Risky Bond Pricer 평가로직 설명서

1. Notation

   Survive Probability: $\Pr[\tau > T] = e^{-\int_0^T \lambda_t dt}$

   Marginal Hazard Rate: $\Pr[t < \tau < t + dt] = \lambda_t e^{-\int_0^t \lambda_s ds} dt$

   Default Probability: $\Pr[\tau \leq T] = \int_0^T \lambda_t e^{-\int_0^t \lambda_s ds} dt$

2. 리커버리가 없는 Risky Zero Coupon Bond

$$\hat{B}(0,T) = E\left[e^{-\int_0^T r_t dt} \cdot 1_{\tau>T}\right] = E\left[E\left[e^{-\int_0^T r_t dt} \cdot 1_{\tau>T} | \{r_t, \lambda_t\}_{t\in[0,T]}\right]\right]$$
$$= E\left[e^{-\int_0^T r_t dt} E\left[1_{\tau>T} | \{r_t, \lambda_t\}_{t\in[0,T]}\right]\right]$$
$$= E\left[e^{-\int_0^T (r_s+\lambda_s) ds}\right] \approx Z(0,T)Q(0,T)$$

3. 파산시 1을 주는 증권의 가치

$$\hat{D}(0,T) = E\left[e^{-\int_0^T r_t dt} \cdot 1_{\tau<T}\right] = E\left[E\left[e^{-\int_0^T r_t dt} \cdot 1_{\tau<T} | \{r_t, \lambda_t\}_{t\in[0,T]}\right]\right] = E\left[\int_0^T \lambda_t e^{-\int_0^t (r_s+\lambda_s)\, ds} dt\right]$$
$$= \int_0^T Z(0,t) E\left[\lambda_t e^{-\int_0^t \lambda_s ds}\right] dt = \int_0^T Z(0,t) d\big(-Q(0,t)\big)$$

4. 리커버리가 존재하는 제로쿠폰채의 가치

$$B(0,T) = Z(0,T)Q(0,T) + RR \cdot \int_0^T Z(0,t) d\big(-Q(0,t)\big)$$

5. CDS 평가모듈 내 부도확률 추정 Calibration 로직

$$B(0,T) = e^{-RiskyZeroRate \cdot T} = Z(0,T)Q(0,T) + RR \cdot \int_0^T Z(0,t) d\big(-Q(0,t)\big)$$

위 식을 통해 λ를 역산

6.