IRStructuredSwapOneCurve 평가 설명서

1. 모듈설명

해당 모듈은 한 개의 커브를 사용하는 Callable 스왑(Range Accrual 제외)의 가격을 평가하는 모듈이다. 평가로직으로 HW 1F, 2F FDM를 사용한다.

2. 평가가능상품

1.1 Simple Callable Swap - Hull White 1Factor

1.2 Range 충족시 쿠폰 지급 스왑(Accrual 제외) - Hull White 1Factor

1.3 Spread Callable Swap - Hull White 2Factor

1.4 Spread Range 충족시 쿠폰 지급 스왑(Accrual 제외) - Hull White 2Factor

3. 평가방법1 – HW 1Factor IFDM

Discount Factor

$$\mathrm{P(t,T)} = \hat{E}\left(\mathrm{e}^{-\int_{\mathrm{t}}^{\mathrm{T}} r(u)du}\right)$$

The instantaneous spot rate, called “short rate”, is the interest rate r(t).

HW 1F Dynamics of Short Rate

$$\mathrm{dr(t)} = [\theta(\mathrm{t}) - \kappa \cdot r(t)]dt + \sigma(t)dW = \kappa\left[\frac{\theta(\mathrm{t})}{\kappa} - r(t)\right]dt + \sigma(t)dW$$

if $r(t) > \frac{\theta(\mathrm{t})}{\kappa}$ Then Drift have (-) else (+)

$$\mathrm{r(t)} = \alpha(\mathrm{t}) + x(t)$$

$$\mathrm{d}\alpha(\mathrm{t}) = [\theta(t) - \kappa\alpha(t)]dt$$

$$\mathrm{d}x(t) = -\kappa x(t)dt + \sigma dW$$

HW 1F PDE

$$\mathrm{U_t} = \kappa x \mathrm{U}_x - \frac{1}{2}\sigma^2 \mathrm{U}_{xx} + [\alpha(t) + x]\mathrm{U}$$

$$\alpha(\mathrm{t}) = -\frac{\delta P(0,t)}{\delta t} + \frac{\sigma^2}{2\kappa^2}(1 - e^{-\kappa t})^2$$

※ $[\alpha(t) + x]$ 대신 $F(t, t + dt)$를 사용

Implicit Finite Difference Method

$$\frac{\mathrm{u_i^{k+1} - u_i^k}}{\mathrm{h_t}} = \kappa \mathrm{x} \cdot \frac{\left(\mathrm{u_{i+1}^k - u_{i-1}^k}\right)}{2\mathrm{h_x}} - \frac{1}{2}\sigma^2 \frac{\left(\mathrm{u_{i+1}^k + u_{i-1}^k - 2u_i^k}\right)}{\mathrm{h_x^2}} + [\alpha(t_{k+1}) + \mathrm{x}_i]\mathrm{u_i^{k+1}}$$

$$\mathrm{u_i^{k+1} - u_i^k} = \mathrm{h_t}\kappa \mathrm{x} \cdot \frac{\left(\mathrm{u_{i+1}^k - u_{i-1}^k}\right)}{2\mathrm{h_x}} - \frac{\mathrm{h_t}}{2}\sigma^2 \frac{\left(\mathrm{u_{i+1}^k + u_{i-1}^k - 2u_i^k}\right)}{\mathrm{h_x^2}} + \mathrm{h_t}[\alpha(t_{k+1}) + \mathrm{x}_i]\mathrm{u_i^{k+1}}$$

$$\mathrm{u_i^{k+1}}(1 - h_t[\alpha(t_{k+1}) + \mathrm{x}_i]) = \left[-\frac{\mathrm{h_t}\kappa x_i}{2h_x} - \frac{h_t\sigma^2}{2h_x^2}\right]u_{i-1}^k + \left[1 + \frac{\mathrm{h_t}\sigma^2}{\mathrm{h_x^2}}\right]u_i^k + \left[\frac{\mathrm{h_t}\kappa x_i}{2h_x} - \frac{h_t\sigma^2}{2h_x^2}\right]u_{i+1}^k$$

$$\mathrm{u_i^{k+1}}W_i = A_i u_{i-1}^k + B u_i^k + C_i u_{i+1}^k$$

$$\mathrm{W_i} = (1 - h_t[\alpha(t_{k+1}) + \mathrm{x}_i])$$

$$A_i = \left[-\frac{\mathrm{h_t}\kappa x_i}{2h_x} - \frac{h_t\sigma^2}{2h_x^2}\right]$$

$$B = \left[1 + \frac{\mathrm{h_t}\sigma^2}{\mathrm{h_x^2}}\right]$$

$$C_i = \left[\frac{\mathrm{h_t}\kappa x_i}{2h_x} - \frac{h_t\sigma^2}{2h_x^2}\right]$$

