

# 模拟与数字电路

## Analog and Digital Circuits



课程主页 扫一扫

### 第二十一讲：滤波器、运算放大器

Lecture 21: Filters and Operational Amps

主 讲：陈 迟 晓

Instructor : Chixiao Chen

# 提纲

- 复习
  - 从时域到频域的变化?
  - 无源滤波器
  - 运算放大器
  - 运算放大器的应用

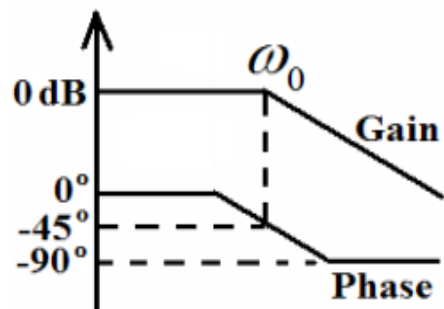


# 波特图的总结

总结:

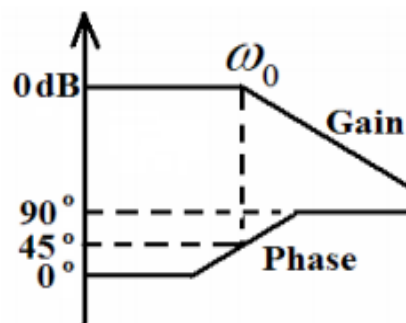
LHP:

$$\frac{1}{1 + s / \omega_0}$$
$$\frac{1}{1 + j(\omega / \omega_0)}$$



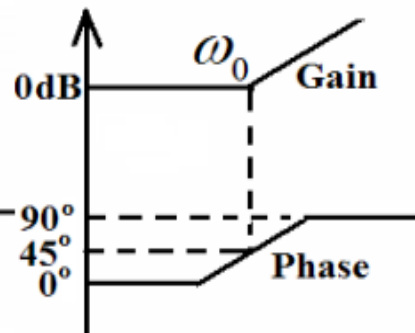
RHP:

$$\frac{1}{1 - s / \omega_0}$$
$$\frac{1}{1 - j(\omega / \omega_0)}$$



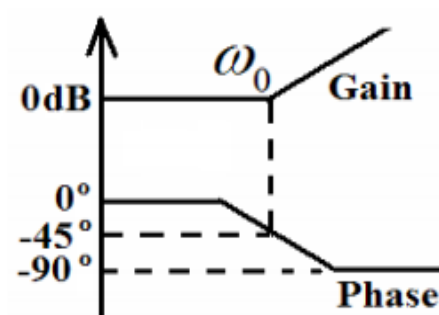
LHZ:

$$1 + s / \omega_0$$
$$1 + j(\omega / \omega_0)$$



RHZ:

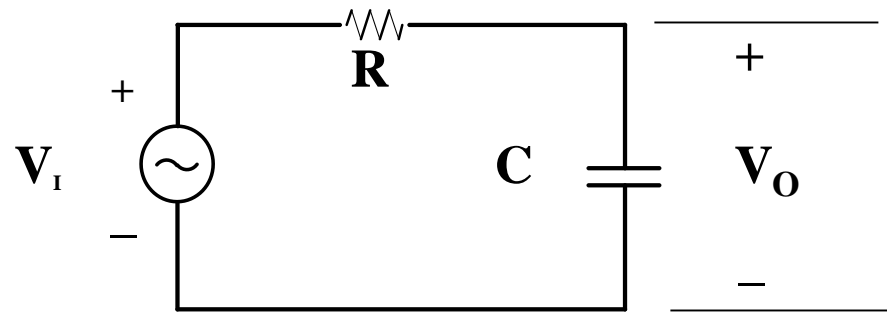
$$1 - s / \omega_0$$
$$1 - j(\omega / \omega_0)$$



规律非常明显，即便不记得了，推一遍也并不难。

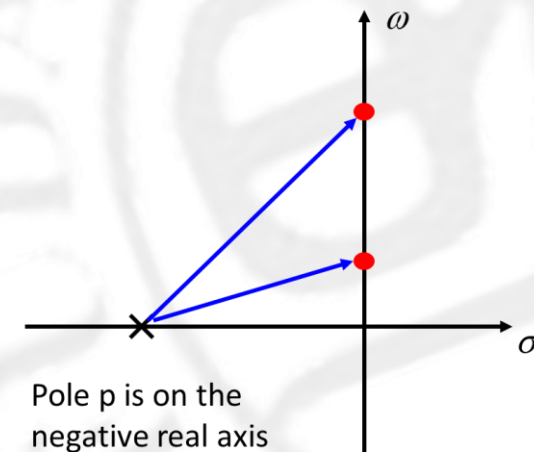
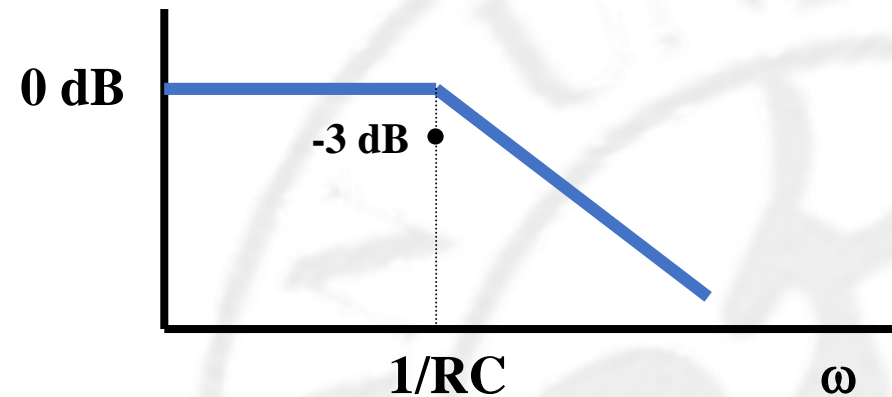
# 低通滤波器

- 一阶无源低通滤波器 (first-order passive low-pass filter)



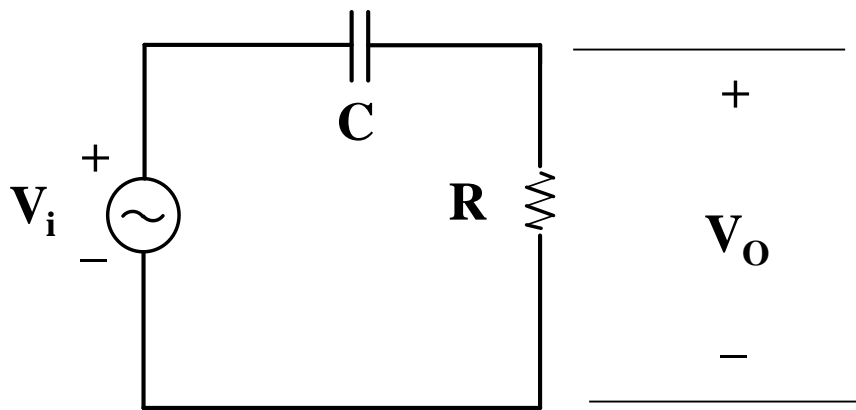
Low pass filter circuit

$$\frac{V_o(j\omega)}{V_i(j\omega)} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$



