模拟与数字电路

Analog and Digital Circuits

课程主页 扫一扫

第 八 讲： **布尔逻辑的化简**

Lecture 8: **Boolean Logic and Simplification**

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提纲

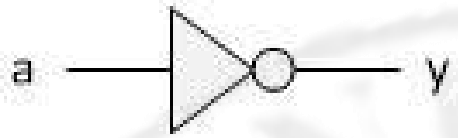
- 复习
  - 最典型逻辑门包括那些？？

- 数字电路的二进制标达
- 二进制 反相器电路
- 基本逻辑门，及其CMOS拓扑电路结构

逻辑关系的组和

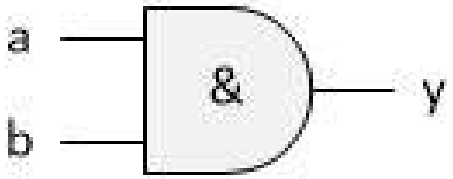
- 通过“与”、“或”、“非”的组和，形成一些新的逻辑
- 与非门
- 或非门
- 异或门?

NOT



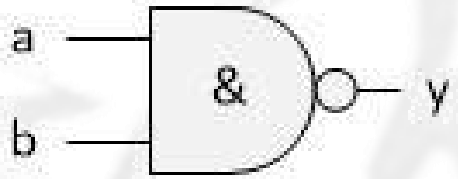
a	y
0	1
1	0

AND



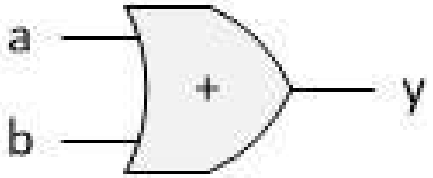
a	b	y
0	0	0
0	1	0
1	0	0
1	1	1

NAND



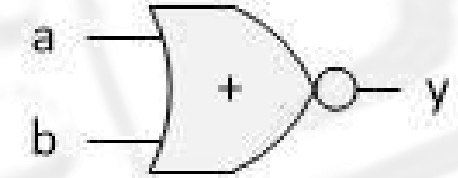
a	b	y
0	0	1
0	1	1
1	0	1
1	1	0

OR



a	b	y
0	0	0
0	1	1
1	0	1
1	1	1

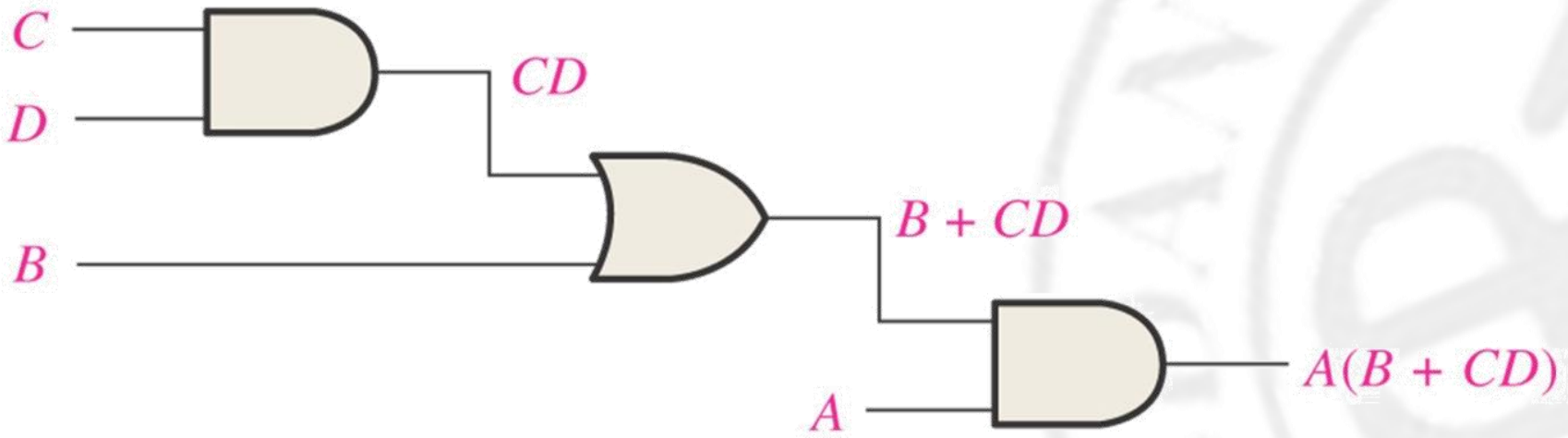
NOR



a	b	y
0	0	1
0	1	0
1	0	0
1	1	0

布朗代数表达式与逻辑电路

- 逻辑电路的布尔表达式
- 书写规则为从左到右，依次写出每个门的输出



布朗代数表达式与逻辑电路

- 逻辑电路的真值表
- $A(B + CD)$

INPUTS				OUTPUT
A	B	C	D	A(B + CD)
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

布朗代数基本定律

- 交换律: $A + B = B + A$
  $AB = BA$
- 结合律: $A + B + C = (A + B) + C = A + (B + C)$
  $ABC = (AB)C = A(BC)$
- 分配律: $A(B + C) = AB + AC$
  $A + BC = (A + B)(A + C)$

分配律证明

- 求证分配律: $A + BC = (A + B)(A + C)$
- 证明:

A	B	C	BC	A+BC	A+B	A+C	(A+B)(A+C)
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

