Data Driven Engineering II: Advaced Topics

State Space Models II

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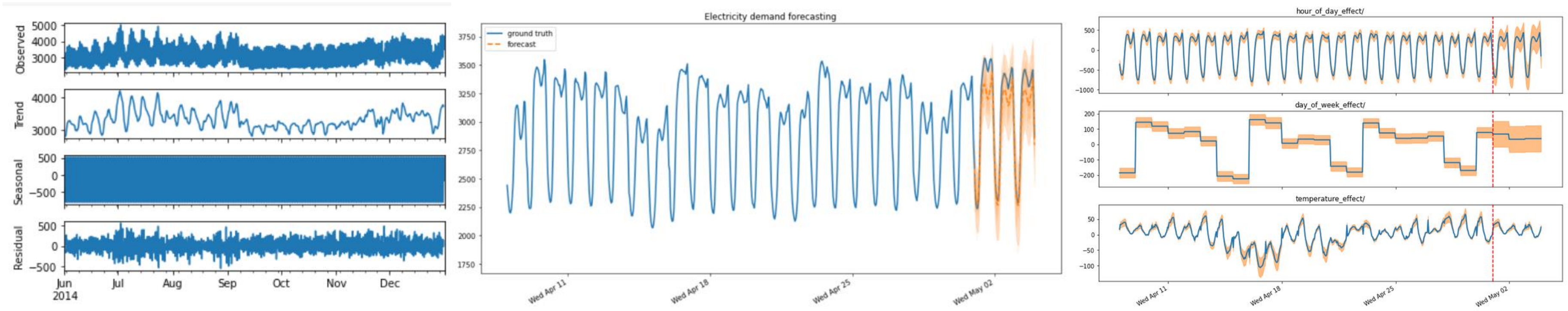


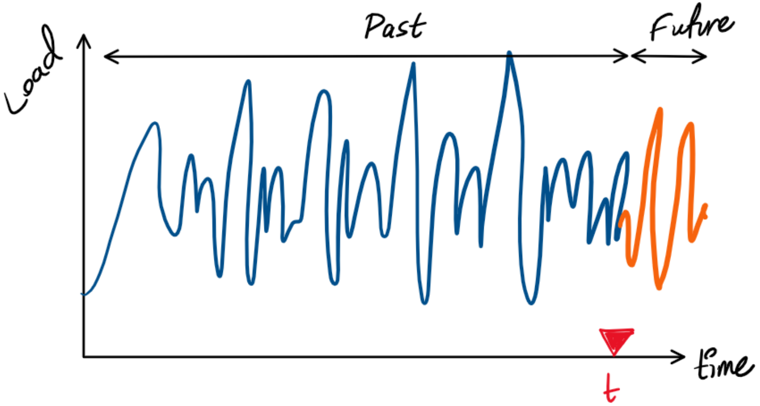
Dynamical Datasets: Time Series

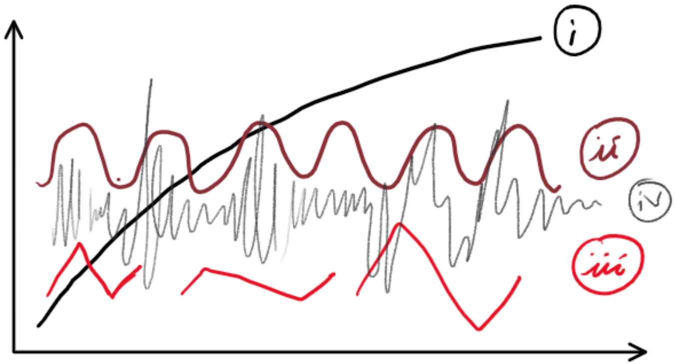
Outline

* Time Series: Overview
* Statistical Models for time series
* State space models ⇒ DDE II
* Machine Learning Part I
* Machine Learning Part II