Boundary Condition

$u_{-1} = 2u_0 - u_1$

$u_{N+1} = 2u_N - u_{N-1}$

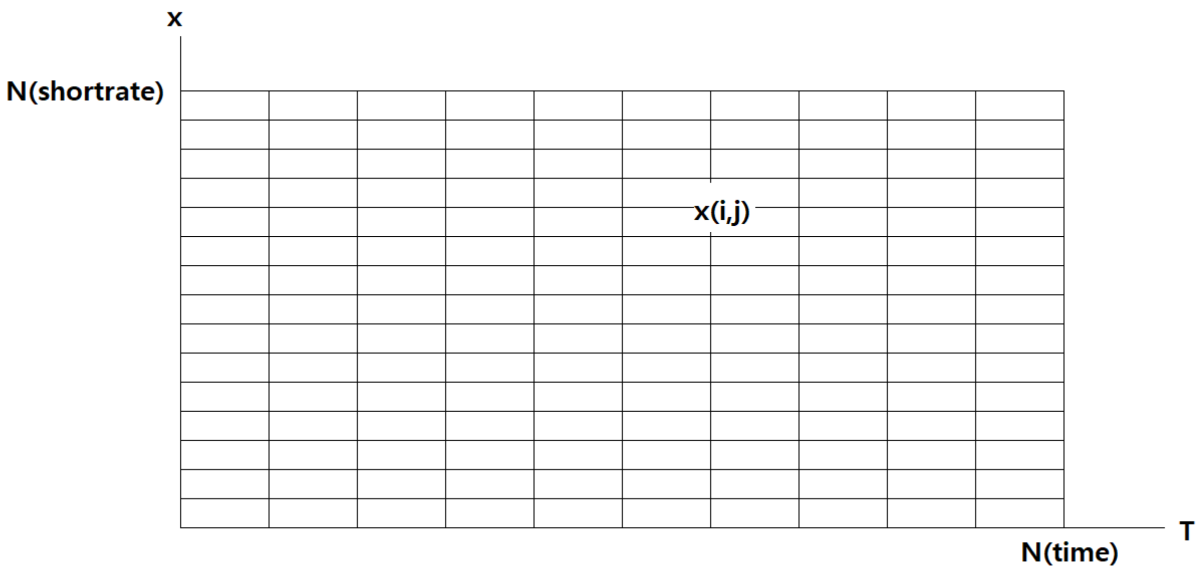
$\mathrm{u_0^{k+1}}W_0 = A_0 u_{-1}^k + B u_0^k + C_i u_1^k = A_0(2u_0^k - u_1^k) + B u_0^k + C_0 u_1^k$

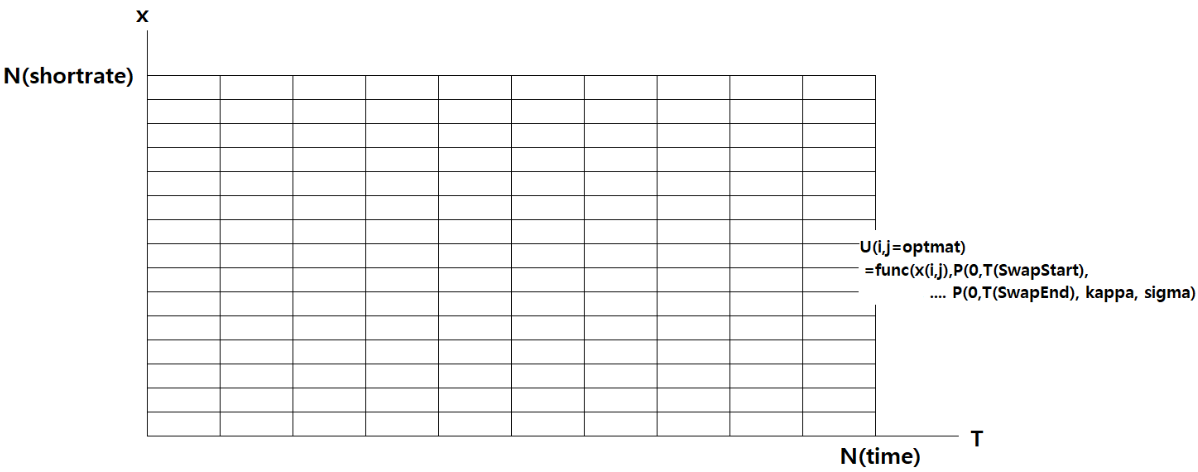
$\mathrm{u_0^{k+1}}W_0 = u_0^k(2A_0 + B) + u_1^k(C_0 - A_0)$

