옵션 평가로직 설명서

1. 옵션타입 [0], 바닐라 옵션 Plain Vanilla Option

① 콜옵션

$$C = Se^{-q \cdot t}N(d_1) - Xe^{-rt}N(d_2)$$

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r - q + \frac{\sigma^2}{2}\right) \cdot t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

② 풋옵션

$$P = Xe^{-rt}N(-d_2) - Se^{-q \cdot t}N(-d_1)$$

2. 옵션타입 [1], 디지털 옵션 Digital Option

① 콜옵션

$$C = e^{-rt}N(d_2)$$

$$d_2 = \frac{\ln\left(\frac{S}{X}\right) + \left(r - q - \frac{\sigma^2}{2}\right) \cdot t}{\sigma\sqrt{t}}$$

② 풋옵션

$$P = e^{-rt}N(-d_2)$$

3. 옵션타입 [2], 아시안 옵션(시뮬레이션) Asian Option(Simulation)

① Path 생성

$$dS_n = \left(R_n - d_n - \rho_{(S_n, FX_n)}\sigma_{S(n)}\sigma_{FX(n)}\right) \cdot S\Delta t + \sigma_{(S_n)}\sqrt{\Delta t} \times \epsilon$$

② Average 주가 계산

③ Payoff 생성 및 할인

$$C = E\left\{\max\left(\hat{S} - K, 0\right) e^{-rt}\right\}$$

4. 옵션타입 [3], 아메리칸 옵션 American Option(FDM)

① **Black Scholes PDE**

$$F_{\mathrm{t}} + (r-d)SF_S + \frac{1}{2}(S\sigma)^2 F_{SS} = rF$$

② **Difference Format**

$$\frac{F(t+\Delta t)-F(t)}{\Delta t} + (r-d)\frac{F(S+\Delta S)-F(S-\Delta S)}{2\Delta S} + \frac{1}{2}(S\sigma)^2\frac{F(S+\Delta S)+F(S-\Delta S)-2F(S)}{(\Delta S)^2} = rF(t,S)$$

③ $S = n \times \Delta S$라고 가정하고 방정식을 풀면 다음과 같다

$$\frac{F_n^{t+\Delta t}-F_n^t}{\Delta t} + (r-d)n\frac{F_{n+1}^t - F_{n-1}^t}{2} + \frac{(n\sigma)^2}{2}\left(F_{n+1}^t - 2F_n^t + F_{n-1}^t\right) = rF_n^t$$

④ **방정식을 정리하면,**

$$F_n^{t+\Delta t} = A_n F_n^t + B_n F_n^t + C_n F_n^t$$

$$A_n = 0.5\Delta t \times \left((r-d)\cdot n - (\sigma\cdot n)^2\right)$$

$$B_n = 1 + \Delta t(r + (\sigma\cdot n)^2)$$

$$C_n = 0.5\Delta t \times (-(r-d)\cdot n - (\sigma\cdot n)^2)$$

$$\begin{bmatrix} 2A_0+B & C_0-A_0 & 0 & 0 & \cdots & \cdots & 0 & 0 & 0 \\ A_1 & B & C_1 & 0 & \cdots & \cdots & 0 & 0 & 0 \\ 0 & A_2 & B & C_2 & \cdots & \cdots & 0 & 0 & 0 \\ 0 & 0 & A_3 & B & \cdots & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & 0 & 0 & A_{N-2} & B & C_{N-2} & 0 \\ \cdots & \cdots & \cdots & 0 & 0 & 0 & A_{N-1} & B & C_{N-1} \\ \cdots & \cdots & \cdots & 0 & 0 & 0 & 0 & A_N - C_N & B+2C_N \end{bmatrix} \begin{bmatrix} F_0^k \\ F_1^k \\ F_2^k \\ F_3^k \\ \cdots \\ \cdots \\ F_{N-2}^k \\ F_{N-1}^k \\ F_N^k \end{bmatrix} = \begin{bmatrix} F_0^{k+1} \\ F_1^{k+1} \\ F_2^{k+1} \\ F_3^{k+1} \\ \cdots \\ \cdots \\ F_{N-2}^{k+1} \\ F_{N-1}^{k+1} \\ F_N^{k+1} \end{bmatrix}$$

⑤ $F = F^{Maturity} = Payoff^{Maturity}$부터 시작하여 현재시점까지 가치를 계산한다.

5. 옵션타입 [4], 아메리칸 디지털 옵션 American Digital Option

(4와 동일한 평가모델)