Time Series Analysis

* Components of time series





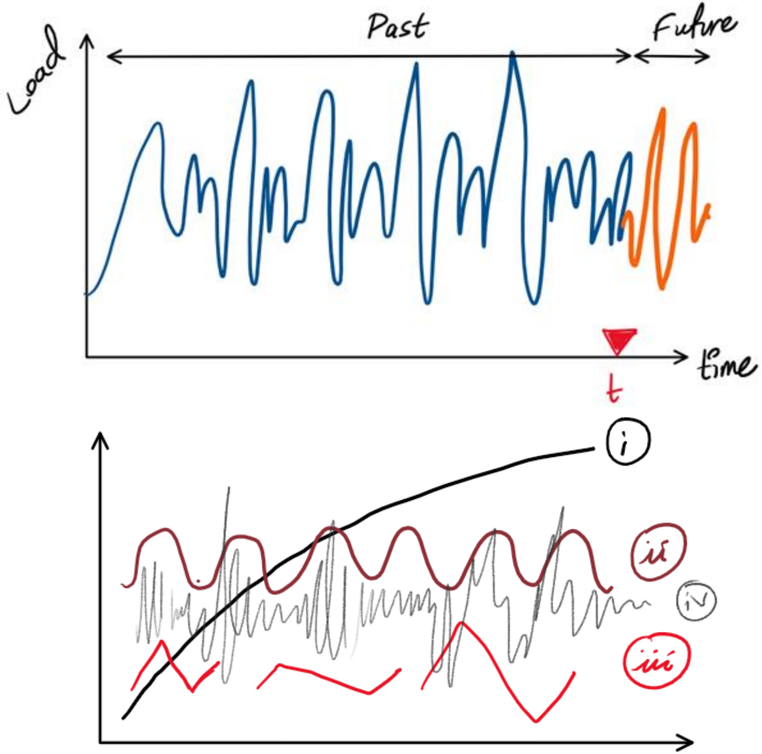
i) Long term trends

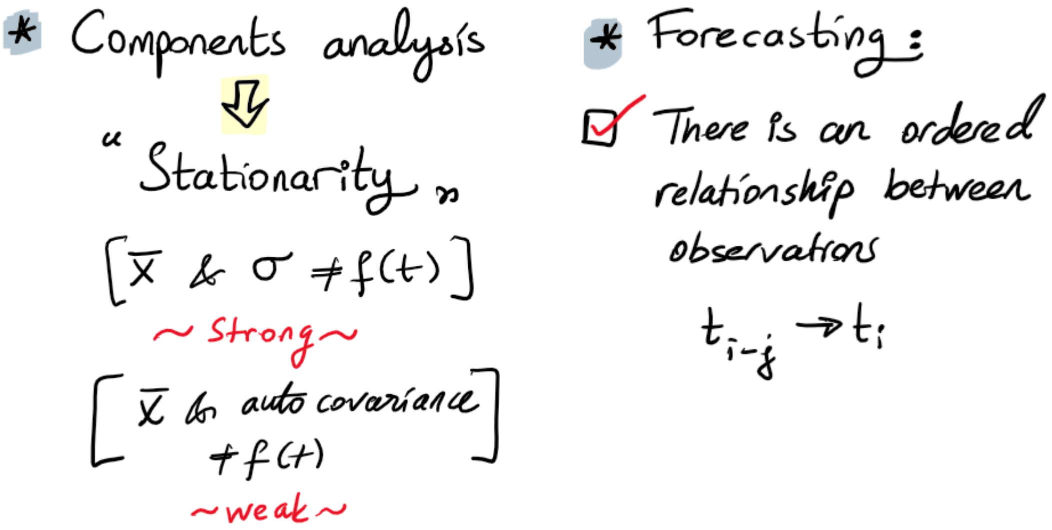
ii) ST seasonal variations

iii) Cyclic variations

