TwoCurveCallableSwap 평가 설명서

1. 모듈설명

해당 모듈은 두 개의 커브를 사용하는 Callable 스왑의 가격을 평가하는 모듈이다. 평가로직으로 HW 1F 2D OS FDM를 사용한다.

2. 평가가능상품

1.1 Quanto Callable Swap

1.2 Domestic Two Curve Swap

1.3 Callable CRS

3. 평가방법1 – HW 1Factor 2D Operator Split IFDM

$$\mathrm{U_t^x} = \kappa_1 x \mathrm{U}_x - \frac{1}{2}\sigma_1^2 \mathrm{U}_{xx} + \frac{1}{2}F(t, t+dt)\mathrm{U} - \frac{1}{2}\rho_{12}\sigma_1\sigma_2 U_{xy}$$

$$\mathrm{U_t^y} = \left(\rho_{2,fx}\sigma_2\sigma_{fx} + \kappa_2 y\right)\mathrm{U}_y - \frac{1}{2}\sigma_2^2 \mathrm{U}_{yy} + \frac{1}{2}F(t, t+dt)\mathrm{U} - \frac{1}{2}\rho_{12}\sigma_1\sigma_2 U_{xy}$$

Implicit Finite Difference Method to Xt(Domestic Short Rate)

$$\mathrm{u_{i,j}^{k+\frac{1}{2}}} + \left(u^{k+\frac{1}{2}}{}_{i+1,j+1} + u^{k+\frac{1}{2}}{}_{i-1,j-1} - u^{k+\frac{1}{2}}{}_{i+1,j-1} - u^{k+\frac{1}{2}}{}_{i-1,j+1}\right)\frac{\sigma_1\sigma_2\rho_{12}h_t}{2}\frac{1}{4h_x h_y}$$
$$= \mathrm{A_i} u_{i-1}^k + \mathrm{B_i} u_i^k + \Gamma_\mathrm{i} u_{i+1}^k$$

$$A_i = \left[-\frac{\mathrm{h_t}\kappa x_i}{2h_x} - \frac{h_t\sigma_1^2}{2h_x^2}\right]$$

$$B = \left[1 + \frac{\mathrm{h_t}\sigma_1^2}{\mathrm{h_x^2}}\right] + \frac{1}{2}F\left(t_k, t_{k+\frac{1}{2}}\right)dt$$

$$C_i = \left[\frac{\mathrm{h_t}\kappa x_i}{2h_x} - \frac{h_t\sigma_1^2}{2h_x^2}\right]$$

Boundary Condition

$u_{-1} = 2u_0 - u_1$

$u_{N+1} = 2u_N - u_{N-1}$

$\mathrm{u_0^{k+1}}W_0 = A_0 u_{-1}^k + Bu_0^k + C_i u_1^k = A_0(2u_0^k - u_1^k) + Bu_0^k + C_0 u_1^k$

$\mathrm{u_0^{k+1}}W_0 = u_0^k(2A_0 + B) + u_1^k(C_0 - A_0)$

$\mathrm{u_N^{k+1}}W_N = A_N u_{N-1}^k + Bu_N^k + C_i u_{N+1}^k = A_N u_{N-1}^k + Bu_N^k + C_N\left(2u_N^k - u_{N-1}^k\right)$

$\mathrm{u_N^{k+1}}W_N = \mathrm{u_{N-1}^k}(A_N - C_N) + u_N^k(B + 2C_N)$

➔ Solve Tridiagonal by Xt(Domestic Leg)

