  10.171  < 2e-16 ***
## failures_mat  -0.7558     0.2211  -3.419 0.000701 ***
## failures_por  -0.6845     0.3143  -2.178 0.030072 *
## higher_yes     1.9206     0.7032   2.731 0.006618 **
## Dalc          -0.5468     0.1952  -2.802 0.005359 **
## studytime      0.4769     0.1664   2.866 0.004399 **
## Walc           0.0275     0.1367   0.201 0.840629
## sex_M         -0.5150     0.2840  -1.813 0.070605 .
## Medu           0.2870     0.1268   2.264 0.024192 *
## address_U      1.0420     0.3199   3.257 0.001234 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.5 on 360 degrees of freedom
## Multiple R-squared:  0.2962, Adjusted R-squared:  0.2786
## F-statistic: 16.83 on 9 and 360 DF,  p-value: < 2.2e-16
```

Model bez ocjena za portugalski objašnjava skoro 30% varijance u promatranoj varijabli. Značajno poboljšanje u odnosu na model bez ocjena za matematiku.