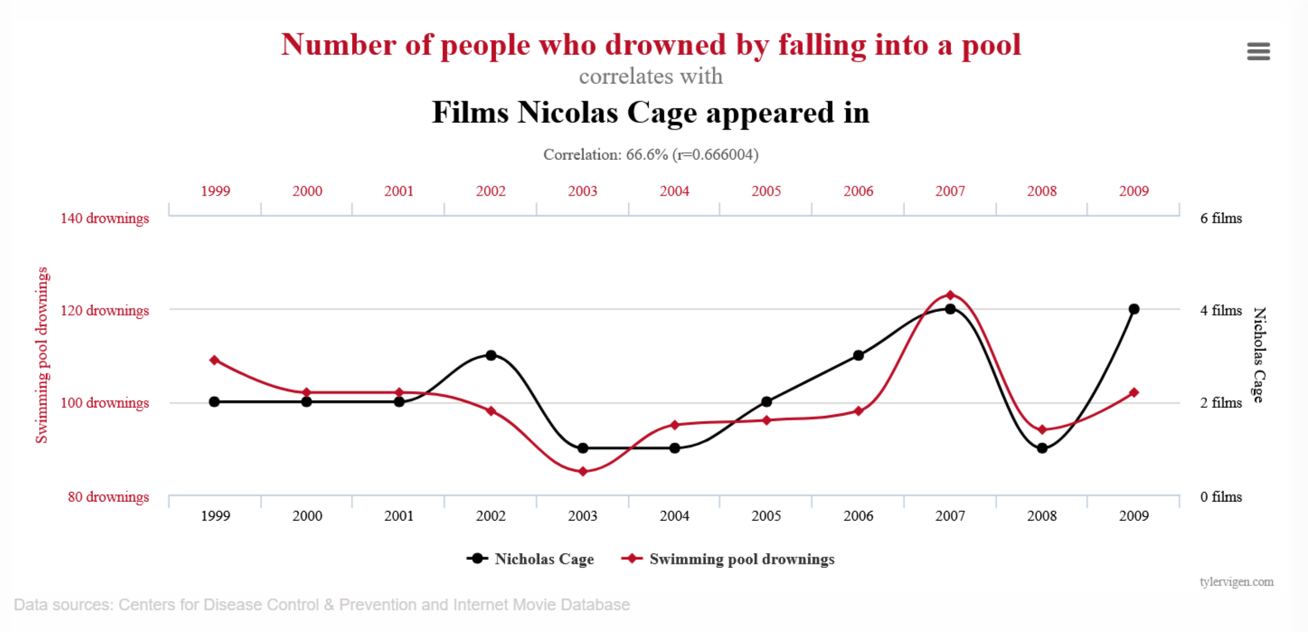
iv) Random fluctuations

Time Series Analysis

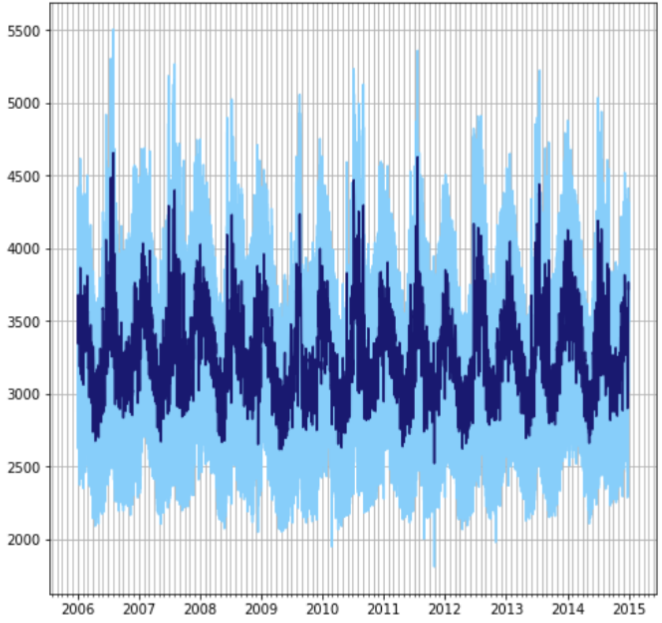




Spurious Correlations



Case: Energy Demand Forecasting



* 8 years data of Temp & Load ($\Delta t = hr$)