Implicit Finite Difference Method to Yt(Foreign Short Rate)

$$\mathrm{u_{i,j}^{k+1}} + \left({u^{k+1}}_{i+1,j+1} + {u^{k+1}}_{i-1,j-1} - {u^{k+1}}_{i+1,j-1} - {u^{k+1}}_{i-1,j+1}\right)\frac{\sigma_1\sigma_2\rho_{12}h_t}{2}\frac{1}{4h_x h_y}$$

$$= \mathrm{A_j} u_{j-1}^{k+\frac{1}{2}} + \mathrm{B_j} u_j^{k+\frac{1}{2}} + \Gamma_\mathrm{j} u_{j+1}^{k+\frac{1}{2}}$$

$$A_i = \left[-\frac{\mathrm{h_t}\left(\rho_{2,fx}\sigma_2\sigma_{fx} + \kappa_2 y_j\right)}{2h_y} - \frac{h_t\sigma_2^2}{2h_y^2}\right]$$

$$B = \left[1 + \frac{\mathrm{h_t\sigma_2^2}}{\mathrm{h_y^2}}\right] + \frac{1}{2}\hat{F}dt$$

$$C_i = \left[\frac{\mathrm{h_t}\left(\rho_{2,fx}\sigma_2\sigma_{fx} + \kappa_2 y_j\right)}{2h_y} - \frac{h_t\sigma_2^2}{2h_y^2}\right]$$

➔ Solve Tridiagonal by Yt(Foreign Leg)

4. 미래 ShortRate Greed에 따른 2F 커브 및 페이오프 산출

$\boldsymbol{P_{HW}(t,T) = E\left(\mathrm{e}^{-\int_t^T r_u du}\middle|F_t\right) = \mathrm{E}\left(\mathrm{e}^{-\int_t^T (x_u+\phi_u)du}|F_t\right)}$**이다.**

$\mathrm{x_t}$는 정규분포를 따르고, 위 수식은 정규분포 적률생성함수(MGF) 형태이다.

$$\mathbf{E(e^{tx}) = e^{\mu t + \frac{1}{2}(\sigma t)^2}}$$

여기서 $\int_{\mathrm{t}}^{\mathrm{T}} x(u)du$는 평균 x(t)B(t,T), 분산 V(t,T)인 정규분포를 따른다.

$$\mathrm{B(t,T)} = \int_t^T e^{-\kappa(\bar{T}-u)}du = \frac{1-e^{-\kappa(T-t)}}{\kappa}$$

$$\mathrm{V(t,T)} = \int_{\mathrm{t}}^{\mathrm{T}} \sigma^2(u)B^2(u,T)du \approx \int_{\mathrm{t}}^{\mathrm{T}} \frac{\sigma^2\left[1-2e^{-\kappa(T-u)}+e^{-2\kappa(T-u)}\right]}{\kappa^2}du$$

$$= \frac{\sigma^2}{\kappa^2}\left(\mathrm{T}-\mathrm{t}+2\frac{\mathrm{e}^{-\kappa(\mathrm{T-t})}-1}{\kappa} - \frac{\mathrm{e}^{-2\kappa(\mathrm{T-t})}-1}{2\kappa}\right)$$

따라서 위험 중립 측도에서 $\mathrm{P(t,T)}$ 는 정규분포의 적률생성함수(Moment Generate Function)에 따라 다음과 같다.

※ $\mathrm{E(e^{tx})} = e^{\mu t+\frac{1}{2}(\sigma t)^2}$ MGF of Normal Distribution 따라서,

$$\mathrm{P(t,T)} = \hat{E}\left(\mathrm{e}^{-\int_{\mathrm{t}}^{\mathrm{T}}\phi(u)du - x(t)B(t,T)+\frac{1}{2}V(t,T)}\right)$$

0시점 시장에서 관측된 만기 T인 Zero Bond $P^{\mathrm{M}}(0,T)$가 다음을 만족한다.

$$\mathrm{P^M}(0,T) = \mathrm{e}^{-\int_0^{\mathrm{T}}\alpha(u)du+\frac{1}{2}V(0,T)}$$

$$\mathrm{e}^{-\int_0^{\mathrm{T}}\alpha(u)du} = \mathrm{P^M}(0,T)\mathrm{e}^{-\frac{1}{2}V(0,T)}$$