$\mathrm{u_N^{k+1}}W_N = A_N u_{N-1}^k + B u_N^k + C_i u_{N+1}^k = A_N u_{N-1}^k + B u_N^k + C_N\left(2u_N^k - u_{N-1}^k\right)$

$$\mathrm{u}_{\mathrm{N}}^{\mathrm{k+1}} W_N = \mathrm{u}_{\mathrm{N-1}}^{k}(A_N - C_N) + u_N^k (B + 2C_N)$$

$$\begin{bmatrix} 2A_0+B & C_0-A_0 & 0 & 0 & \dots & \dots & 0 & 0 & 0 \\ A_1 & B & C_1 & 0 & \dots & \dots & 0 & 0 & 0 \\ 0 & A_2 & B & C_2 & \dots & \dots & 0 & 0 & 0 \\ 0 & 0 & A_3 & B & \dots & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & 0 & 0 & A_{N-2} & B & C_{N-2} & 0 \\ \dots & \dots & \dots & 0 & 0 & 0 & A_{N-1} & B & C_{N-1} \\ \dots & \dots & \dots & 0 & 0 & 0 & 0 & A_N - C_N & B+2C_N \end{bmatrix} \begin{bmatrix} u_0^k \\ u_1^k \\ u_2^k \\ u_3^k \\ \dots \\ \dots \\ u_{N-2}^k \\ u_{N-1}^k \\ u_N^k \end{bmatrix} = \begin{bmatrix} W_0 u_0^{k+1} \\ W_1 u_1^{k+1} \\ W_2 u_2^{k+1} \\ W_3 u_3^{k+1} \\ \dots \\ \dots \\ W_{N-2} u_{N-2}^{k+1} \\ W_{N-1} u_{N-1}^{k+1} \\ W_N u_N^{k+1} \end{bmatrix}$$





HW Forward Discount Factor

$$P(t,T) = \hat{E}\left(e^{-\int_t^T r(u)du}\right) = \hat{E}\left(e^{-\int_t^T (x(u)+\alpha(u))du}\right)$$

여기서 $\int_t^T x(u)du$는 평균 x(t)B(t,T), 분산 V(t,T)인 정규분포를 따른다.

$$B(t,T) = \frac{1-e^{-\kappa(T-t)}}{\kappa}$$

$$V(t,T) = \int_t^T \sigma^2(u)B^2(u,T)du \approx \int_t^T \frac{\sigma^2\left[1-2e^{-\kappa(T-u)}+e^{-2\kappa(T-u)}\right]}{\kappa^2}du$$

$$= \frac{\sigma^2}{\kappa^2}\left(T-t+2\frac{e^{-\kappa(T-t)}-1}{\kappa}-\frac{e^{-2\kappa(T-t)}-1}{2\kappa}\right)$$

따라서 위험 중립 측도에서 $P(t,T)$는 정규분포의 적률생성함수(Moment Generate Function)에 따라 다음과 같다.