布朗代数基本定律

- 0 1律: $A \cdot 1 = A$ $A \cdot 0 = 0$
  $A + 1 = 1$ $A + 0 = A$
- 互补律: $A \cdot \bar{A} = 0$
  $A + \bar{A} = 1$
- 重叠律: $A + A = A$ $A \cdot A = A$
- 还原律: $\bar{\bar{A}} = A$
- Demorgan定理: $\overline{A \cdot B} = \bar{A} + \bar{B}$
  $\overline{A + B} = \bar{A} \cdot \bar{B}$

Demorgan定律及其证明

- 求证Demorgan定律: $\overline{A \cdot B} = \bar{A} + \bar{B}$ $\overline{A + B} = \bar{A} \cdot \bar{B}$

- 证明:

A	B	$\overline{A \cdot B}$	$\bar{A} + \bar{B}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

A	B	$\overline{A + B}$	$\bar{A} \cdot \bar{B}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

布朗代数定律汇总

01律	(1) $A \cdot 1 = A$ (3) $A \cdot 0 = 0$	(2) $A + 0 = A$ (4) $A + 1 = 1$
交换律	(5) $A \cdot B = B \cdot A$	(6) $A + B = B + A$
结合律	(7) $A \cdot (B \cdot C) = (A \cdot B) \cdot C$	(8) $A + (B+C) = (A+B) + C$
分配律	(9) $A \cdot (B+C) = A \cdot B + A \cdot C$	(10) $A + (BC) = (A+B)(A+C)$
互补律	(11) $A \cdot \overline{A} = 0$	(12) $A + \overline{A} = 1$
重叠律	(13) $A \cdot A = A$	(14) $A + A = A$
反演律	(15) $\overline{AB} = \overline{A} + \overline{B}$	(16) $\overline{A+B} = \overline{A} \cdot \overline{B}$
还原律	(17) $\overline{\overline{A}} = A$	