6. 옵션타입 [5~8], 배리어 옵션 Barrier Option

$$S : Spot, \quad K : Stike, \quad H : Barrier$$

$$T : Expiry, \quad \sigma : Volatility, \quad r : discount\ rate, \quad q : dividend\ rate$$

$$b = r - q$$

$$\eta = Down\ Up\ Flag(1 = Down, -1 = Up)$$

$$\phi = Call\ Put\ Flag(1 = Call, -1 = Put)$$

- Down And Out Call

$$C_{do}(X > H) = A - C + F$$

$$C_{do}(X < H) = B - D + F$$

- Up And Out Call

$$C_{uo}(X > H) = F$$

$$C_{uo}(X < H) = A - B + C - D + F$$

- Down And Out Put

$$P_{do}(X > H) = A - B + C - D + F$$

$$P_{do}(X < H) = F$$

- Up And Out Put

$$P_{uo}(X > H) = B - D + F$$

$$P_{uo}(X < H) = A - C + F$$

$$A = \phi S e^{(b-r)T} N(\phi x_1) - \phi X e^{-rT} N\left(\phi x_1 - \phi\sigma\sqrt{T}\right)$$

$$B = \phi S e^{(b-r)T} N(\phi x_2) - \phi X e^{-rT} N\left(\phi x_2 - \phi\sigma\sqrt{T}\right)$$

$$C = \phi S e^{(b-r)T} \left(\frac{H}{S}\right)^{2(\mu+1)} N(\eta y_1) - \phi X e^{-rT} \left(\frac{H}{S}\right)^{2\mu} N\left(\eta y_1 - \eta\sigma\sqrt{T}\right)$$

$$D = \phi S e^{(b-r)T} \left(\frac{H}{S}\right)^{2(\mu+1)} N(\eta y_2) - \phi X e^{-rT} \left(\frac{H}{S}\right)^{2\mu} N\left(\eta y_2 - \eta\sigma\sqrt{T}\right)$$

$$E = K e^{-rT} \left[N\left(\eta y_2 - \eta\sigma\sqrt{T}\right) - \left(\frac{H}{S}\right)^{2\mu} N\left(\eta y_2 - \eta\sigma\sqrt{T}\right) \right]$$

$$F = K \left[\left(\frac{H}{S}\right)^{\mu+\lambda} N(\eta z) + \left(\frac{H}{S}\right)^{\mu-\lambda} N\left(\eta z - 2\eta\lambda\sigma\sqrt{T}\right) \right]$$

$$x_1 = \frac{ln\left(\frac{S}{X}\right)}{\sigma\sqrt{T}} + (1+\mu)\sigma\sqrt{T}$$

$$x_2 = \frac{ln\left(\frac{S}{H}\right)}{\sigma\sqrt{T}} + (1+\mu)\sigma\sqrt{T}$$

$$y_1 = \frac{ln\left(\frac{H^2}{SX}\right)}{\sigma\sqrt{T}} + (1+\mu)\sigma\sqrt{T}$$

$$y_2 = \frac{ln\left(\frac{H}{S}\right)}{\sigma\sqrt{T}} + (1+\mu)\sigma\sqrt{T}$$

$$z = \frac{ln\left(\frac{H}{S}\right)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}$$

$$\mu = \frac{b - \frac{\sigma^2}{2}}{\sigma^2}$$

$$\lambda = \sqrt{\mu^2 + \frac{2r}{\sigma^2}}$$

- Knock In Put/Call Option

$$KI + KO = Vanilla$$

$$KI = Vanilla - KO$$

7. 옵션타입 [9], 아시안 옵션 Asian Option

$$m_1 = \sum_{i=0}^{n-1} \beta_i$$

$$m_2 = 2\sum_{i=0}^{n-1} \beta_i \epsilon_i \sum_{j=i}^{n-1} \beta_j - \sum_{i=0}^{n-1} \beta_i^2 \epsilon_i$$

$$m_3 = 2\sum_{i=0}^{n-1} \beta_i \epsilon_i^2 \left(\beta_i^2 \epsilon_i - 3\beta_i \epsilon_i \sum_{j=i}^{n-1} \beta_j - 3\sum_{j=i}^{n-1} \beta_j^2 \epsilon_j + 6\sum_{j=i}^{n-1} \beta_j \epsilon_j \sum_{j=k}^{n-1} \beta_k\right)$$

$$\beta_i = w_i F_i, \quad e_i = e^{\sigma_i^2 t_i}$$

$$w_i : Weight\ of\ Total\ Average\ Days$$

$$\mu_1 = m_1$$

$$\mu_2 = m_2 - \mu_1^2$$

$$\mu_3 = m_3 - 3\mu_1\mu_2 - \mu_1^3$$

$$y_1 = \mu_1 \left(if\ shift\ normal : y_1 = \frac{\mu_2}{z - \frac{\mu_2}{z}} \right)$$

$$y_{11} = \mu_2 + y_1^2$$

$$\epsilon = \mu_1 - y_1$$

$$z = \left(\frac{\mu_3 + \sqrt{(\mu_3^2 + 4\mu_2^3)}}{2} \right)^{\frac{1}{3}}$$

$$C = e^{-rT}\big(y_1 N(d_1) - (K - \phi A - \epsilon) N(d_2)\big)$$

$$P = e^{-rT}\big(-y_1 N(-d_1) + (K - \phi A - \epsilon) N(-d_2)\big)$$

$$d_1 = \ln \frac{\left(\frac{\sqrt{y_{11}}}{K - \phi A - \epsilon} \right)}{\sqrt{ln \frac{y_{11}}{y_1^2}}}, d_2 = d_1 - \sqrt{ln \frac{y_{11}}{y_1^2}}$$

$$\phi = PrevCummulativeWeight, \qquad A = PrevAverage$$

만약 $(K - \phi A - \epsilon) \leq 0$ 이면

$$C = e^{-rT}\big(y_1 - (K - \phi A - \epsilon)\big), \qquad P = 0$$

8.