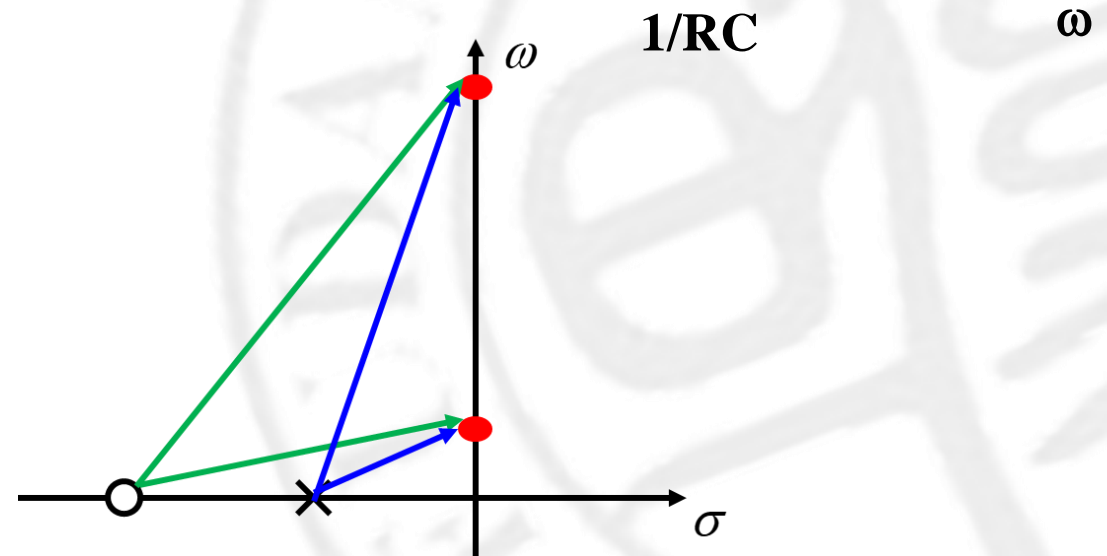
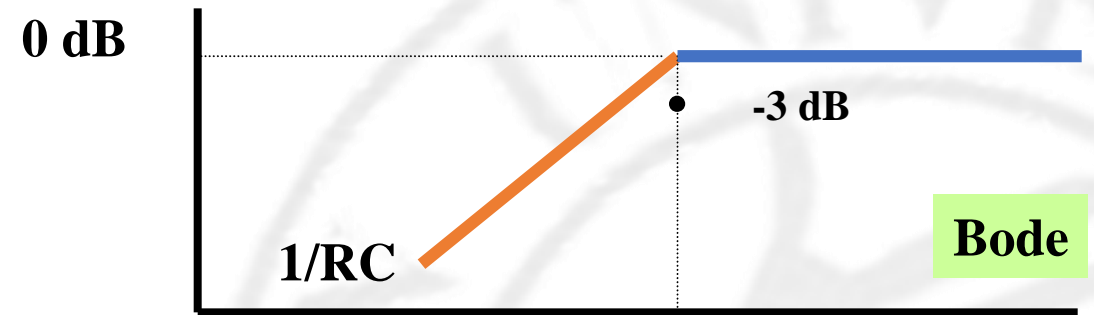
# 高通滤波器

- 一阶无源高通滤波器 (first-order passive high-pass filter)



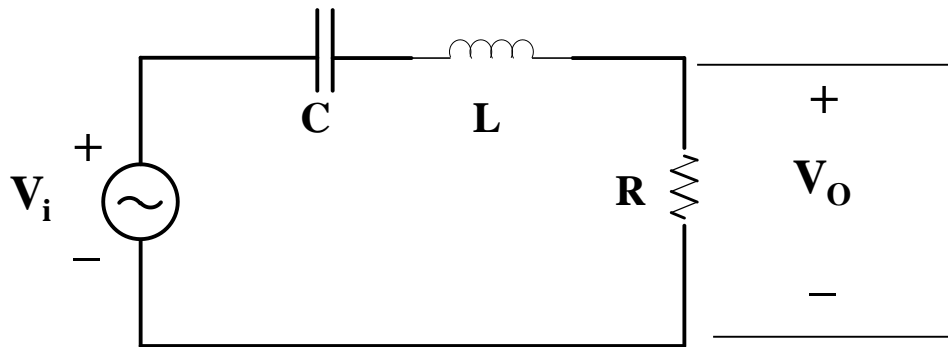
High Pass Filter

$$\frac{V_o(j\omega)}{V_i(j\omega)} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega RC}{1 + j\omega RC}$$

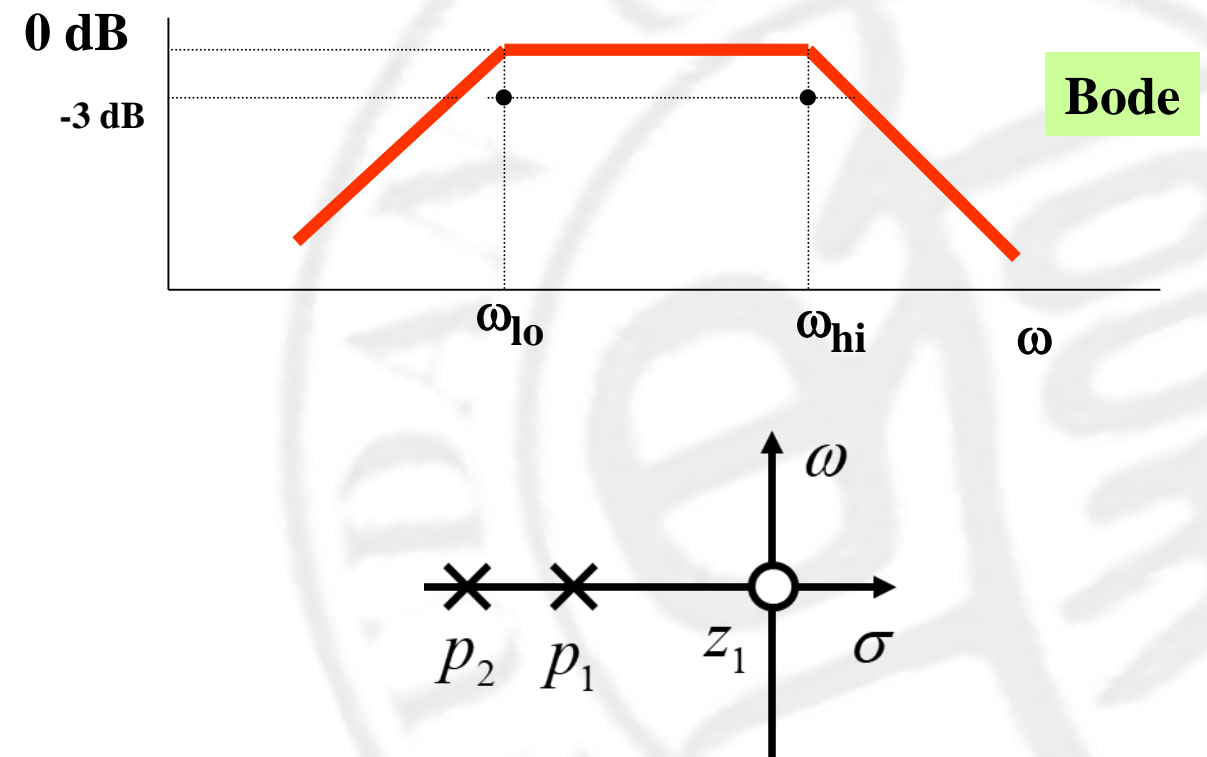


# 帶通濾波器

- 无源帶通濾波器 (first-order passive band-pass filter)