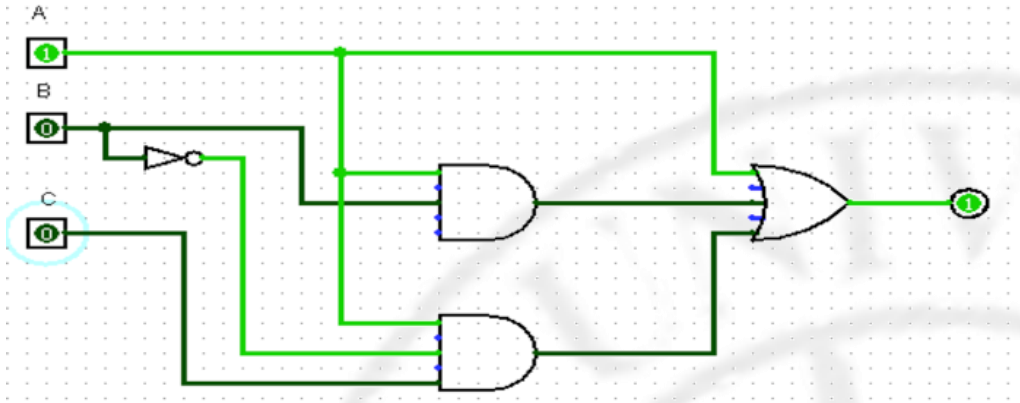
布朗代数常用公式及证明

常用公式	证　明
① $AB+A\bar{B}=A$	$AB+A\bar{B}=A(B+\bar{B})=A\cdot 1=A$
② $A+AB=A$	$A+AB=A(1+B)=A\cdot 1=A$
③ $A+\bar{A}B=A+B$	$A+\bar{A}B=(A+\bar{A})(A+B)$ $=1\cdot(A+B)=A+B$
④ $AB+\bar{A}C+BC=AB+\bar{A}C$ 推论： $AB+\bar{A}C+BCDE=AB+\bar{A}C$	原式$=AB+\bar{A}C+BC(A+\bar{A})$ $=AB+\bar{A}C+ABC+\bar{A}BC$ $=AB(1+C)+\bar{A}C(1+B)$ $=AB+\bar{A}C$

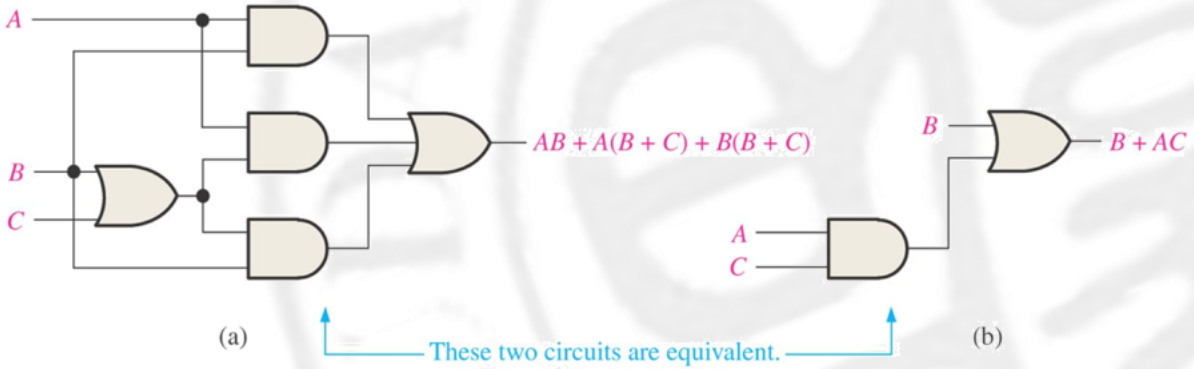
布朗代数化简例题-i

- 化简　$A + AB + A\bar{B}C$
- $\rightarrow A + A\bar{B}C$
- $\rightarrow A$

$A + AB = A$



- 化简　$AB + A(B + C) + B(B + C)$
- $\rightarrow AB + AB + AC + BB + BC$
- $\rightarrow AB + AC + B + BC$
- $\rightarrow AB + B + AC$
- $\rightarrow B + AC$



布朗代数化简例题-i

- 根据公式 $A + A = A$,为某项配上其所能合并的项

- 化简　$ABC + A\bar{B} + A\bar{C}$
- $\rightarrow ABC + A(\bar{B} + \bar{C})$
- $\rightarrow ABC + A\overline{BC}$
- $\rightarrow A(BC + \overline{BC})$
- $\rightarrow A$

$$
\begin{aligned}
Y &= ABC + AB\bar{C} + A\bar{B}C + \bar{A}BC \\
&= (ABC + AB\bar{C}) + (ABC + A\bar{B}C) + (ABC + \bar{A}BC) \\
&= AB(C + \bar{C}) + AC(B + \bar{B}) + BC(A + \bar{A}) \\
&= AB + AC + BC
\end{aligned}
$$