※ $E(e^{tx}) = e^{\mu t+\frac{1}{2}(\sigma t)^2}$ MGF of Normal Distribution 따라서,

$$P(t,T) = \hat{E}\left(e^{-\int_t^T \alpha(u)du - x(t)B(t,T)+\frac{1}{2}V(t,T)}\right)$$

0시점 시장에서 관측된 만기 T인 Zero Bond $P^M(0,T)$가 다음을 만족한다.

$$P^M(0,T) = e^{-\int_0^T \alpha(u)du+\frac{1}{2}V(0,T)}$$

$$e^{-\int_0^T \alpha(u)du} = P^M(0,T)e^{-\frac{1}{2}V(0,T)}$$

따라서

$$\mathrm{e}^{-\int_{\mathrm{t}}^{\mathrm{T}}\alpha(u)du} = \frac{P^{\mathrm{M}}(0,T)}{P^{M}(0,t)}e^{-\frac{1}{2}\left(V(0,T)-V(0,t)\right)}$$

$$\mathrm{P}_{\mathrm{HW}}^{x_i}(\mathrm{t},\mathrm{T}) = \frac{P^{\mathrm{M}}(0,T)}{P^{M}(0,t)}\exp\left(-x_iB(t,T)+\frac{1}{2}\left(V(t,T)-V(0,T)+V(0,t)\right)\right)$$

$$\mathrm{P}_{\mathrm{HW}}^{x_i}(\mathrm{t},\mathrm{T}) = \frac{P^{\mathrm{M}}(0,T)}{P^{M}(0,t)}\exp\left(-x_i\times B(t,T)+QVTerm(t,T)\right)$$

4. 평가방법2 – HW 2Factor IFDM

$$\mathrm{U}_{\mathrm{t}}^{\mathrm{x}} = \kappa_1 x\mathrm{U}_x - \frac{1}{2}\sigma_1^2\mathrm{U}_{xx} + \frac{1}{2}F(t,t+dt)\mathrm{U} - \frac{1}{2}\rho_{12}\sigma_1\sigma_2 U_{xy}$$

$$\mathrm{U}_{\mathrm{t}}^{\mathrm{y}} = \kappa_2 y\mathrm{U}_y - \frac{1}{2}\sigma_2^2\mathrm{U}_{yy} + \frac{1}{2}F(t,t+dt)\mathrm{U} - \frac{1}{2}\rho_{12}\sigma_1\sigma_2 U_{xy}$$

Implicit Finite Difference Method to Xt

$$\mathrm{u}_{\mathrm{i,j}}^{\mathrm{k}+\frac{1}{2}} + \left(u^{k+\frac{1}{2}}{}_{i+1,j+1} + u^{k+\frac{1}{2}}{}_{i-1,j-1} - u^{k+\frac{1}{2}}{}_{i+1,j-1} - u^{k+\frac{1}{2}}{}_{i-1,j+1}\right)\frac{\sigma_1\sigma_2\rho_{12}h_t}{2}\frac{1}{4h_xh_y}$$

$$= \mathrm{A_i}u_{i-1}^k + \mathrm{B_i}u_i^k + \Gamma_{\mathrm{i}}u_{i+1}^k$$

$$A_i = \left[-\frac{\mathrm{h_t}\kappa_1 x_i}{2h_x} - \frac{h_t\sigma_1^2}{2h_x^2}\right]$$

$$B = \left[1+\frac{\mathrm{h_t}\sigma_1^2}{\mathrm{h_x^2}}\right] + \frac{1}{2}F\left(t_k, t_{k+\frac{1}{2}}\right)dt$$

$$C_i = \left[\frac{\mathrm{h_t}\kappa_1 x_i}{2h_x} - \frac{h_t\sigma_1^2}{2h_x^2}\right]$$

Boundary Condition

$$u_{-1} = 2u_0 - u_1$$

$$u_{N+1} = 2u_N - u_{N-1}$$

$$\mathrm{u}_0^{\mathrm{k+1}}W_0 = A_0u_{-1}^k + Bu_0^k + C_iu_1^k = A_0(2u_0^k - u_1^k) + Bu_0^k + C_0u_1^k$$

$$\mathrm{u}_0^{\mathrm{k+1}}W_0 = u_0^k(2A_0+B) + u_1^k(C_0 - A_0)$$

$$\mathrm{u}_{\mathrm{N}}^{\mathrm{k+1}}W_N = A_Nu_{N-1}^k + Bu_N^k + C_iu_{N+1}^k = A_Nu_{N-1}^k + Bu_N^k + C_N\left(2u_N^k - u_{N-1}^k\right)$$

$$\mathrm{u}_{\mathrm{N}}^{\mathrm{k+1}} W_N = \mathrm{u}_{\mathrm{N-1}}^{k}(A_N - C_N) + u_N^k(B + 2C_N)$$