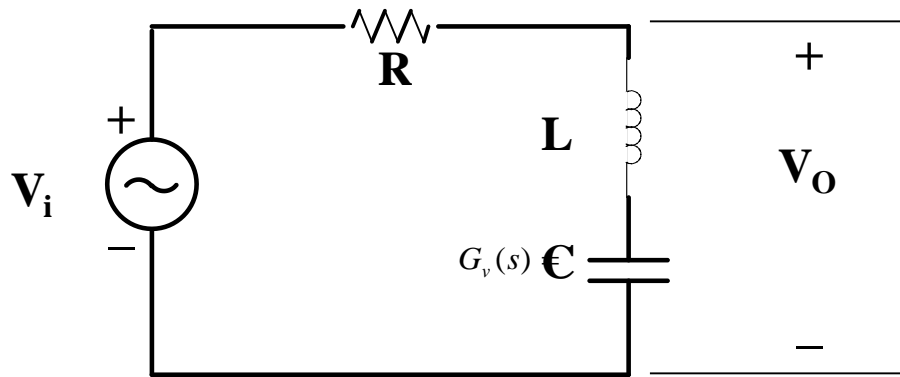


$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{R}{L}s}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

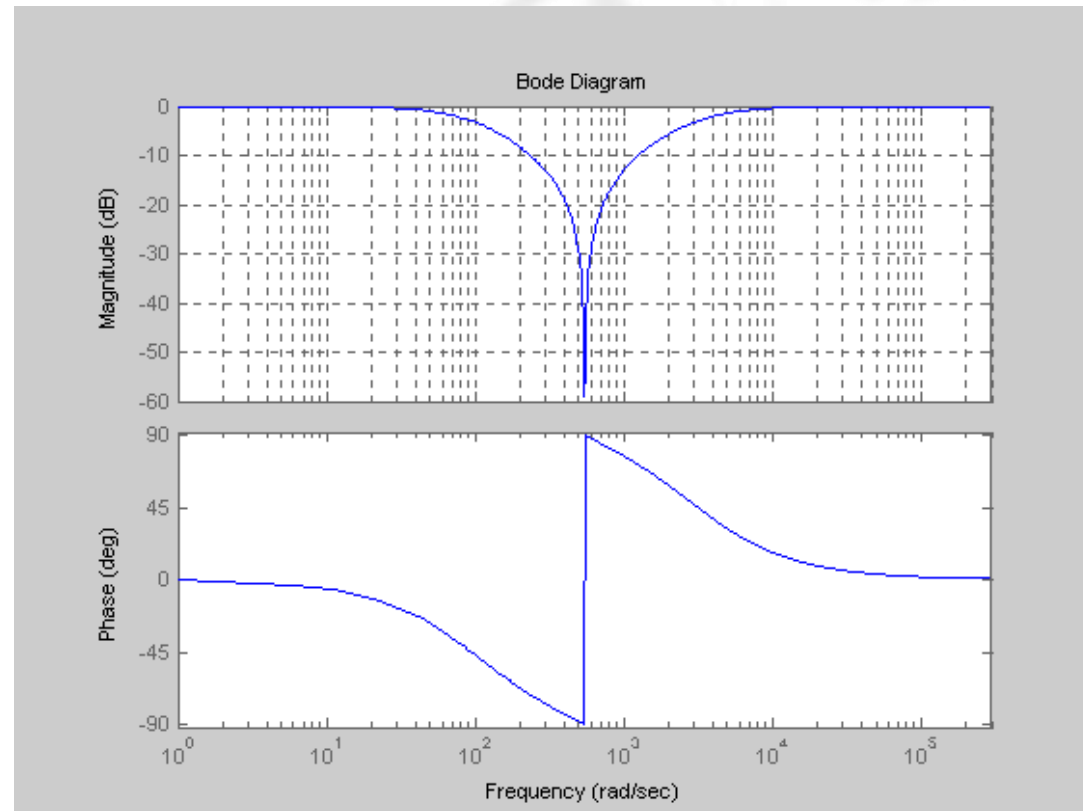


# 帶阻濾波器

- 无源帶阻濾波器 (first-order passive band-stop filter)

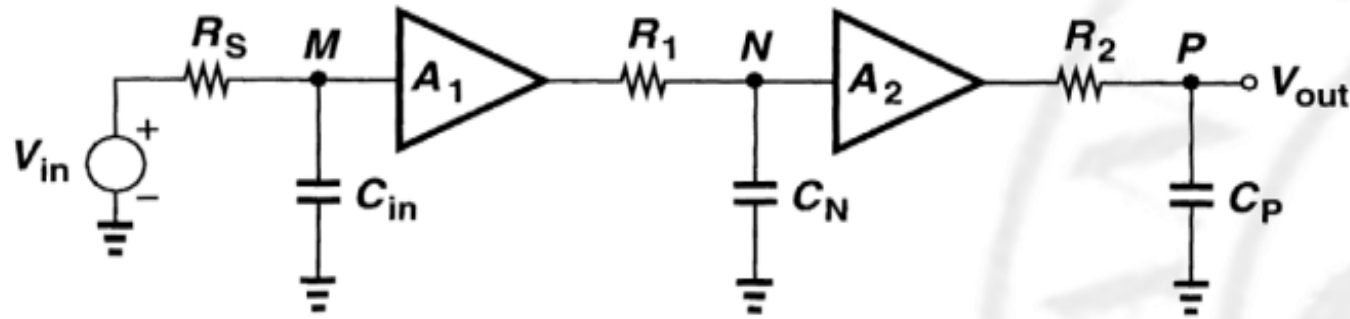


$$G_v(s) = \frac{s^2 + \frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$



# 高阶有源滤波器

- 对于一个电路的结点，如果有电容和电阻连接在这个结点上，那么通常这个结点会给最终的表达式贡献一个极点：



$$\begin{aligned}\frac{V_{out}}{V_{in}}(s) &= \frac{1}{1 + R_S C_{in} s} \cdot \frac{A_1}{1 + R_1 C_N s} \cdot \frac{A_2}{1 + R_2 C_P s} \\ &= 1 \cdot A_1 A_2 \frac{1}{1 + s / \omega_{pM}} \cdot \frac{1}{1 + s / \omega_{pN}} \cdot \frac{1}{1 + s / \omega_{pP}}\end{aligned}$$



# 放大器的非线性

- 典型建模方案：泰勒展开

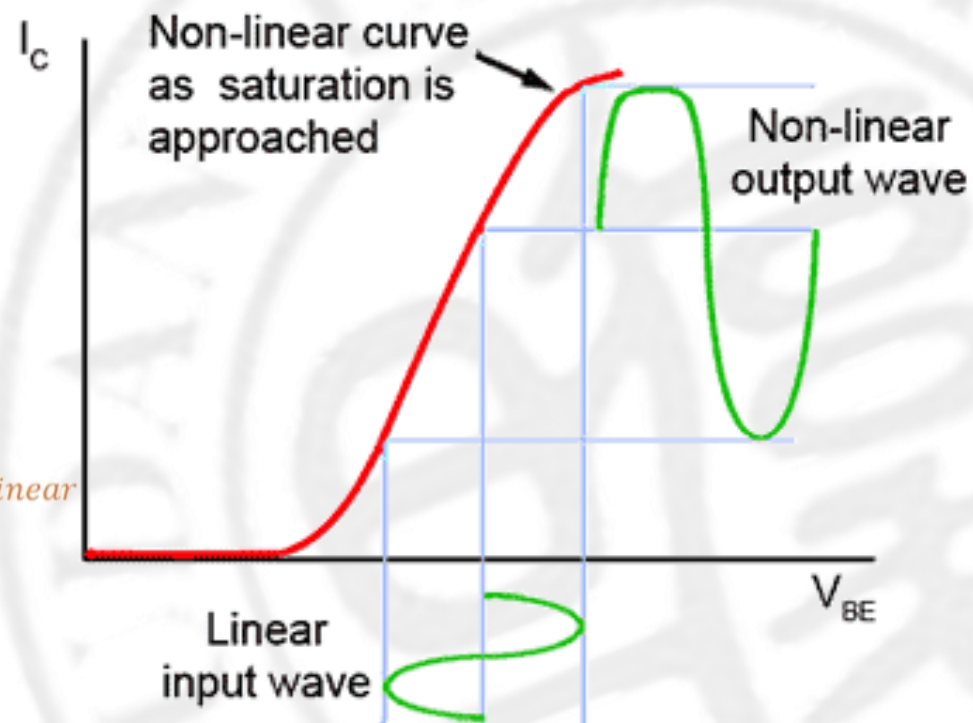
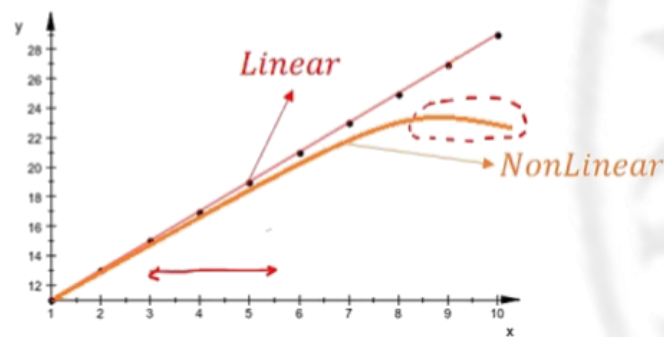
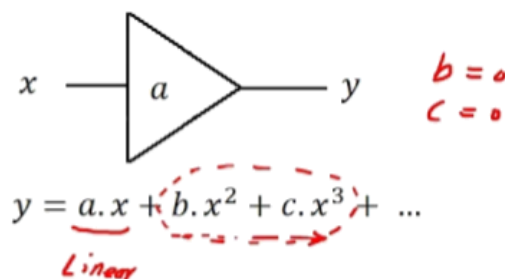
## Non-Linear Amplifier

