UCK 439E – Introduction to Optimal Control: Term Project

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Policy: You can form a team up to three students or study alone to perform the term project. Your team should write your own answer/code. Cheating is highly discouraged; it could mean a zero or negative grade from the study. If you have a team, you should clearly indicate the assigned tasks for each team member and present the work distribution in your report.

Submission Instructions: Please submit your results through the Ninova website. Each team member must independently upload the team report and source codes to the system. Please zip and upload all your files using filename studentID.rar. You must provide all source codes you wrote with your zipped file. Source codes you do not submit will cause you to lose a portion of your grade. Please make sure that you comment on your code. Make also sure that the plots you produce are readable and they have labels and legends. You must include the report.pdf file containing the description of the method and implementation results.

Project:
In the project, you are expected to implement optimized flight planning algorithms under wind fields. The aircraft dynamic model can be presented by the set of equations below:

$$\dot{x} = v\cos(\psi)\cos(\gamma) + W_x(x,y) \tag{1}$$

$$\dot{y} = v\sin(\psi)\cos(\gamma) + W_y(x,y) \tag{2}$$

$$\dot{h} = v\sin(\gamma) \tag{3}$$

$$\dot{v} = \frac{Thr_{max}\delta}{m} - g\sin(\gamma) - \frac{c_D S\rho v^2}{2m} \tag{4}$$

$$\dot{\psi} = \frac{c_L S\rho v}{2m}\frac{\sin(\mu)}{\cos(\gamma)} \tag{5}$$

$$\dot{m} = -f \tag{6}$$

where the position (x, y), altitude h, speed v, heading angle ψ, and mass of the aircraft m are the state variables, and the flight path angle γ, bank angle μ and thrust lever position (throttle) δ are the control inputs. $W_x(x,y)$ and $W_y(x,y)$ are the wind speeds in the x-axis and y-axis, respectively.

The following optimal control problem should be solved to obtain an optimized trajectory under wind fields in cruise phase:

$$\min\ \left(x(t_f) - x_d\right)^2 + \left(y(t_f) - y_d\right)^2 + \left(h(t_f) - h_d\right)^2 + \int_0^{t_f} (0.05 + f)\, dt$$

subject to: (7)

$$aircraft\ dynamics\ (1) - (6)$$

$$\rho = \rho_0 (1 - (2.2257e - 5)h)^{4.2586}$$

$$C_L = \frac{2mg}{\rho S v^2 \cos(\mu)}$$

$$C_D = C_{d0} + kC_L^2$$

$$Thr_{max} = C_{Tcr} C_{Tc,1} \left(1 - \frac{3.28h}{C_{Tc,2}} + C_{Tc,3}(3.28h)^2\right)$$

$$\eta = \frac{C_{f1}}{60000}\left(1 + \frac{1.943v}{C_{f2}}\right)$$

$$f = \delta\, Thr_{max} \eta C_{fcr}$$

where the aircraft-specific performance coefficients $C_{fcr}, C_{f1}, C_{f2}, C_{Tcr}, C_{Tc,1}, C_{Tc,2}, C_{Tc,3}, C_{d0}$, and k are obtained from a database (BADA). And, the performance coefficients of B737-800, which will be the default aircraft type in the project, are presented in Table 1. In the presented optimal control problem, the aircraft aims to reach the destination point (x_d, y_d, h_d) while minimizing the time and fuel costs.

Table 1. Performance Coefficients of B737-800

Fuel Consumption		Engine Thrust		Aerodynamics	
C_{fcr}	0.92958	C_{Tcr}	0.95	C_{d0}	0.025452
C_{f1}	0.70057	$C_{Tc,1}$	146590	k	0.035815
C_{f2}	1068.1	$C_{Tc,2}$	53872	S	124.65 m^2
		$C_{Tc,3}$	3.0453e-11		

As presented in the aircraft model, the trajectory is also affected by the wind. Assume that the wind speeds in x-axis and y-axis can be presented via high-degree polynomials in the following form:

$$W_x(\lambda, \phi) = c_{x0} + c_{x1}\lambda + c_{x2}\phi + c_{x3}\lambda\phi + c_{x4}\lambda^2 + c_{x5}\phi^2 + c_{x6}\lambda^2\phi + c_{x7}\lambda\phi^2 + c_{x8}\lambda^2\phi^2$$

$$W_y(\lambda, \phi) = c_{y0} + c_{y1}\lambda + c_{y2}\phi + c_{y3}\lambda\phi + c_{y4}\lambda^2 + c_{y5}\phi^2 + c_{y6}\lambda^2\phi + c_{y7}\lambda\phi^2 + c_{y8}\lambda^2\phi^2$$

where λ and ϕ are longitude and latitude, respectively. And, the polynomial coefficients determine the wind fields. In the project, you should use the coefficients presented in Table 2 to specify the wind speeds, which are illustrated in Figure 1.