Implicit Finite Difference Method to Yt

$$\mathrm{u}_{\mathrm{i,j}}^{\mathrm{k+1}} + \left({u^{k+1}}_{i+1,j+1} + {u^{k+1}}_{i-1,j-1} - {u^{k+1}}_{i+1,j-1} - {u^{k+1}}_{i-1,j+1}\right)\frac{\sigma_1\sigma_2\rho_{12}h_t}{2}\frac{1}{4h_x h_y}$$

$$= \mathrm{A_j} u_{j-1}^{k+\frac{1}{2}} + \mathrm{B_j} u_j^{k+\frac{1}{2}} + \Gamma_{\mathrm{j}} u_{j+1}^{k+\frac{1}{2}}$$

$$A_i = \left[-\frac{\mathrm{h_t}\kappa_2 y_j}{2h_y} - \frac{h_t\sigma_2^2}{2h_y^2}\right]$$

$$B = \left[1 + \frac{\mathrm{h_t}\sigma_2^2}{\mathrm{h_y^2}}\right] + \frac{1}{2}\hat{F}dt$$

$$C_i = \left[\frac{\mathrm{h_t}\kappa_2 y_j}{2h_y} - \frac{h_t\sigma_2^2}{2h_y^2}\right]$$

5. 미래 ShortRate Greed별 커브 및 페이오프 계산

$\boldsymbol{P_{HW}(t,T) = E\left(\mathrm{e}^{-\int_t^T r_u du}\middle|F_t\right) = \mathrm{E}\left(\mathrm{e}^{-\int_t^T (x_u+\phi_u)du}|F_t\right)}$**이다.**

x_t는 정규분포를 따르고, 위 수식은 정규분포 적률생성함수(MGF) 형태이다.

$$\mathbf{E(e^{tx})} = \boldsymbol{e^{\mu t + \frac{1}{2}(\sigma t)^2}}$$

여기서 $\int_{\mathrm{t}}^{\mathrm{T}} x(u)du$는 평균 x(t)B(t,T), 분산 V(t,T)인 정규분포를 따른다.

$$\mathrm{B(t,T)} = \int_t^T e^{-\kappa(\bar{T}-u)}du = \frac{1-e^{-\kappa(T-t)}}{\kappa}$$

$$\mathrm{V(t,T)} = \int_{\mathrm{t}}^{\mathrm{T}} \sigma^2(u)B^2(u,T)du \approx \int_{\mathrm{t}}^{\mathrm{T}} \frac{\sigma^2\left[1-2e^{-\kappa(T-u)}+e^{-2\kappa(T-u)}\right]}{\kappa^2}du$$

$$= \frac{\sigma^2}{\kappa^2}\left(\mathrm{T - t + 2\frac{e^{-\kappa(T-t)}-1}{\kappa} - \frac{e^{-2\kappa(T-t)}-1}{2\kappa}}\right)$$

따라서 위험 중립 측도에서 $\mathrm{P(t,T)}$는 정규분포의 적률생성함수(Moment Generate Function)에 따라 다음과 같다.

※ $\mathrm{E}(\mathrm{e}^{\mathrm{tx}}) = e^{\mu t+\frac{1}{2}(\sigma t)^2}$ MGF of Normal Distribution 따라서,

$$\mathrm{P(t,T)} = \hat{E}\left(\mathrm{e}^{-\int_{\mathrm{t}}^{\mathrm{T}}\phi(u)du-x(t)B(t,T)+\frac{1}{2}V(t,T)}\right)$$

0시점 시장에서 관측된 만기 T인 Zero Bond $P^{\mathrm{M}}(0,T)$가 다음을 만족한다.

$$\mathrm{P^M}(0,T) = \mathrm{e}^{-\int_0^{\mathrm{T}}\alpha(u)du+\frac{1}{2}V(0,T)}$$

$$\mathrm{e}^{-\int_0^{\mathrm{T}}\alpha(u)du} = \mathrm{P^M}(0,T)\mathrm{e}^{-\frac{1}{2}V(0,T)}$$