- $x$  = input
- $y$  = output

$$y = a.x + b.x^2 + \textcircled{c}.x^3 + \dots$$

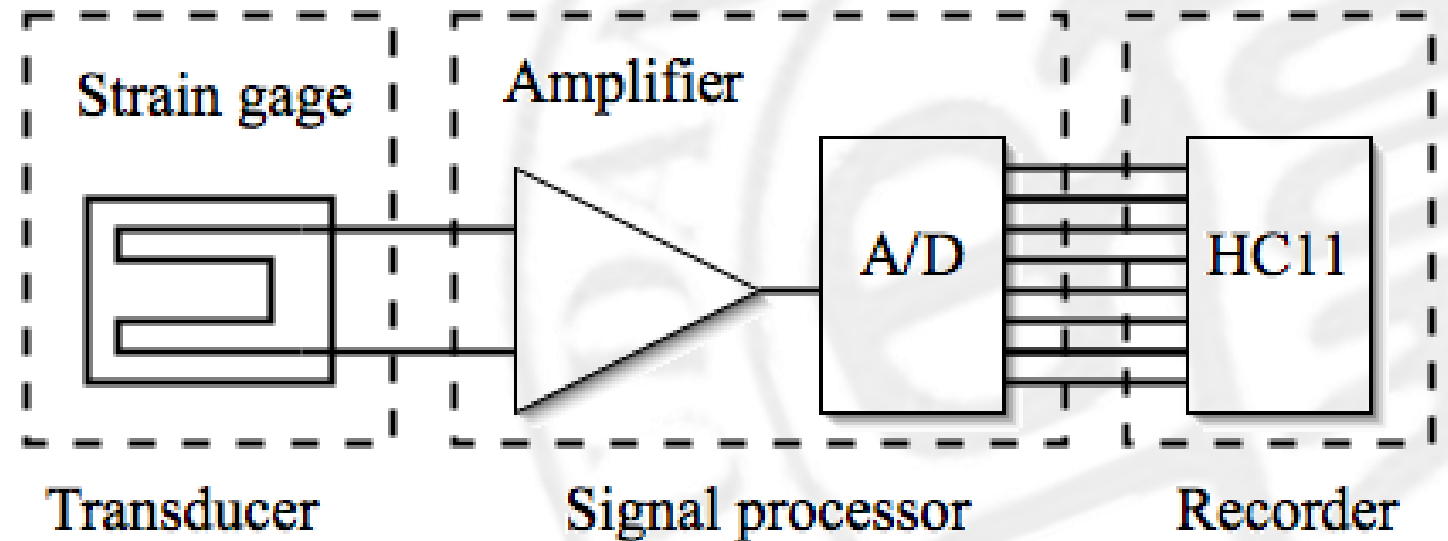
- Generally  $c < 0$

$x \rightarrow 10\mu$



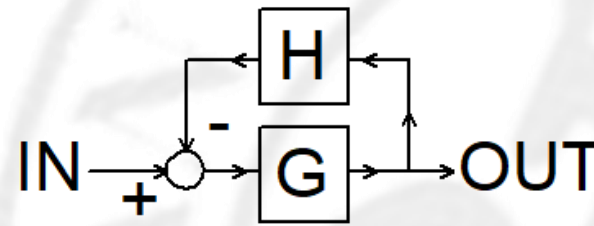
# 运算放大器

- Sensor signals are often too weak or too noisy
- 目标1： 以固定的比例线性放大信号
- 目标2： 将某些频率滤去



# 负反馈原理

- 负反馈：将输出按一定比例反馈到输入
- 通过信号流图得到负反馈的表达式
- 当增益G不恒定，但保持远远大于1时
- 增益由H的大小决定
- 实际上当， $GH \gg 1$ 时  
G的输入摆幅非常小



$$\text{Gain} = \frac{G}{1 + GH}$$

$$\text{Gain} = \frac{1}{H} \times \frac{GH}{1 + GH}$$

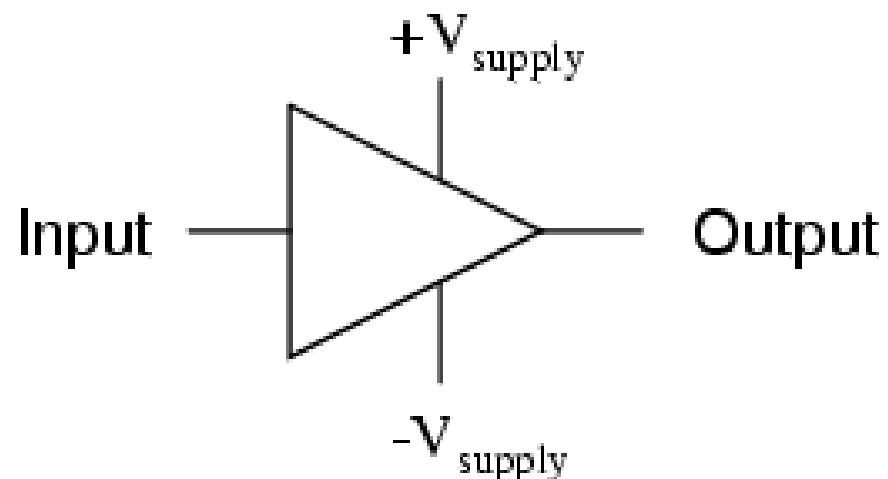
Loop gain is not GH.

Loop gain is -GH.