布朗代数化简例题

将$Y=\bar{A}B\bar{C}+\bar{A}BC+A\bar{B}C+AB\bar{C}+ABC$

化简为最简与或式。

$Y=\bar{A}B\bar{C}+\bar{A}BC+A\bar{B}C+AB\bar{C}+ABC$

$=\bar{A}B(\bar{C}+C)+A\bar{B}C+AB(\bar{C}+C)$

$=\bar{A}B+A\bar{B}C+AB$ 利用$\bar{C}+C=1$

$=(\bar{A}+A)B+A\bar{B}C$

$=B+\bar{B}AC$ 利用$\bar{A}+AB=A+B$

$=B+AC$

将Y化简为最简与或式。

$Y=A\bar{B}+(\bar{A}+B)CD$

解：$Y=A\bar{B}+(\bar{A}+B)CD$ ；$A=\bar{\bar{A}}$

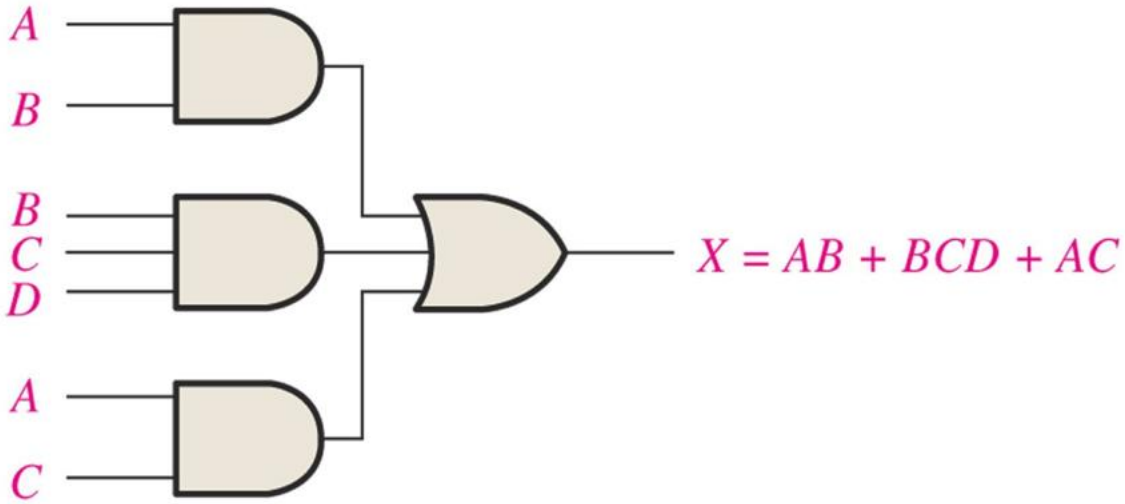
$=A\bar{B}+(\bar{A}+B)CD$;利用摩根定理

$=A\bar{B}+A\bar{B}\,CD$;将$A\bar{B}$当成一个变量，

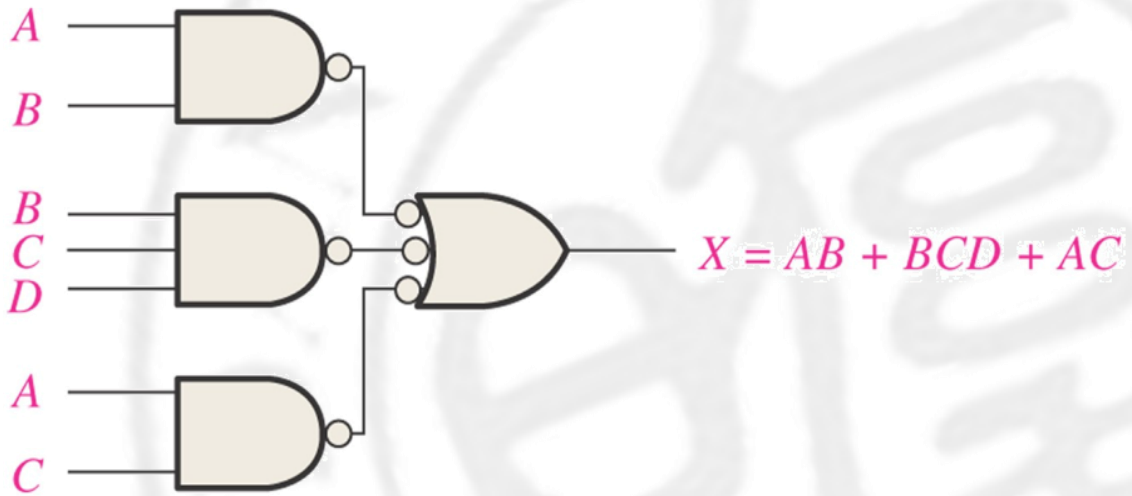
$=A\bar{B}+CD$ 利用公式$A+\bar{A}B=A+B$

SOP表达法及其电路

- SOP(Sum-of-Product) Form
  - e.g. $AB + ABC, ABC + CDE + \bar{B}CD$
  - SOC表达式的AND/OR实现



AND/OR实现



NAND/NOR实现

SOP表达法及其电路

- 将一般表达式转化为SOP形式
  - e.g. AB + B(CD + EF) → AB + BCD +BEF
- 标准SOP形式
  - 所有变量都要出现在表达式中
  - e.g. $A\bar{B}CD + \bar{A}\bar{B}CD$