따라서

$$\mathrm{e}^{-\int_{\mathrm{t}}^{\mathrm{T}}\alpha(u)du} = \frac{P^{\mathrm{M}}(0,T)}{P^M(0,t)}e^{-\frac{1}{2}(V(0,T)-V(0,t))}$$

$$\mathrm{P}_{\mathrm{HW}}^{x_i}(\mathrm{t,T}) = \frac{P^{\mathrm{M}}(0,T)}{P^M(0,t)}\exp\left(-x_iB(t,T)+\frac{1}{2}\big(V(t,T)-V(0,T)+V(0,t)\big)\right)$$

$$\mathrm{P}_{\mathrm{HW}}^{x_i}(\mathrm{t,T}) = \frac{P^{\mathrm{M}}(0,T)}{P^M(0,t)}\exp\big(-x_i\times B(t,T)+QVT(t,T)\big)$$

HW 2F Dynamics of Short Rate

$$\mathrm{r(t)} = \alpha(\mathrm{t}) + x(t) + y(t)$$

$$d\alpha(t) = [\theta(t)-\kappa_1\alpha(t)-\kappa_2\alpha(t)]dt$$

$$dx(t) = -\kappa_1 x(t)dt + \sigma_1 dW_1$$

$$dy(t) = -\kappa_2 y(t)dt + \sigma_2 dW_2$$

$$\mathrm{P}_{\mathrm{HW2F}}^{x_i y_i}(\mathrm{t},\mathrm{T}) = \frac{P^{\mathrm{M}}(0,T)}{P^{M}(0,t)} \exp\Big(-x_i B_x(t,T) + QVT_x(t,T) - y_i B_y(t,T) + QVT_y(t,T) + \mathrm{CrossTerm}_{2\mathrm{F}}(\mathrm{t},\mathrm{T})\Big)$$

$$\mathbf{CrossTerm_{2F}(t,T)} = \mathbf{2\rho\frac{1}{2}}\Big(\mathbf{VT_{2F}(t,T,\kappa_1,\kappa_2,\sigma_1,\sigma_2) - VT_{2F}(0,T,\kappa_1,\kappa_2,\sigma_1,\sigma_2) + VT_{2F}(0,t,\kappa_1,\kappa_2,\sigma_1,\sigma_2)}\Big)$$

$$\mathbf{VT_{2F}(T_1,T_2,\kappa_1,\kappa_2,\sigma_1,\sigma_2)} \approx \frac{\boldsymbol{\sigma_1\sigma_2}}{\kappa_1\kappa_2}\left(\mathrm{T}-\mathrm{t}+\frac{\mathrm{e}^{-\kappa_1(\mathrm{T}-\mathrm{t})}-1}{\kappa_1}+\frac{\mathrm{e}^{-\kappa_2(\mathrm{T}-\mathrm{t})}-1}{\kappa_2}-\frac{\mathrm{e}^{-(\kappa_1+\kappa_2)(\mathrm{T}-\mathrm{t})}-1}{(\kappa_1+\kappa_2)}\right)$$

· $B(t,T,\kappa)$와 $VT(t,T,\kappa)$은 다음과 같이 구현한다.

```
double B_s_to_t(
    double kappa,
    double s,
    double t
)
{
    return (1.0 - exp(-kappa * (t - s))) / kappa;
}
```

```
double V_t_T(
    double kappa,
    double kappa2,
    double t,
    double T,
    double vol,
    double vol2
)
{
    return vol * vol2 / (kappa * kappa2) * (T - t + (exp(-kappa * (T - t)) - 1.0) / kappa + (exp(-kappa2 * (T - t)) - 1.0) / kappa2 - (exp(-(kappa + kappa2) * (T - t)) - 1.0) / (kappa + kappa2));
}
```

ShortRate별 커브를 구현하여 기초금리를 추정하고 Greed별 Payoff를 계산한다.

5.