# 如何实现减法？

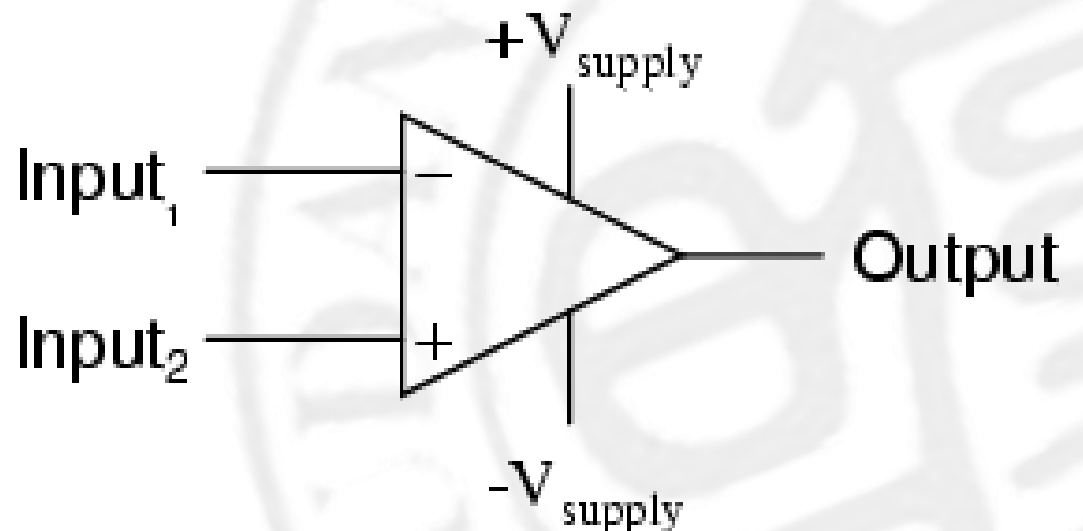
- 单端放大器

*General amplifier circuit symbol*



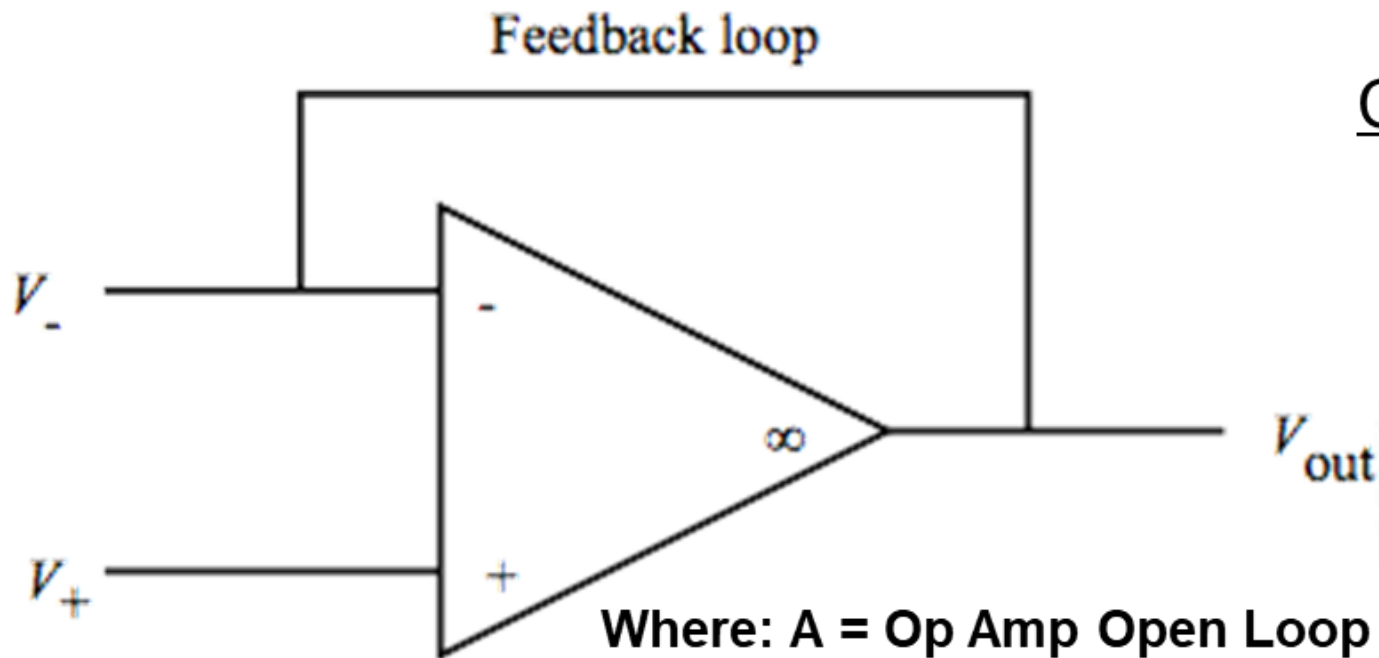
- 差分放大器

*Differential amplifier*



# 放大电路 应用实例

- 将输出直接连接到负端输入



Where:  $A$  = Op Amp Open Loop Gain

$F$  = Feedback Loop Gain

Closed Loop Transfer Function

$$H(s) = A / (1 + AF)$$

When  $AF \gg 1$ ...