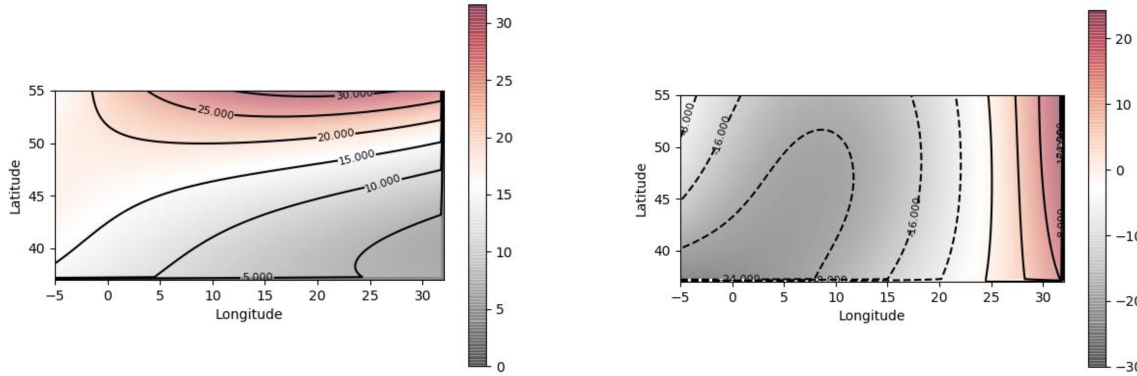

Figure 1. Illustrated wind speeds [m/s] in x-axis (left) and y-axis (right) for the European Region based on the coefficients in Table 2.

Table 2. Polynomial Coefficients for the wind speeds in the focused time window.

	c_{*0}	c_{*1}	c_{*2}	c_{*3}	c_{*4}	c_{*5}	c_{*6}	c_{*7}	c_{*8}
W_x	-21.151	10.0039	1.1081	-0.5239	-0.1297	-0.006	0.0073	0.0066	-0.0001
W_y	-65.3035	17.6148	1.0855	-0.7001	-0.5508	-0.003	0.0241	0.0064	-0.000227

Table 3. Flight Plans

Flight 1		Flight 2		Flight 3	
Initial Conditions	Destination Waypoint	Initial Conditions	Destination Waypoint	Initial Conditions	Destination Waypoint
$\lambda_0 = 5°$ $\phi_0 = 40°$ $h_0 = 8000m$ $v_0 = 210m/s$ $\psi_0 = 0°$ $m_0 = 68000kg$	$\lambda_d = 32°$ $\phi_d = 40°$ $h_d = 8000m$	$\lambda_0 = 30°$ $\phi_0 = 55°$ $h_0 = 7000m$ $v_0 = 220m/s$ $\psi_0 = 40°$ $m_0 = 67000kg$	$\lambda_d = 15°$ $\phi_d = 40°$ $h_d = 9000m$	$\lambda_0 = 32°$ $\phi_0 = 45°$ $h_0 = 8000m$ $v_0 = 210m/s$ $\psi_0 = 180°$ $m_0 = 65000kg$	$\lambda_d = 5°$ $\phi_d = 45°$ $h_d = 7000m$

You are expected to obtain the optimized trajectories for three different flights whose flight plans are given in Table 3. Solve the optimal control problem (7) for each flight by coding the required algorithms and generate the optimized trajectories via

a) A direct method or indirect method
b) A dynamic programming approach or Reinforcement Learning

Present the $x - y$, $h - Time$, $V_{tas} - Time$, $Mass - Time$, $Thrust - Time$, $Throttle - Time$, $Bank\ angle - Time$, and $Flight\ path\ angle - Time$ graphs, and also indicate the total fuel consumption and flight duration for each case. Discuss your findings, and compare the pros and cons of the implemented methods.