  - $A\bar{B}C + \bar{A}\bar{B} + AB\bar{C}D$转化为标准SOP形式

↓

$$A\bar{B}C = A\bar{B}C(D+\bar{D}) = A\bar{B}CD + A\bar{B}C\bar{D}$$
$$\bar{A}\bar{B} = \bar{A}\bar{B}(C+\bar{C}) = \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}$$
$$\bar{A}\bar{B} = \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} = \bar{A}\bar{B}C(D+\bar{D}) + \bar{A}\bar{B}\bar{C}(D+\bar{D}) = \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D}$$
$$A\bar{B}C + \bar{A}\bar{B} + AB\bar{C}D = A\bar{B}CD + A\bar{B}C\bar{D} + \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D} + AB\bar{C}D$$

真值表与SOP表达法

- 将SOP表达式转化为真值表
  - e.g. 求表达式$\bar{A}B\bar{C} + A\bar{B}C$ 的真值表

Inputs			Output	Product Term
A	B	C	X	
0	0	0	0	
0	0	1	0	
0	1	0	1	$\overline{A}\,B\,\overline{C}$
0	1	1	0	
1	0	0	0	
1	0	1	1	$A\,\overline{B}\,C$
1	1	0	0	
1	1	1	0	

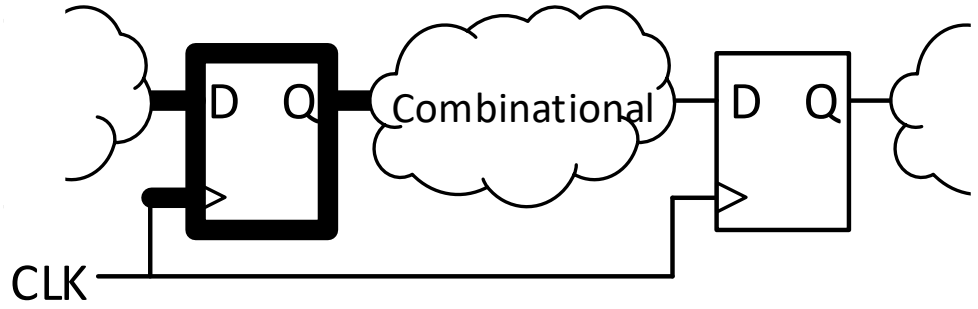
真值表与SOP表达法(例题)