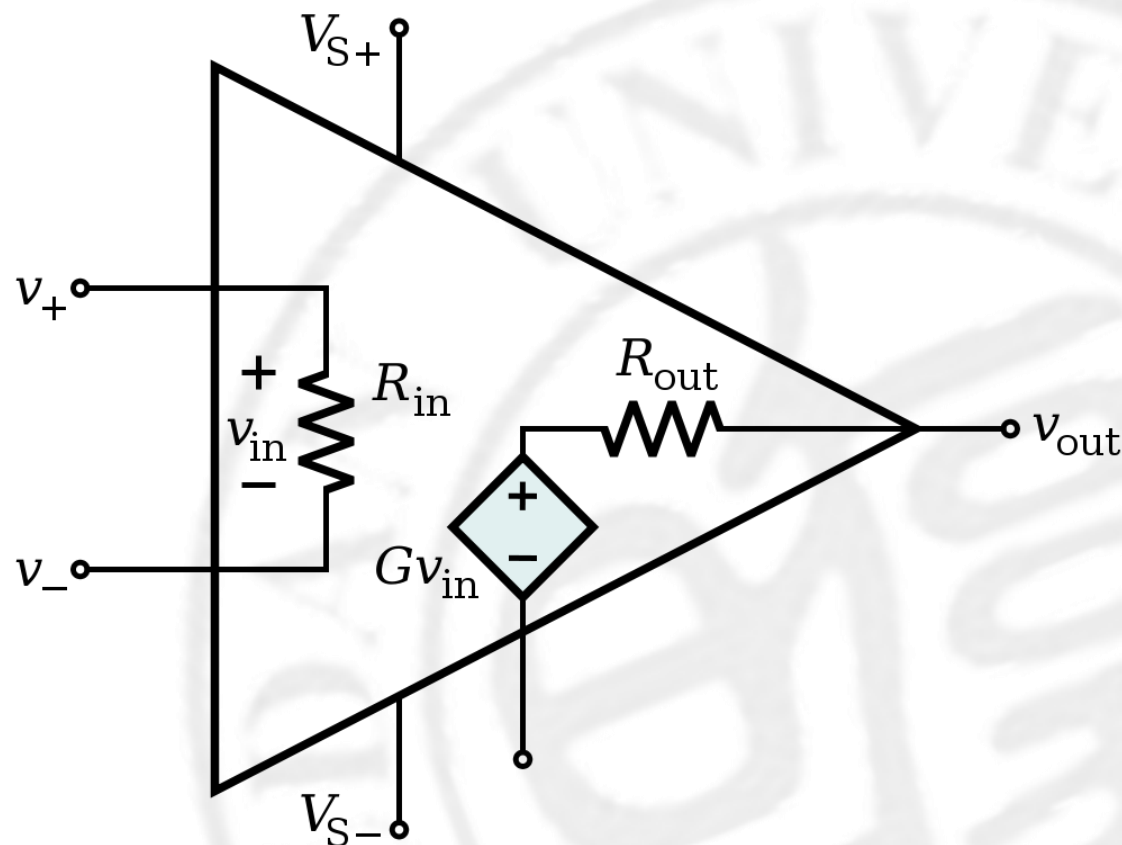
$$H(s) = 1 / F$$

# 理想运算放大器 参数

- 电压增益：无限大（大于 10000）
- 输入阻抗：无限大（开路）
- 输出等效电阻：零（实际上如果负载也是无限大可以不满足）
- 频率响应为固定值

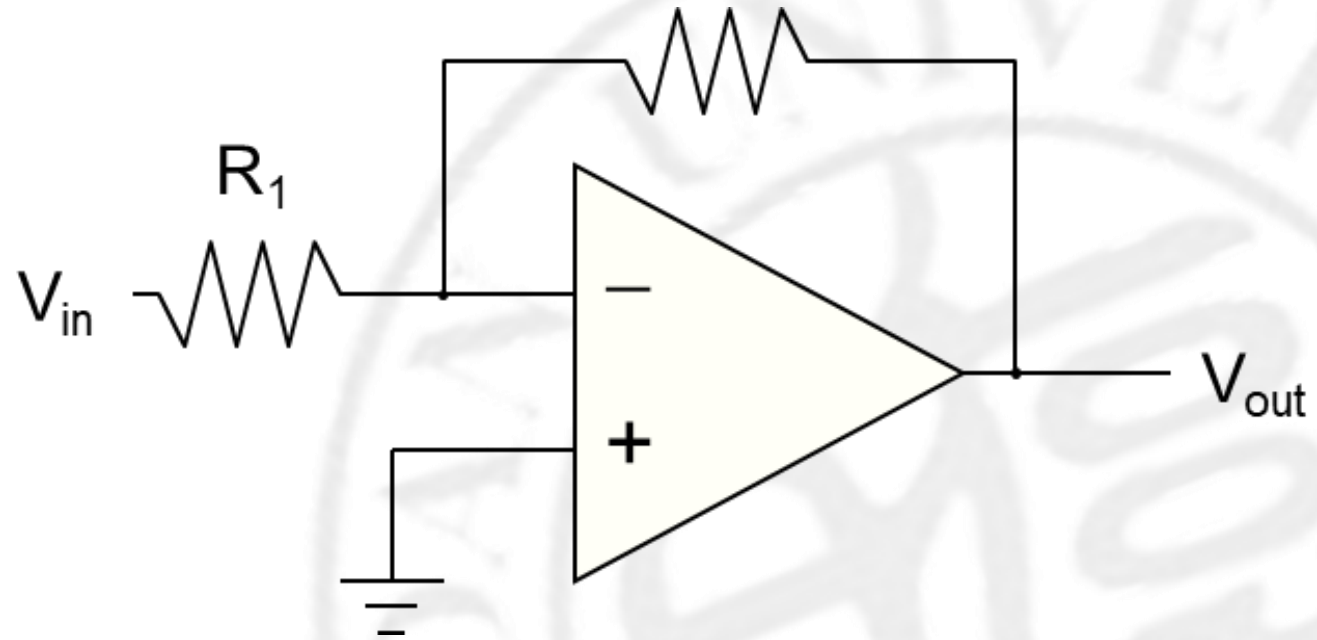
→ 虚短

→ 虚断



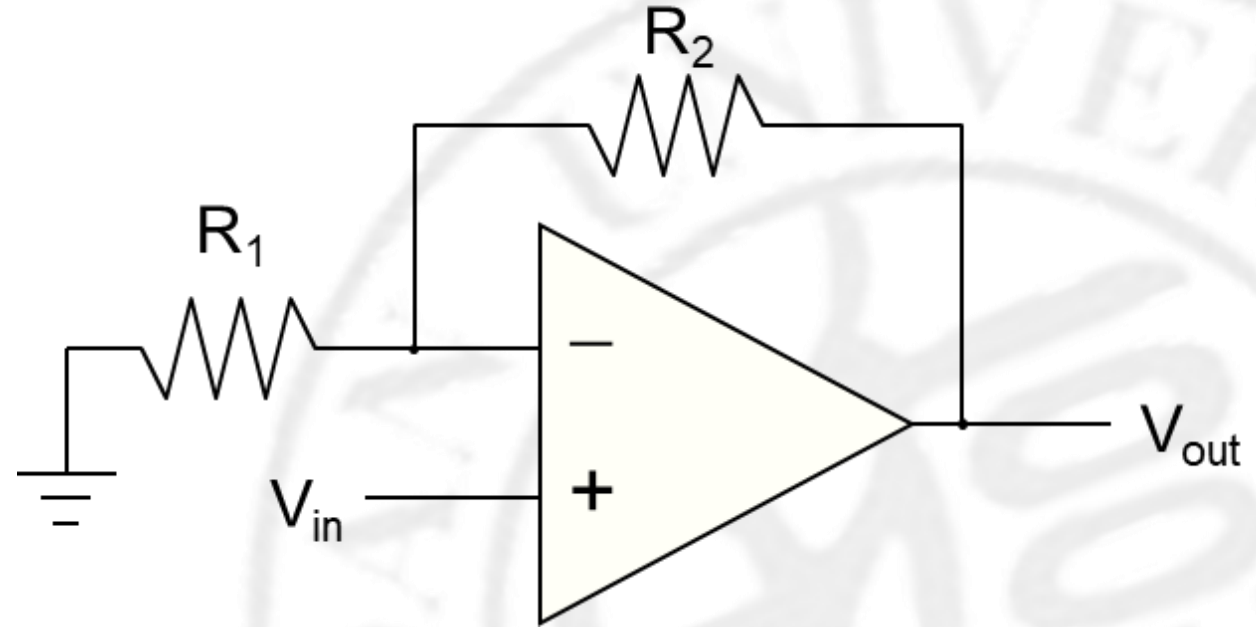
# 反向放大器

- Current through  $R_1$  is  $I_f = V_{in}/R_1$
- the current through  $R_1$  must go through  $R_2$
- Voltage drop across  $R_2$  is then  $I_f R_2 = V_{in} \times (R_2/R_1)$
- $V_{out} = 0 - V_{in} \times (R_2/R_1) = -V_{in} \times (R_2/R_1)$
- Amplify  $V_{in}$  by factor  $-R_2/R_1$ 
  - negative sign  $\rightarrow$  “inverting”