? Power Demand forecasting

Short Term

Long Term

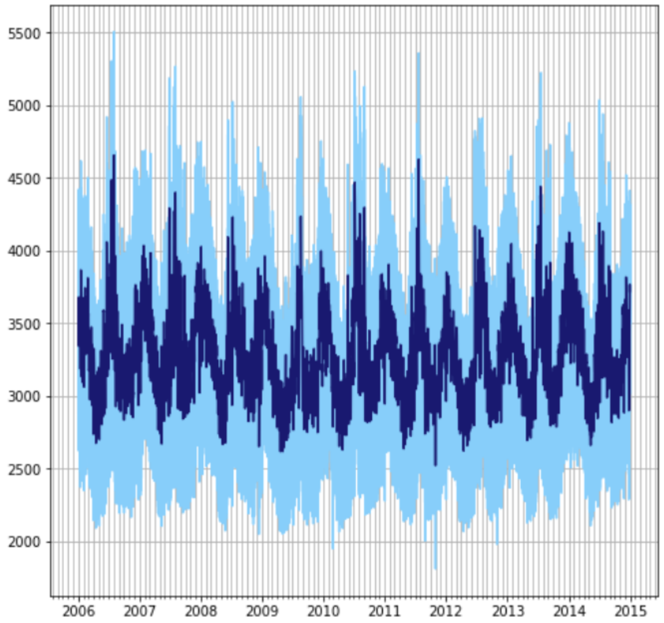
Case: Energy Demand Forecasting

* Short term load forecasting : ~1 hr to 24 hr ⇐ near past is used
  ~demand / supply ⇐ Temperature is an important feature

* Long term LF : ~1 week to months } Planning & investment ⇐ Seasonal patterns
  ~ years ⇐ Long term trends
  ⇐ Climate Models

Case: Energy Demand Forecasting

Typical	STLF	LTLF
Horizon	1 hr – 2 days	≥ 1 months
Granularity	~ hr	~ hr – day
History Range	~ 2 years	~≥ 5 years
Accuracy	≤ 5% error	≤ 25% error
Forecasting freq.	~hr to day	≥ month



Data Exploration: Temporal Nature of data

① How to handle "time stamps"

	Date	Hour	load	T
0	01/01/2004	1	NaN	37.33
1	01/01/2004	2	NaN	37.67
2	01/01/2004	3	NaN	37.00
3	01/01/2004	4	NaN	36.33
4	01/01/2004	5	NaN	36.00

⇨

	load	T
2012-01-05 00:00:00	3167.0	19.00
2012-01-05 01:00:00	3014.0	22.33
2012-01-05 02:00:00	2921.0	22.33
2012-01-05 03:00:00	2874.0	22.00
2012-01-05 04:00:00	2876.0	21.67

Data Exploration: Temporal Nature of data

② Temporal data decomposition → Trend, Seasonality, Noise

Stationarity

⇓

how 'stable' your system

- ☑ Intuition
- ☐ Tests

⇓

how much we should expect the past reflects itself on future?

⇒ "Self Correlations"

Bayesian Structural Time Series

* BSTS $\Leftrightarrow$ linear Gaussian Model << Kalman F.

* Idea: $\left. \begin{matrix} f_1 & f_2 & \\ & & f_3 \\ f_4 & f_i & \end{matrix} \right\}$ 'f, := Hypothesis library

$X = [\, . \; . \; . \,]$ $\rightleftarrows$ $\boxed{f}$ → Decribes & predicts system behavior

... Fitting ...

Bayesian Structural Time Series

* **General psudo-algorithm:**

  (1) Define a structural model

  (2) Specify priors.