따라서

$$\mathrm{e}^{-\int_{\mathrm{t}}^{\mathrm{T}}\alpha(u)du} = \frac{P^{\mathrm{M}}(0,T)}{P^{M}(0,t)}e^{-\frac{1}{2}(V(0,T)-V(0,t))}$$

$$\mathrm{P}_{\mathrm{HW}}^{x_i}(\mathrm{t,T}) = \frac{P^{\mathrm{M}}(0,T)}{P^{M}(0,t)}\exp\left(-x_iB(t,T)+\frac{1}{2}\big(V(t,T)-V(0,T)+V(0,t)\big)\right)$$

$$\mathrm{P}_{\mathrm{HW}}^{x_i}(\mathrm{t,T}) = \frac{P^{\mathrm{M}}(0,T)}{P^{M}(0,t)}\exp\big(-x_i\times B(t,T)+QVT(t,T)\big)$$

HW 2F Dynamics of Short Rate

$$\mathrm{r(t)} = \mathrm{\alpha(t)} + x(t) + y(t)$$

$$d\alpha(t) = [\theta(t)-\kappa_1\alpha(t)-\kappa_2\alpha(t)]dt$$

$$dx(t) = -\kappa_1x(t)dt+\sigma_1dW_1$$

$$dy(t) = -\kappa_2y(t)dt+\sigma_2dW_2$$

$$\begin{aligned}\mathrm{P}_{\mathrm{HW2F}}^{x_iy_i}(\mathrm{t,T}) = \frac{P^{\mathrm{M}}(0,T)}{P^{M}(0,t)}\exp\Big(&-x_iB_x(t,T)+QVT_x(t,T)-y_iB_y(t,T)\\ &+QVT_y(t,T)+\mathrm{CrossTerm_{2F}(t,T)}\Big)\end{aligned}$$

$$\begin{aligned}&\mathbf{CrossTerm_{2F}(t,T)}\\ &\qquad= \mathbf{2\rho\frac{1}{2}}\big(\mathbf{VT_{2F}(t,T,\kappa_1,\kappa_2,\sigma_1,\boldsymbol{\sigma}_2)}-\mathbf{VT_{2F}(0,T,\kappa_1,\kappa_2,\sigma_1,\boldsymbol{\sigma}_2)}\\ &\qquad+\mathbf{VT_{2F}(0,t,\kappa_1,\kappa_2,\sigma_1,\boldsymbol{\sigma}_2)}\big)\end{aligned}$$

$$
\mathbf{VT_{2F}}(\mathbf{T_1},\mathbf{T_2},\boldsymbol{\kappa_1},\boldsymbol{\kappa_2},\boldsymbol{\sigma_1},\boldsymbol{\sigma_2}) \approx \frac{\boldsymbol{\sigma_1\sigma_2}}{\kappa_1\kappa_2}\left(\mathrm{T}-\mathrm{t}+\frac{\mathrm{e}^{-\kappa_1(\mathrm{T}-\mathrm{t})}-1}{\kappa_1}+\frac{\mathrm{e}^{-\kappa_2(\mathrm{T}-\mathrm{t})}-1}{\kappa_2}-\frac{\mathrm{e}^{-(\kappa_1+\kappa_2)(\mathrm{T}-\mathrm{t})}-1}{(\kappa_1+\kappa_2)}\right)
$$

· $B(t,T,\kappa)$와 $VT(t,T,\kappa)$은 다음과 같이 구현한다.

```
double B_s_to_t(
    double kappa,
    double s,
    double t
)
{
    return (1.0 - exp(-kappa * (t - s))) / kappa;
}
```

```
double V_t_T(
    double kappa,
    double kappa2,
    double t,
    double T,
    double vol,
    double vol2
)
{
    return vol * vol2 / (kappa * kappa2) * (T - t + (exp(-kappa * (T - t)) - 1.0) / kappa + (exp(-kappa2 * (T - t)) - 1.0) / kappa2 - (exp(-(kappa + kappa2) * (T - t)) - 1.0) / (kappa + kappa2));
}
```

ShortRate별 커브를 구현하여 기초금리를 추정하고 Greed별 Payoff를 계산한다.

6.