# 非反向放大器

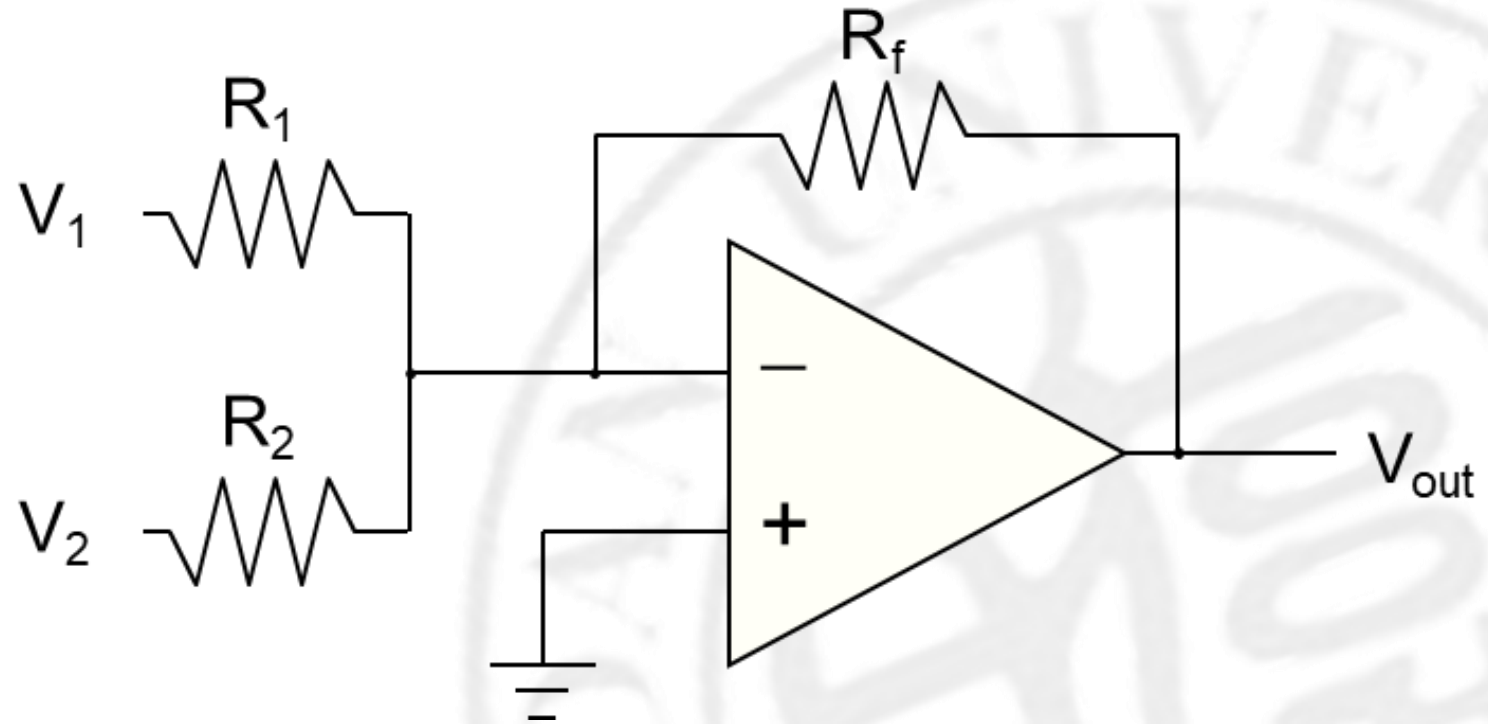
- Current through  $R_1$  is  $I_f = V_{in}/R_1$  (to left, into ground)
- Voltage drop across  $R_2$  is  $I_f R_2 = V_{in} \times (R_2/R_1)$
- $V_{out} = V_{in} + V_{in} \times (R_2/R_1)$   
 $= V_{in} \times (1 + R_2/R_1)$
- The gain is  $(1 + R_2/R_1)$ , and is positive (non-inverting)





# 模拟加权加法器

- inverting input still held at virtual ground
- $I_1$  and  $I_2$  are added together to run through  $R_f$
- Get the (inverted) sum:  
 $V_{out} = -R_f \times (V_1/R_1 + V_2/R_2)$
- 加权系数由电阻的比例决定