  (3) Update estimates of states based on observation data

  (4) Perform variable selection (eg. spike & slab method)

  (5) Apply Bayesian model averaging

  (6) Forecasting

How fitting works?

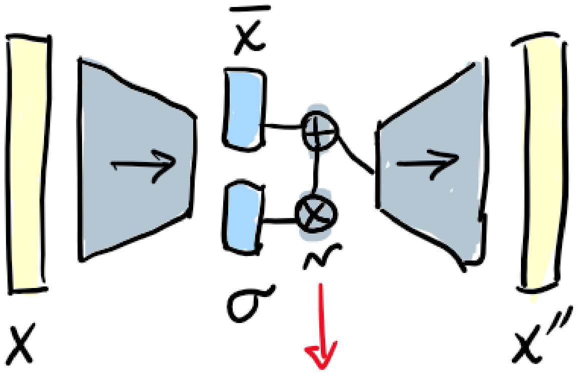
* Minimize Kullback – Leibler divergence (see VAE, DDE-I)

Variational Encoders:

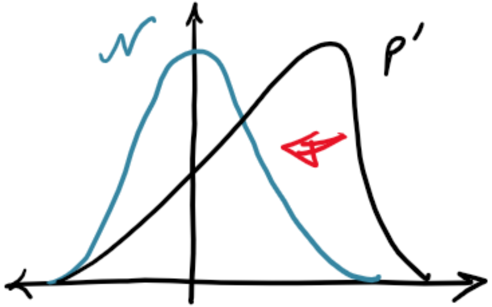
* Gaussian Sampling ⇒ [enforced via cost function]

$\underbrace{\text{[enforced via cost function]}}$

Kullback – Leibler (KL) Divergence ($\mathcal{N}$ & p')



Sampled Randomly ($\mathcal{N}$)



$$D_{KL}\left[p'(\bar{x},\sigma) \,||\, \mathcal{N}(0,1)\right]$$

- $\mathcal{L}_{KL} = -\frac{1}{2}\sum_i 1 + \gamma_i - \exp(\gamma_i) - \bar{x}_i^2$
- $\gamma_i = \ln(\sigma_i^2)$

How fitting works?

* Minimize Kullback–Leibler divergence (see VAE, DDE-I)

- $x :=$ observable ; $z :=$ 'latent' variables ;

- $$p(z|x) = \frac{p(x|z)}{p(x)} \Rightarrow p(x) = \int p(x,z)\,dz \;?$$

- Variational Inference $:=$ $$q^*(z) = \min_{\underbrace{q \in \mathcal{D}}_{\text{candidate approximations}}} KL\big(q(z)\,\|\,p(z|x)\big)$$

How fitting works?

- $$KL\left(q(z) \| p(z|x)\right) = \underbrace{E\left[\log(q(z))\right] - E\left[\log(p(z|x))\right]}_{(-ELBO):\ \text{evidence lower band}} + \log(p(x))$$!

$$ELBO(q) = E\left[\log(p(z,x))\right] - E\left[\log(q(z))\right]$$

$$= E\left[\log(p(x|z))\right] - KL\left(q(z) \| p(z)\right)$$

~ balance btw. expected likelihood ~ prior

How fitting works?

- Variational Inference $:= q^*(z) = \underbrace{\min_{q \in \mathcal{D}}}_{\text{candidate approximations}} \text{ELBO}(q)$

Tensorflow

* Create variational posteriors;

- Sample prior distributions
- Select an optimizer
- Fit posteriors by min. ELBO

```python
# Building the variational surrogate posteriors `qs`:
#https://www.tensorflow.org/probability/api_docs/python/tfp/sts/build_f
variational_posteriors = tfp.sts.build_factored_surrogate_posterior(
    model=load_model)
```

```python
num_variational_steps = 120
optimizer = tf.optimizers.Adam(learning_rate=.1)
```

```python
elbo_loss_curve = tfp.vi.fit_surrogate_posterior(
  target_log_prob_fn=load_model.joint_log_prob(
      observed_time_series=train),
  surrogate_posterior=variational_posteriors,
  optimizer=optimizer,
  num_steps=num_variational_steps)
```

colab

Markov Chain Monte Carlo (MCMC)