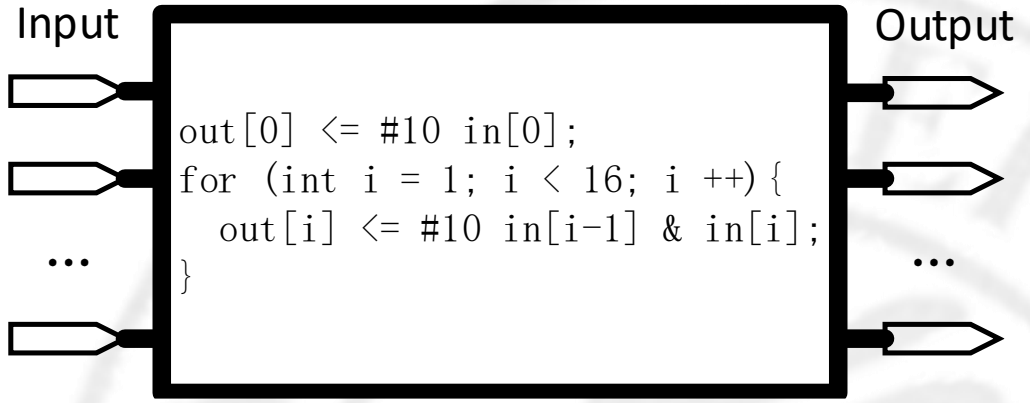
- 举重比赛A、B、C三个裁判，判杠铃完全举起为成功。按一下按钮，只有当两个或两个以上裁判判断成功才表明成功，表决电路灯亮。

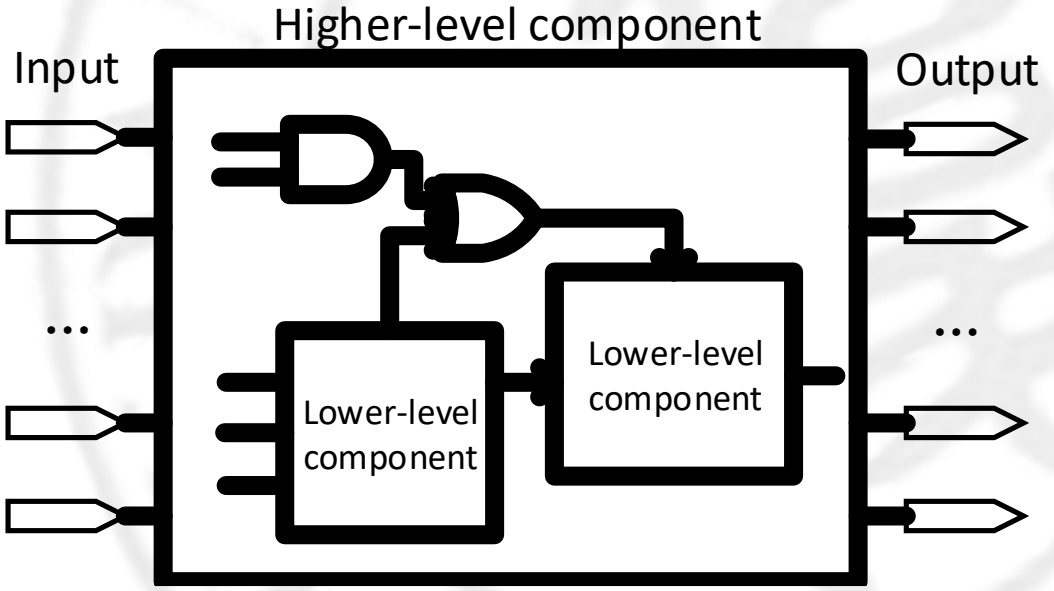
A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Hardware Description Language

- HDL=Hardware Description Language
- RTL=Register-Transfer-Level
- Behavior vs. Structural Modeling
- Combinational vs. Sequential logic
- Verilog HDL vs. VHDL vs. SystemVerilog







Arithmetic and Logistic Expression in Verilog

- Logical operation vs. Bit-wise operation : `a && b || !c` `a & b | ~c`
- Logical shift vs. arithmetic shift : `a >> 1` `a >>> 1`
- Concatenation operation: `{a[2:0], b[4:3]}, {{3{a}}, b}`
- Logistic operation: `&a` `^b` `|c`
- Ternary conditional operation: `is_zero ? 0 : A`
- Conversion between signed and unsigned: `$signed(a), $unsigned(b)`
- Vector slicing: `a[3:0], b[7-:4], b[4+:4]`
- Use Assignment to perform combinational logic:
  - `assign {a[2], a[0]} = 2'b10;`
  - `begin a = b; c = a; end`

Verilog LAB

- Design Digital Systems using FPGA and Vivado

- 请自行下载Vivado (Xilinx官方网站需要下载过程科学上网，可搜索百度网盘等，10G左右)