# 模拟加权减法器

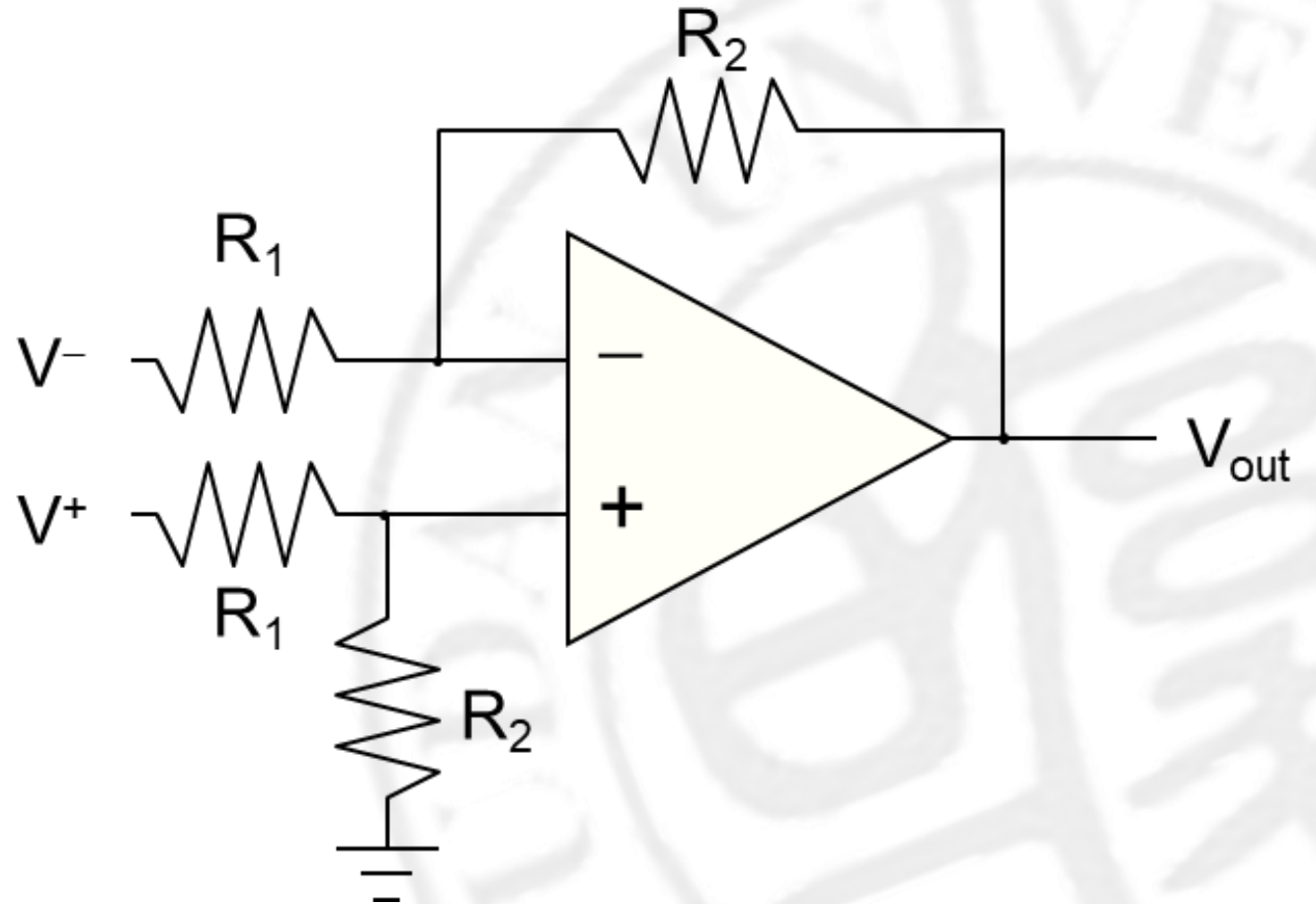
- 运放正端输入

$$V_{\text{node}} = V^+ R_2 / (R_1 + R_2)$$

- 运放负端输入

- $I_f = (V^- - V_{\text{node}}) / R_1$

- $V_{\text{out}} = (R_2 / R_1)(V^+ - V^-)$

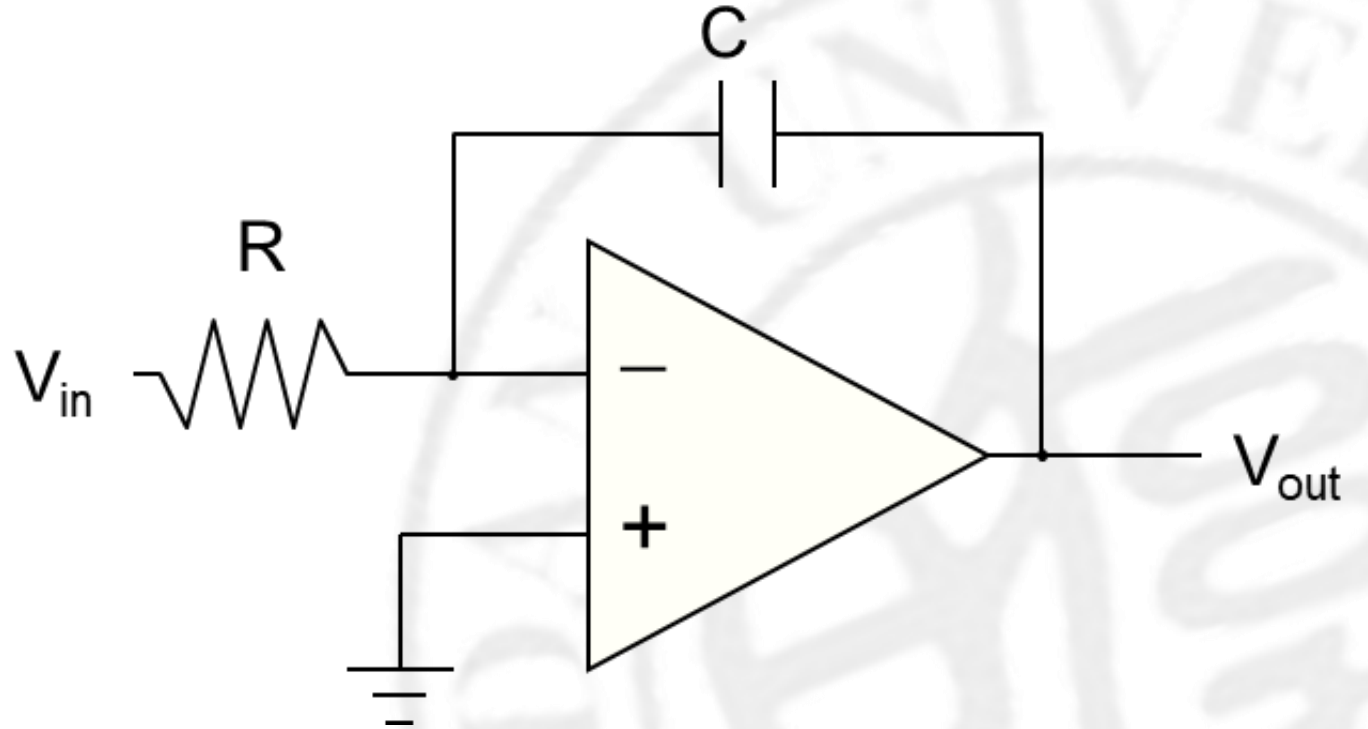


# 积分器（低通滤波器）

- $I_f = V_{in}/R$
- $C \cdot dV_{cap}/dt = V_{in}/R$

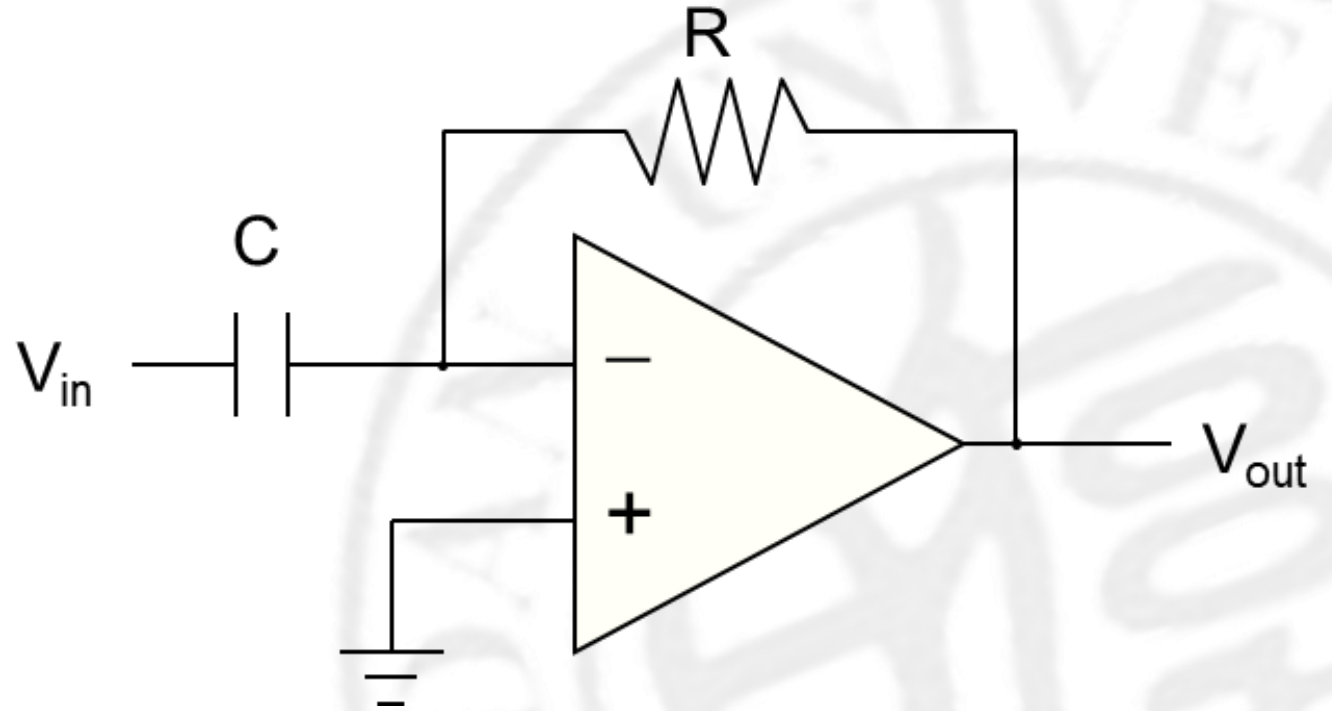
$$V_{out} = -\frac{1}{RC} \int V_{in} dt$$

- 将电容的等效阻值  $1/sC$   
代入反向放大器

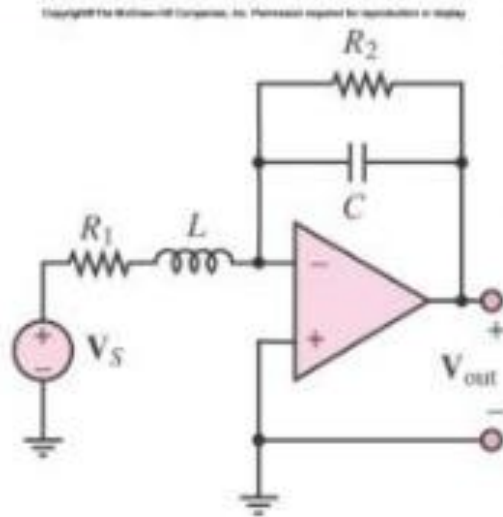


# 微分器（高通滤波器）

- $Q = CV$
- $I_{\text{cap}} = dQ/dt = C \cdot dV/dt$
- $V_{\text{out}} = -I_{\text{cap}}R = -RC \cdot dV/dt$
- 将电容的等效阻值  $1/sC$  代入反向放大器



# 二阶低通滤波器



$R_1$ ,  $R_2$ ,  $C$  and  $L$  are specially chosen so that:

$$\omega_0 = 1/(CR_2) = R_1/L.$$

$$H_v(j\omega) = \frac{V_{out}}{V_S}(j\omega) = -\frac{R_2}{R_1(1 + j\omega/\omega_0)^2}$$

Above  $\omega_0$ ,  $H_v$  is reduced by a factor of **100** for a ten fold increase in  $\omega$  (40dB drop per decade)

First-order filter:  $H_v$  is reduced by a factor of **10** for a ten fold increase in  $\omega$  (20dB drop per decade)

# 高阶低通滤波器

Third-Order Low pass Butterworth Filter Circuit