* invented after MC @ Los Alamos (1953)

  ↳ liq. phase eq. with its vapor (Eq. Thermodynamics)

  ↳ Metropolis Algorithm

  ↳ M-Hasting

  ↳ Gibbs Sampler

} Simulates posterior dist.

Enables full Bayesian interface

Markov Chain Monte Carlo (MCMC)

* Markov Chains ⇒ early 20^{th} Cent; alternations of letters in a poem; "Onegin" by Poeshkin

- Characterization of Sequence of random variables;

- $X_1, X_2, X_3, X_4 \dots X_n, X_{n+1}$ $\left\{ \begin{array}{l} (n+1) \text{ depends only} \\ (n) \end{array} \right.$

Markov Chain Monte Carlo (MCMC)

* Hidden Markov Models $\Leftrightarrow$ Kalman Filter
  - variables we observe may not be the most descriptive
  - "unsupervised learning"
    - ↳ we have an idea about how system works.

Eg

State	L	G
L	0.6	0.4
G	0.2	0.8

? (0) state → L
(1) 60% ⇒ L
40% ⇒ G

Markov Chain Monte Carlo (MCMC)

(Hidden) X_1 X_2 X_3 X_4 X_5 X_n

$\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$

(Observ.) y_1 y_2 y_3 y_4 y_5 y_n

$\lambda(\pi_{ij}, b_{ij}, a_{ij})$

$\Updownarrow$

y_i

* We can describe a system if we know:

- initial state of the system (π_{ij})
- emission probability; prob. of observing y_i given X_i (b_{ij})
- Transition probability; $X_i \rightarrow X_{i+n}$ (a_{ij})

iterative solution for λ

Markov Chain Monte Carlo (MCMC)

Karlsruhe Institute of Technology

(Hidden) X_1 X_2 X_3 X_4 X_5 X_n

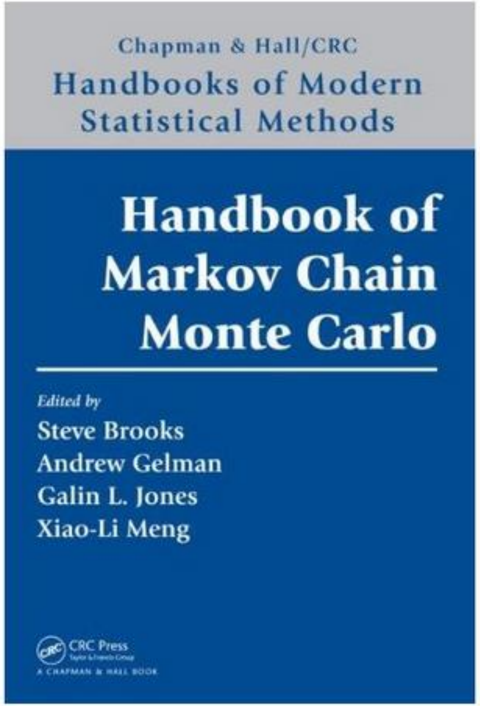
(Observ.) y_1 y_2 y_3 y_4 y_5 y_n

$\lambda\,(\pi_{ij},\ b_{ij},\ a_{ij})$

λ — Frequentist inference — "with enough sampling, we can find it"
- Expectation Maximization (EM)
- ...
- Baum–Welch Algorithm

λ — Bayesian inference — "continuously update the posterior for best λ"
- MCMC
- Gibbs Sampling
- VT Algorithm


Ateliers & Saveurs in Montreal



Variational Inference: A Review for Statisticians

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Additional Notes